REINFORCED CONCRETE

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INTRODUCTION

Reinforced concrete is a rational union of concrete and steel (reinforcing bars) combined to act jointly. Plain concrete (without steel bars) possesses great resistance to compression but is much weaker in tension, and when tested under a load that produces tensile stresses, will have very little carrying capacity. But reinforced concrete, with steel bars in the tension zone to take advantage of their great resistance to tensile stresses, will sustain enormous loads.

For example, in a beam freely supported at its ends (Fig. 1, *a*) the tension created by the load *P* in the lower half is borne by the steel bars, while the corresponding compression in the upper part is sustained by the concrete. In a cantilever beam (Fig. 1, *b*) the tension is in the upper part where the reinforcement must therefore be placed.

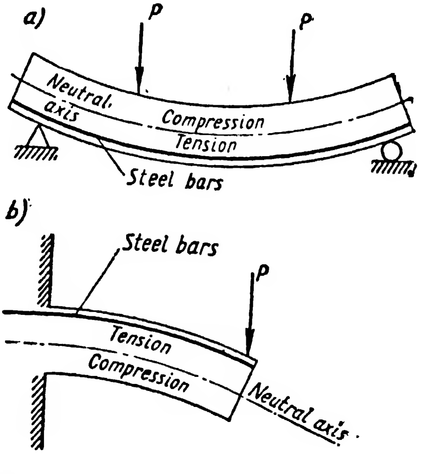

Fig. 1. Sketch showing position of bars in reinforced concrete beams

Steel resists compression as well as tension, for which reason it is rationally employed to increase the carrying capacity of reinforced concrete units undergoing compression, such as columns.

Joint action of the steel and concrete in reinforced concrete is made possible because of the following factors:

1) *Bond between the concrete and bars* after hardening of the concrete. This assures mutual deformation of both components under load-carrying conditions.

2) *Absence of corrosion of embedded bars* if the mix contains a correct amount of cement and is of adequate density.

3) *Similar rates of thermal expansion for both components* (0.00001 to 0.000015 for concrete and 0.000012 for steel), resulting in negligible incipient stresses from temperature changes up to 100° C and therefore excluding the possibility of slipping between the steel and concrete.

A feature of loaded reinforced concrete is the creation of fissures in the tension zone. Nevertheless, under normal load conditions such cracks will progress very little and not interfere with the work of the given member.

When high-strength steel is used for reinforcing bars (which are thus subject to considerable tensile deformation), or when the concrete must be watertight (in reservoirs, piping, etc.) crack formation may be either prevented or limited by the process of pretensioning the bars and consequently compressing the concrete before the designated load is applied. This method produces what is known as prestressed reinforced concrete.

Reinforced concrete is popular because of its durability, fire and weather resistance, great strength under static and dynamic loads, ability to arrest radioactive radiation, low upkeep, and also because its component aggregate is readily available everywhere.

However, a number of objectionable features are involved in its use: it is very heavy, has low sound and heat insulating value, requires forms within which it must be retained until it hardens, and must be heat cured while hardening in winter to avert its freezing.

But thanks to modern prefabricated reinforced concrete elements and prestressing methods, the above objections are either being gradually eliminated or their negative qualities neutralised, resulting in an ever-growing demand for reinforced concrete.

Many types of structures include reinforced concrete as their principal material: factories (Fig. 2), trestlework, warehouses (Fig. 3), garages, silos, hoppers, reservoirs, and smokestacks. It is also widely used in hydraulic works, powerhouses, underground urban railways, and bridges. In recent years it has become the mainstay of housing development (Fig. 4), civic buildings, and a number of types of farm structures. In the Soviet Union, electric plants powered by nuclear energy use reinforced concrete shields to guard their personnel against harmful radiation.

The adoption of reinforced concrete in place of structural steel makes a two-fold cut in steel expenditure, which is very important for the national economy.

Reinforced concrete is classified according to methods of fabrication: it is *in-situ* when cast in place and *precast* if the units are produced in a plant or in a casting yard and subsequently delivered to the site for assembly.

Precast reinforced concrete is well suited to industrialised means of construction, which happily combines this factory-made product with mechanised handling methods.

In comparison to in-situ construction, precast reinforced concrete reduces manual labour three- and four-fold on the site, cuts erection time and saves material that would otherwise go into formwork and scaffolding. All this has placed precast reinforced concrete in the fore and elevated the building trades to a new technical level. Its efficient use requires typified plans and standardised reinforced concrete structural details for all types of buildings.

The adoption of reinforced concrete began in the 2nd half of the 19th century, when industry and transport first began their expansion. Certain kinds of construction combining concrete and steel were already known in the 1850's. The first patent for simple articles made of this material was issued in 1867 to a Frenchman, Monier.

Fig. 2. Reinforced concrete factory building
a—one-storey; *b*—multi-storey

The year 1886 marks the initial appearance of reinforced concrete in Russia, and in 1891 professor N. A. Belelyubsky conducted some public tests in St. Petersburg on a number of types of reinforced concrete construction which helped to encourage its use. The first theoretical treatises on the design and practical application of this material were the work of the French scientists Considere and Hennebique, the German engineer Mörsch, and others.

At the end of the 19th century the main principles of safe stress computation had already been formulated, based on Strength of Materials and Structural Mechanics. The first codes of practice on reinforced concrete were compiled in 1904-1908 in Germany, France, and Russia, and during the ensuing period the material became widespread in the erection of industrial and civic structures.

Considerable impetus was lent to the development of reinforced concrete by the Russian scientists I. S. Podolsky, A. F. Loleit, G. P. Peredery, and others.

The Volkhov hydropower plant, built in 1921-1926, was the first large project in the U.S.S.R. to adopt reinforced concrete. Both the plain and reinforced kinds were employed in later construction of hydropower stations on the rivers Dnieper and Svir.

The colossal scale of construction in the U.S.S.R. and the need to save steel gave reinforced concrete a dominant position in indus-

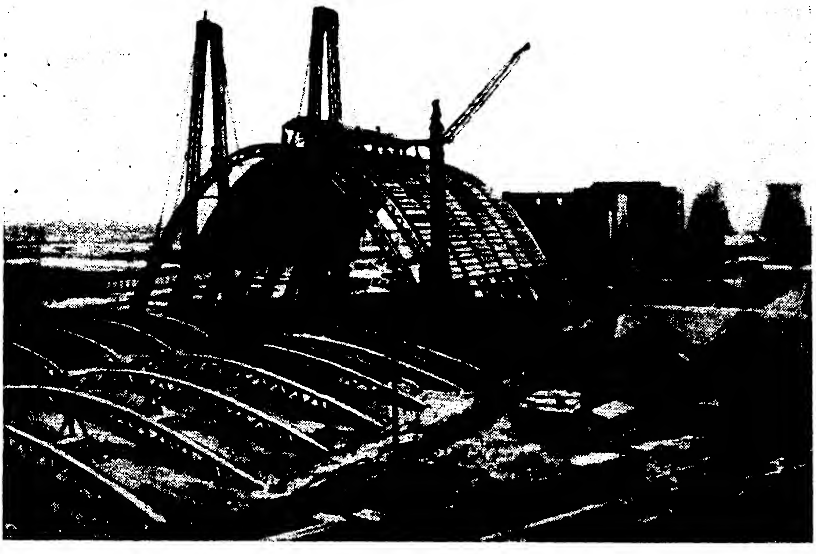

Fig. 3. A reinforced concrete warehouse in Hungary

trial structures, where it gradually began taking the lead in the 1930's. Although in-situ reinforced concrete was used in the main, some sites pioneered with precast methods. Research was also begun by a number of laboratories and other scientific institutions.

In the 1920's the idea of prestressed reinforced concrete, first suggested at the end of the last century, became a reality due to the work of E. Freyssinet, a noted French engineer. In the U.S.S.R., V. V. Mikhailov was the first to contribute to its study.

In 1931-1933 the Russian scientist, Professor Loleit, developed the basic principles of a new method, according to which reinforced concrete is computed on the basis of failure stresses. This method, which approaches closer than before to the material's actual elasto-plastic nature, was thoroughly tested and received its finishing

touches under the direction of A. A. Gvozdyov in the Reinforced Concrete Laboratories of the Central Institute of Industrial Structures, which issued new designing codes and specifications in 1938.

Important contributions to the new theory of reinforced concrete were also made by the Soviet scientists P. L. Pasternak, V. I. Murashov, Y. V. Stolyarov, and others.

The last decade has been especially fruitful of rapid progress in this field. Radical alterations have been made in the design of reinforcement, prefabricated into spot-welded blocks and mats for precast members. In the realm of theory, calculating methods have been evolved for rigidity and fissure resistance in construction and for determining stresses along diagonal planes. Advances have also been made in the design of prestressed units.

Building construction is going over more decidedly to prefabricated precast reinforced concrete. Further developments must aim at structural types that will promote mass mechanised output, decrease weights of elements, and reduce costs by introducing thin-walled prestressed members made of high-density concretes and high-strength bars.

CHAPTER I

PHYSICO-MECHANICAL PROPERTIES OF CONCRETE, STEEL BARS, AND REINFORCED CONCRETE

Sec. 1. CONCRETE

1. Role of Concrete in Reinforced Concrete

The concrete in reinforced concrete must be durable and capable of bonding well with the steel bars. It must be sufficiently dense (impermeable) to protect the bars against corrosion and must contain at least 225-275 kilograms of cement (depending upon the intended use of the structure) to each cubic metre of concrete. The cement, as the binding agent, can be either ordinary portland, trass, slag-portland, or alumina cement, not less than 200-grade, the choice being determined by the character of the given construction.

The stiffness of the concrete batch is determined by the water-cement ratio (w-c), measured by weight of the water in relation to the cement, and is selected to secure workability of the mixture. The w-c of a plastic batch ranges from 0.7 to 0.5, while that of a stiff one is from 0.4 to 0.3.

Stiff mixes produce concrete of great density and strength, require less cement, and can be stripped in a shorter time. Such mixes are rational for precast units produced in a plant where the use of mechanised vibrating equipment (vibrating tables, vibrating tampers, etc.) results in a highly compact mix. Good density is obtained from correct choice of aggregate, a minimum w-c ratio, and an efficient vibrating process.

2. Structure of Concrete

Chemical interaction of the cement and water in the batch causes a reaction known as hydration, in which the cement minerals become chemically united with the water. The greatest portion of the resulting compound turns into a gel, but a certain part attains a crystalline structure. Warmth and moisture cause the doughlike paste to

harden: the crystals penetrate the gel and join with each other, while the gel itself gradually thickens, hardens, and decreases in volume. Mechanical agitation of the mass directly after placing causes the cement grout to envelope every particle of inert aggregate and bind all into a monolithic whole as the mass hardens. The hardened cement grout is known as neat cement.

As already stated, the amount of water for the mixture is determined by its required workability. But only a minor portion of this water, or about 20% by weight of the cement, enters into the chemical reaction. The remainder is superfluous. It dilutes the gel, and together with the air, partly fills the micropores (capillaries) of the hardening concrete and slowly evaporates.

Thus it may be said that concrete in structure is a coarse non-homogeneous body of unsymmetrically arranged aggregate of varied size and form, held firmly together by the neat cement, the whole containing pores and voids filled with moisture and air. This structure determines the physico-mechanical properties of the concrete, the physico-chemical phenomena that go on within it during the hardening stage (crystallisation, decrease in gel volume, evaporation of surplus water, etc.), and the subsequent alteration of its properties in the course of time.

3. Contraction of Concrete and Initial Stresses

When hardening in the air, concrete lessens in volume, but if the process takes place under water it expands slightly. A decrease in volume is known as shrinkage or contraction, an increase is called swelling. Tests indicate that the amount of contraction is 2 or 3 times greater than a corresponding amount of swelling.

Contraction is caused by a number of circumstances, the main ones being shrinkage of the hardening gel, and the capillary action accompanying the movement and evaporation of water in the micropores. The exact amount of contraction will depend upon the composition of the batch and the environment in which it hardens: an increase of water and cement per unit volume of concrete mixture will make for greater contraction, but shrinkage will be lessened with a greater moisture content of the environment. The most intense contraction takes place during the first hardening stage and in the course of the whole following year. According to experiments, the rate of volumetric deformation resulting from contraction $\varepsilon_{c.c}$ attains a value of 0.0003.

Contraction stems from processes within the neat cement: the latter's free deformation is restrained by the aggregate, and initial stresses are created in the concrete. A number of practical measures have been adopted to counteract this during the hardening process: wetting the surface, increasing the moisture content and temperature of the environment, etc.

4. Strength of Concrete

An external load will create composite stresses within the concrete because of its structure; the existence of pores and voids and dissimilarities of resistance and deformation between the neat cement and individual aggregate particles will cause stresses to concentrate around the pores (Fig. 5, *a*). Perpendicular to the line of pressure, these stresses will be tensile. Stresses in bordering areas will be the sum of adjacent forces.

When the load reaches a given value, the concrete's tensile resistance will be overcome, accompanied by a rupture and the production of microscopic fissures, these latter gradually merging to visibility in the direction parallel with the compressive force of the load (N in Fig. 5, *b*). If the load is further increased, the cracks will open until complete failure is accomplished.

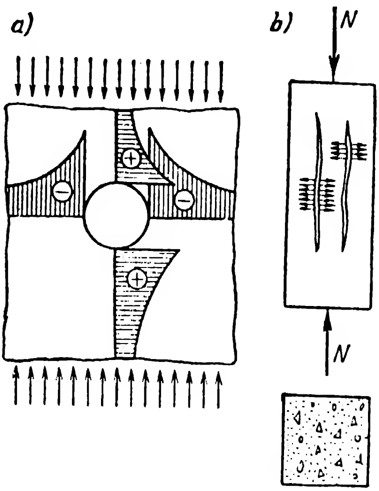

Fig. 5. Sketch showing stresses in a concrete specimen subjected to compression

Hence, the main reason for failure of a concrete specimen subjected to compression is lateral rupture due to inadequate tensile resistance, this being from 10 to 20 times less than that of compression.

The absence of either a definite arrangement of the aggregate, or of order and size of the pores, leads to differences of unit resistance when testing specimens of varying sizes and shapes, even when they are prepared from the same batch. Similar significant fluctuations, known as scattered strength values, are even noted when the specimens are alike in shape and dimensions. Hence, basic resistance values in calculations are taken from mean results procured from a great number of tests.

Concrete betrays great differences in resistance under various forces—compression, tension, and shear. Its strength also depends upon age, hardening conditions, and, as already noted, the shape and size of samples. Its resistance is therefore judged by a standard value or *grade*, which classifies the ultimate compressive resistance in kg per cm² of 28-day-old, 20-cm cubes that have been hardened normally, i. e., at a temperature of $+15°$ and 90% humidity.

State Specifications 123-55 establish the following concrete grades:

a) 50, 75, 100, 150, 200, 300, 400, 500, and 600 for heavy concretes with a bulk weight of $\gamma \geq 1\,800$ kg per cu metre;

b) 35, 50, 75, 100, and 200 for lightweight concretes (of porous aggregate) having a bulk weight of $\gamma < 1{,}800$ kg per cu metre.

Technical and economic prerequisites dictate the grade for any given job. For example, the practical choice for members under compression, such as columns and arches, would be amongst comparatively higher grades (200-300), inasmuch as the carrying capacity of such elements is in proportion to the concrete's strength. For units that are bent by the load (bending members), such as beams and slabs, it is economical to choose the lower grades (150-200).

At present, grades below 100 are used only for structures requiring lightweight concrete members that are safeguarded against moisture and frost. In foundations the 100-grade is employed, and the 150- and 200-grades are for civic and industrial buildings. Grades 200 to 400, also for the latter structures, are for heavy loads, and likewise for thin-walled concrete elements and hydraulic works, while grades 300-600 are mostly for prestressed construction. In ordinary precast members, the 200-grade is the minimum that is usually employed.

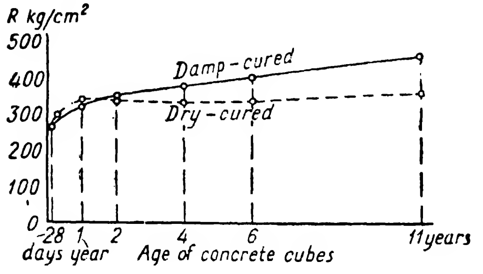

Fig. 6. Diagram showing strength accumulation as concrete ages

Age strengthens concrete; the most intensive increase in strength for portland-cement concrete occurs during the first 28 days; but under favourable conditions (plus-temperatures and sufficient moisture) it will continue to grow stronger for a long time because of further growth of gel compactness and crystalline structure within the neat cement. An increase of moisture and warmth of environment will hasten the hardening process. The diagram in Fig. 6 illustrates this accumulation of concrete resistance.

Resistance of cubes. When a compressive load is applied to an element, it is exposed to longitudinal compressive deformation and lateral tensile strain. When this happens to a concrete cube, failure will occur through lateral rupture because tensile stresses are created in this direction by the lack of homogeneity in the concrete. The character of failure in a concrete cube tested for compression is shown in Fig. 7, *a*. The angle of rupture in relation to the bases of the cube is qualified by the friction between the plates of the press and the bearing surfaces of the cube. This friction interferes with the development of lateral deformation. The countering action of friction is diminished if the distance from the bearing surfaces to

the middle height is augmented. If friction is eliminated by greasing the bearing surfaces, the cracks will be vertical (Fig. 7, *b*) and failure will occur under a lesser load. With a larger cube the influence of friction will be decreased, as also will be the ultimate resistance R.

The resistance index, i.e., the grade of a standard cube specimen, can only be accepted as a comparative figure of unit resistance and cannot be merely incorporated into compression calculations of reinforced concrete members, for these latter differ from cubes both in shape and dimensions. However, cube resistance and resistance values of concrete members have definite empirical relationships.

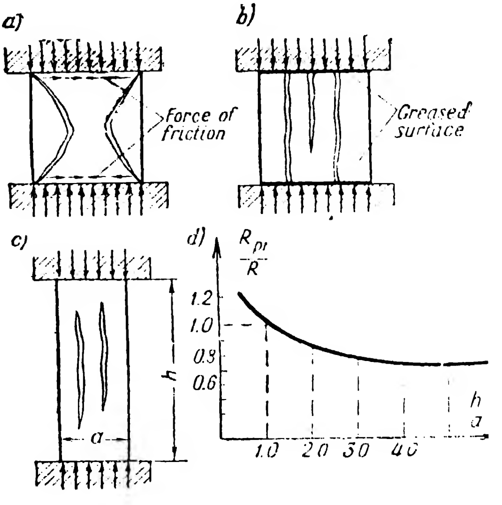

Fig. 7. The effect of friction on resistance in concrete specimens

Resistance of Prisms. Samples in the form of prisms (Fig. 7, *c*) can withstand comparatively less compression than cubes of similar cross-section, because the greater height minimises the effect of friction along bearing surfaces. If the relation of the prism height h to the base a equals $\frac{h}{a} \geq 4$, friction will have no practical effect on ultimate resistance, as can be seen from Fig. 7, *d*. Prism resistance R_{pr} can serve as a strength value for compression members whose lateral dimensions are small in comparison to their height, a typical instance being an axially loaded reinforced concrete column.

Professor Gvozdyev has established by tests the following equation for prism and cube strengths of concrete:

$$R_{pr} = \frac{1,300 + R}{1,450 + 3R} R. \tag{1}$$

Calculations based on this formula coincide closely with prism tests of comparatively low grades of concrete (up to 300). If higher grades are used (from 300 to 600), the following formula of Professor B. G. Scramtayev comes closer to experimental values:

$$R_{pr} = 0.7R. \tag{2}$$

Resistance to compression in bending. In reinforced concrete members that undergo bending, concrete strength in the compression zone (Fig. 8) differs specifically from that in prisms: stresses grow

from the neutral plane towards the extreme fibre of the compressed zone, the lesser stressed parts decreasing the deformation and increasing the resistance of the area subjected to greater stresses. The following tested equation determines R_{bc}, the concrete's ultimate compressive resistance in bending:

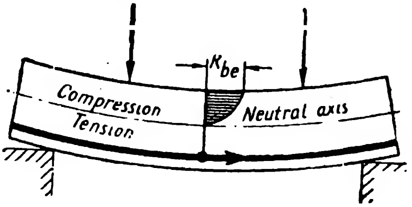

Fig. 8. Compression in a concrete member that is being bent

$$R_{bc}=1.25\ R_{pr}. \tag{3}$$

R_{bc} expresses the resistance of concrete in the compression zone of elements undergoing bending, eccentric compression, and eccentric tension.

Strength in tension. Ultimate tensile resistance R_t of concrete is determined either by pulling apart figure-eight specimens, or by bending tests of beams. R_t can also be computed via cube resistance by means of the formula:

$$R_t=0.5\sqrt[3]{R^2}. \tag{4}$$

R_t will indicate the resistance of reinforced concrete to fissure formation in the tensile zone. In certain cases (reservoirs, piping, etc.) it will determine necessary cross-sections.

Shear resistance. Shear tests of concrete have shown that ultimate shear resistance R_{sh} is expressed by the formula

$$R_{sh}=0.7\sqrt{R_{pr}R_t}. \tag{5}$$

It must be noted that pure shear without a simultaneous bending moment and direct stresses (Fig. 9) are rarely met with in practice.

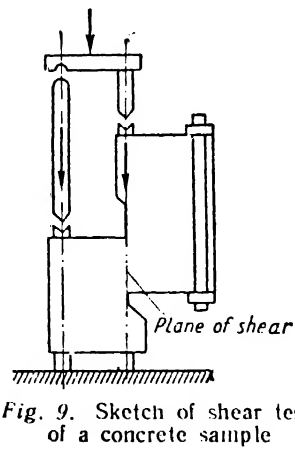

Fig. 9. Sketch of shear test of a concrete sample

Local bearing resistance. If a concrete member is loaded only on part of its area, e.g., the bearing surface of a beam, it will display a heightened resistance due to aid from the unloaded surface, and its ultimate bearing resistance can be expressed as

$$R_{br}=\psi R_{pr},$$
$$\psi=\sqrt[3]{\frac{F}{F_{cr}}} \tag{6}$$

where

F is the cross-section of the member;

F_{cr} is the crushing area.

When the local load is unsymmetrical, the F-area is considered as symmetrical to the centre of gravity of the loaded part. The coefficient ψ must not exceed 1.5 when the local load is concerned, nor be more than 2.0 for combined local and other loads.

Specified resistance of concrete. The above formulae have yielded corresponding resistance limits, known as specified resistance, for each grade of concrete. These values, as given in State Specifications 123-55, are presented in Table 1.*

Table 1

Specified Resistance of Concrete in kg/cm²

Stress	Symbol	Grade of concrete								
		50	75	100	150	200	300	400	500	600
Axial compression (prism resistance)	R^s_{pr}	40	60	80	115	145	210	280	350	420
Compression in bending .	R^s_{be}	50	75	100	140	180	260	350	440	520
Tension	R^s_t	6	8	10	13	16	21	25	28	30

Note. When the concrete is made with alumina cement, a coefficient of 0.7 is attached to the R^s_t value.

5. Deforming Ability of Concrete

The sum of deformation ε_c in concrete as an elasto-plastic material is equal to its elastic deformation ε_{el} (which recovers when the load is removed), plus its plastic, or residual, deformation ε_{pl}:

$$\varepsilon_c = \varepsilon_{pl} + \varepsilon_{el}$$

If a concrete specimen be gradually loaded and its deformation charted twice each time the load is increased (once when the load is added and again after a given interval), the charted line σ-ε in the stress-deformation diagram will be stepped (Fig. 10, *a*). Deformation measured at the moment of load application will be elastic, directly proportional to stresses, and will be charted at a constant angle α_0. Deformation developed from sustained loads will be plastic, inversely proportional to the stresses, and be charted as horizontal segments. The coordinates (σ-ε) from a sufficient number of these segments will form a curve as in Fig. 10,*b*. If stresses increase, plastic deformation will grow. If the σ-stress remains constant but the

* An upper index "s" (specified) has been added to the symbols: R^s_{pr}; R^s_{be}; R^s_t.

loading velocity increases, plastic deformation will diminish; if loading is instantaneous (or as fast as practical), deformation will be elastic. Fig. 10, *c* gives stress-deformation curves σ-ε for various loading velocities $v_1 > v_2 > v_3 > v_4$ accomplished as single momentary loadings.

If the load is applied for a long period, plastic deformation will increase for quite a while and cease when it reaches its ultimate value ε_{pl}^{ult}.

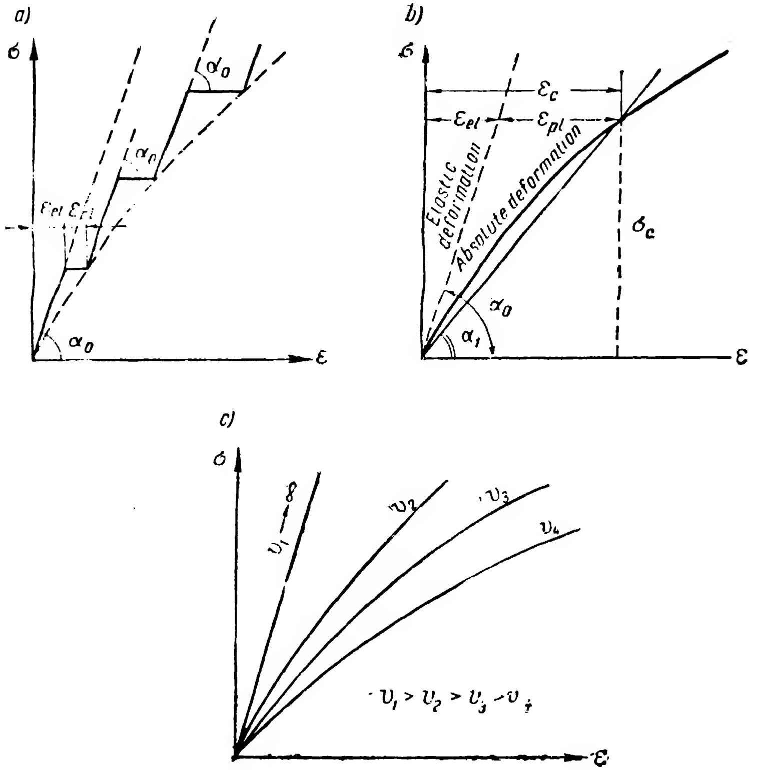

Fig. 10. Stress-deformation diagrams from momentary loadings of concrete
a—single loadings in separate stages; *b*—general σ-ε diagram; *c*—diagram showing influence of speed of deformation on σ-ε

Tests have shown that final deformation of the concrete is not influenced by the velocity of *v*-load application which results in the stress σ (Fig. 11, *a*). The ability of concrete to increase its plastic deformation under a constantly applied load is known as concrete creep. It has been found by experiment that creep deformation advances with a corresponding increase of both stress and duration of the load (Fig. 11,*b*). All other conditions being equal, creep will

be greater with a fresher concrete, a drier environment, and a larger w-c ratio.*

Plastic deformation may exceed elastic deformation two- and three-fold and more. Creep depends upon the structure of concrete and the process of gel hardening. Experiments show that ε_{pl}^{ult} is reached only after the member has been under load for a number of years, the most intensive creep occurring during the first 3-4 months. Loaded reinforced concrete is considerably effected by creep and therefore must be considered in the design.

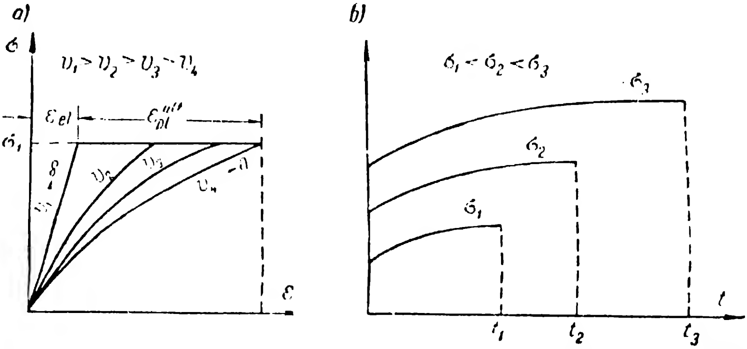

Fig. 11. Stress-deformation diagrams from loads constantly applied to concrete

a—creep deformation as related to velocity of deformation; *b*—influence of duration and value of stress on deformation

Concrete being an elasto-plastic material, the coordinates of its stress and deformation do not form a straight line (Fig. 10, *b*).

If the σ_c stresses were expressed only by elastic deformation ε_{el} and the modulus of elasticity E_c then, according to Hooke's law,

$$\sigma_c = E_c \varepsilon_{el}. \tag{7}$$

But if formulated through the full sum of deformation ε_c, then a variable modulus must be employed, i.e., the modulus of elasto-plasticity of concrete E'_c as follows:

$$\sigma_c = E'_c \varepsilon_c = E'_c (\varepsilon_{el} + \varepsilon_{pl}). \tag{8}$$

The diagram in Fig. 10, *b* indicates that E'_c is a tangent of the angle of the secant running from the initial coordinate to the point corresponding to the given stress:

$$E'_c = \frac{\sigma_c}{\varepsilon_c} = \tan \alpha_1.$$

When changes occur in the σ_c stress and t duration, there will be corresponding alterations in the angle of the secant α_1. Hence the modulus of elasto-plasticity of concrete is a variable quantity.

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* W = c ratio — — water = cement ratio.— — *Tr.*

The modulus of elasticity which establishes a linear dependence between stress and deformation is an arbitrary conception for concrete and possesses a physical value only when instantaneous loading occurs. Therefore, the modulus of elasticity,

$$E_c = \frac{\sigma_c}{\varepsilon_{el}} = \tan \alpha_0 = \text{const.}$$

Values for the modulus of elasticity for concrete under compression are determined by the following empirical formulae:

$$E_c = \frac{1,000,000}{1.7 + \frac{360}{R}} \text{ for heavy concretes;} \tag{9}$$

$$E_c = 11,000\sqrt{R} \text{ for light concretes.} \tag{10}$$

Table 2 presents specified moduli of elasticity for concrete in compression.

Table 2

Specified Moduli of Elasticity E_c^s, in kg/cm² for Concrete in Compression

Grade of concrete	50	75	100	150	200	300	400	500	600
Heavy concretes	110,000	155,000	190,000	240,000	290,000	340,000	380,000	410,000	430,000
Light-weight concretes	70,000	95,000	110,000	130,000	150,000	—	—	—	—

The modulus of elasto-plasticity of concrete E_c' can be formulated by making a separate equation of the right members of equations (7) and (8), since they represent the same stress values of σ_c:

$$E_c \varepsilon_{el} = E_c' (\varepsilon_{el} + \varepsilon_{pl}),$$

from which

$$E_c' = \frac{\varepsilon_{el}}{\varepsilon_{el} + \varepsilon_{pl}} E_c = \frac{\varepsilon_c - \varepsilon_{pl}}{\varepsilon_{el} + \varepsilon_{pl}} E_c = (1 - \lambda) E_c, \tag{11}$$

or

$$E_c' = \nu E_c, \tag{12}$$

where $\lambda = \frac{\varepsilon_{pl}}{\varepsilon_{el} + \varepsilon_{pl}}$ is the coefficient of concrete plasticity,

and

$$\nu = 1 - \lambda = \frac{\varepsilon_{el}}{\varepsilon_{el} + \varepsilon_{pl}}.$$

In elastic deformation ε_{pl} equals zero, hence, $\nu = 1$ and $E_c' = E_c$. In absolute plasticity, when all deformation is beyond recovery, $\varepsilon_{pl} = 0$, hence, $\nu = 0$ and $E_c' = 0$.

Experiments at the Central Scientific Institute of Industrial Structures have shown that for concrete under compression, $\nu=0.2$-0.5, hence, $E'_c=(0.2\text{-}0.5)\,E_c$.

The modulus of elasto-plasticity for concrete in tension $E'_{c.t}$ can also be expressed as a term of the modulus of elasticity:

$$E'_{c.t}=\nu_t E_c. \tag{13}$$

According to experiments, $\nu_t=0.5$ at the ultimate tensile stress of R_t.

Ultimate compression of concrete, i.e., its comparative deformation under compression at the point of failure, is $\varepsilon_c=0.001$-0.003, that is, from 1 to 3 millimetres per linear metre. However, ultimate tension of concrete is from 10 to 20 times less than compression and is equal to $\varepsilon_t=0.0001$-0.00015, i.e., from 0.1 to 0.15 millimetre per linear metre.

Ultimate tension of concrete may also be determined for each grade with a plasticity coefficient for tension $\nu_t=0.5$. Thus,

$$E'_{c.t}=0.5E_c. \tag{14}$$

The dependence of ultimate tension and tensile deformation in concrete then takes the form

$$R_t=E'_{c.t}\varepsilon_t=0.5E_c\varepsilon_t,$$

from which

$$\varepsilon_t=\frac{2R_t}{E_c}. \tag{15}$$

When concrete is subject to repeated loading and unloading, plastic deformation gradually accumulates. When this is repeated several million times, plastic deformation will attain its culmination and the concrete will begin to behave as an elastic material if the stresses do not go beyond the concrete's fatigue-stress limit $R_{f.s}\approx0.5R_{bc}$ (or $R_{f.s}\approx0.5R_{pr}$). After a number of cycles of repeated loadings under stresses of $\sigma_c>R_{f.s}$ plastic deformation will attain the stage of failure.

All the above types of concrete deformation occasioned by external loads are known as load deformation, as differing from volumetric strain which is evoked by such forces as concrete shrinkage and temperature changes.

The rate of thermal expansion of concrete and reinforced concrete α_t due to heating or cooling within the range of 0-100° C, is assumed as 0.00001.

1. General Principles

In reinforced concrete the forces created by the load decide the arrangement of the bars, which are classified either as effective reinforcement or as installation (spacing) bars, depending upon their purpose.

The cross-sectional area of effective reinforcement is computed in accordance with resulting forces.* Installation bars tie the effective reinforcement into *mats* and *blocks* for the prescribed reinforced concrete member, bear such generally uncomputed forces as concrete contraction, temperature changes, etc., and are arranged without theoretical computations to meet requirements of geometric form and the character of the loads.

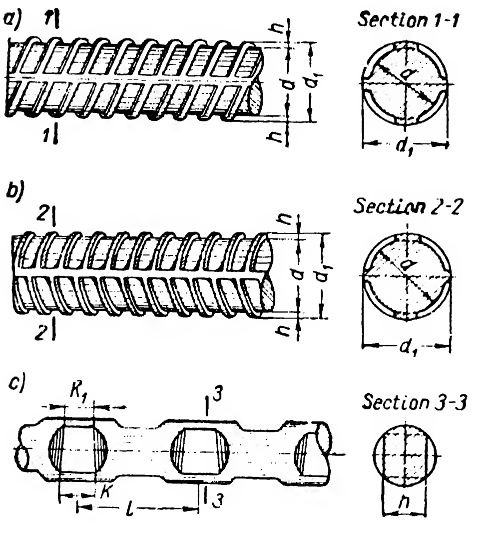

Fig. 12. Intermittently deformed reinforcement
a—hot rolled bars of St-5 steel; *b*—the same of St-25Г2С steel; *c*—cold-notched bars

The common type of reinforcement, known as flexible, consists of 3 to 40 mm steel bars either plain round or intermittently deformed (Fig. 12). In some cases rigid reinforcement is used: I-beams, channels, and angles.

In most cases the reinforcement is assembled into mats and blocks by means of spot-contact welding (Fig. 13, *a*). This is the most industrialised method of handling the work, since fabrication is done by efficient welding machines and subsequent erection requires very little manpower.

Fundamentally, spot welding consists in applying pressure to, and sending an electric current through, the spliced bars (Fig. 13, *b*). The electric current heats the spliced metal to plasticity and the applied pressure makes a reliable joint.

Rigid reinforcement (Fig. 13, *c*) takes much more metal than the flexible type and is used mostly for heavily loaded members in high

* A 5% difference is allowed between theoretically required and actually available reinforcement stock.
A table of bar areas is given in Supplement 1.

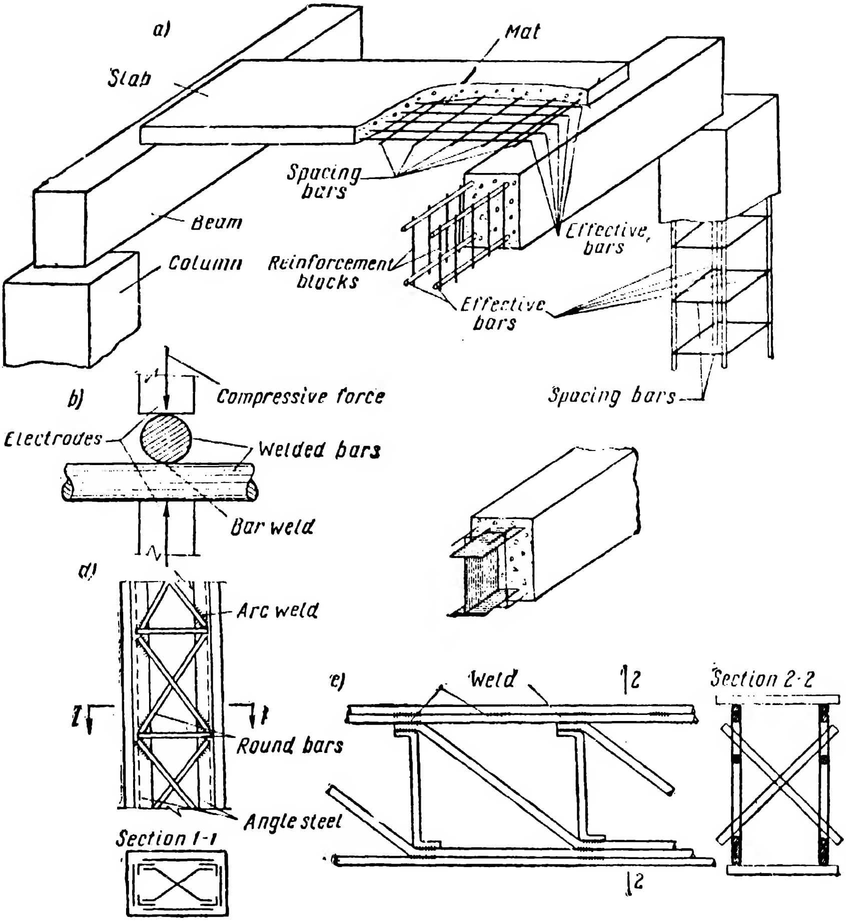

Fig. 13. Methods of reinforcing concrete members

a—flexible reinforcement in the form of blocks and mats; *b*—sketch of spot-contact method of weldings; *c*—rigid reinforcement in a beam; *d*—self-supporting block reinforcement in a column; *e*—welded self-supporting block reinforcement of round bars in a beam

buildings, etc. Such reinforcement carries its own weight and that of the formwork, raw concrete, and miscellaneous erection loads, until the concrete hardens, acting as a self-supporting frame. However, even heavy bearing members are more economical if made of combined angles and round bars, or just round bars, spot- or arc-welded into systems of latticework (Fig. 13, *d* and *e*), known as self-supporting block reinforcement.

2. Bond Between Reinforcement and Concrete

Slipping of the bars within a loaded reinforced concrete member is prevented by the concrete-steel bond created by 1) the gripping properties of the cement grout, 2) friction exerted by the contracted concrete upon the bars, 3) the projections on intermittently deformed bars, and 4) anchoring hooks on the bar ends.

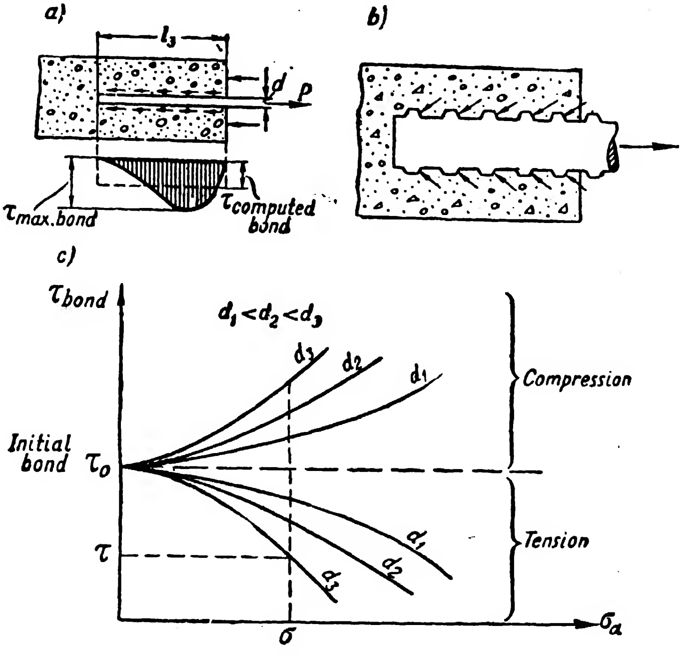

Fig. 14. Bond of bars and concrete

When a bar fixed in concrete is either pulled or pushed, the bond will vary along its buried length (Fig. 14, *a*). The mean bonding resistance τ_{bond} is computed by dividing the force *P* by the buried surface area πdl of the bar:

$$\tau_{bond} = \frac{P}{\pi dl}. \tag{16}$$

The value τ_{mb} depends mainly upon the proportion of the concrete's aggregate and the condition of the bar's surface. Tests have proved that, given a plain round bar within an ordinary grade of concrete $\tau_{mb}=25\text{-}35$ kg/cm². With a concrete richer in cement but with a lower w-c ratio, the τ_{mb} value will increase. Projections on the bar will make it more slip-proof because of direct grip (Fig. 14,*b*). Intermittently deformed bars also minimise fissure widening in the tension zone of a member. Tests have shown that fissure widths in such cases are half of those occurring with plain bars.

Given one grade of concrete, bonding of the bars will be a function of the kind of forces, and the amount of stress in, and the diameters of, the reinforcement (Fig. 14,*c*). Under tension, bond will increase with a lesser bar diameter and a decrease in stresses, whereas under compression there will be greater bond with a larger bar diameter and increased stresses. The explanation for this is that a compressed bar will strive to expand laterally, exert pressure upon the surrounding concrete, and thus increase friction. Just the opposite is true under tension.

3. Mechanical Properties of Reinforcement Steel

Steel bars are manufactured hot-rolled to obtain both plain and deformed surfaces, and are also made by cold deforming methods.

The *hot-rolled process* produces mainly low-carbon, mild steels whose σ-ε diagram (Fig. 15, *a*) reveals a considerable horizontal yield-segment and comparatively great elongation (about 20%) in failure. With the addition of more carbon or the introduction of alloys, the steel grows stronger and its horizontal yield-segment and elongation decreases.

Rupture strength of hot-rolled steels σ_r considerably exceeds the yield point σ_y; however, the latter σ_y is accepted as the criterion for ultimate stresses in reinforcing bars, inasmuch as even such stresses already cause considerable plasticity in the bars and corresponding deformation in construction which hinders its normal service.

Cold-deformed bars are produced by the further processing of hot-rolled steel to raise its yield point (resistance) so that a lesser percentage of it may go into the design.

The essence of strengthening steel by means of cold deforming consists in forcing the bars to bear an additional stress σ_k, higher than the yield point σ_y, and then relieving the stress. The unloading curve will take the form of the dotted line in Fig. 15, *b* and the bar will acquire a residual deformation OO_1. When again loaded, the load line will follow the dotted line to point AD and then trace along the initial curve. Thus the portion O_1AD will show only elastic deformation and the value σ_k will become the new, artificially acquired

yield point, higher than the initial σ_y. After an interval, the additional yield point σ_k will appear with a small horizontal segment of yield, and ultimate rupture strength σ_r will also rise to some extent. In this phenomenon, called *aging of metal*, the steel's resistance is raised through a process of structural toughening, which is a change in the crystal structure of the metal when stressed to σ_y (at the horizontal segment of yield).

There are three principal methods of such cold working: stretch strengthening, drawing, and pressure notching.

Cold stretch strengthening is a process of stretching the bars, either until the desired additional stress σ_k is reached (which exceeds the ultimate yield point σ_y), or until plastic deformation ε_{pl} is attained which exceeds the elongation at the horizontal yield-segment. This method is for raising the yield point only for bars which are to be subjected to tension, but not compression.

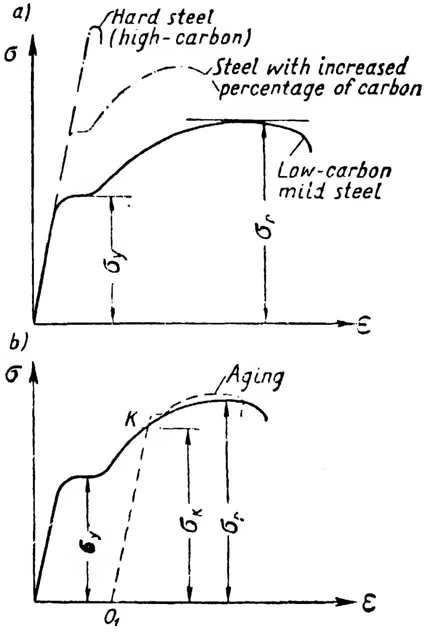

Fig. 15. σ-ε diagram for reinforcement steel

a—for mild and hard steels; *b*—when mild steel is strengthened by cold manipulation

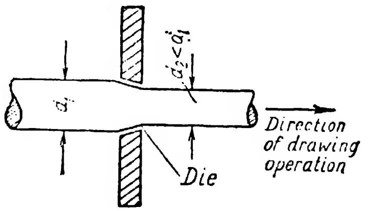

Fig. 16. Sketch illustrating cold drawing of bars

In *cold drawing*, the formerly hot-rolled bars are pulled through dies (openings) whose diameters are less than the bars (Fig. 16). This operation, repeated several times with a smaller die hole for each consecutive die, diminishes the cross-section but elongates the bar. After 4 or 5 such operations a bar can thus be reduced from 6 to 4 mm and elongated 2.25 times.

Cold pressure notching of plain bars is done on a special machine provided with indented rollers which press reciprocally perpendicular notches into the bar, spaced equally along its length (Fig. 12,*c*). This deformed surface grips better with the concrete.

Bars which undergo either drawing or notching acquire a three-dimensional structural deformation which increases their resistance both against tension and compression. The mechanical properties of such bars are close to those of hard steels with no horizontal yield-segment on their σ-ε diagram (Fig. 15, *a*).

When cold-hardened bars are exposed to high temperatures, they reverse to their mild-steel properties. Therefore such bars cannot withstand spot welding without an exacting welding regimen—high current intensity, instant-action welding, and high-pressure gripping. Furthermore, the welded points must not coincide with the flattened notches.

4. Kinds of Steel for Reinforcement

Table 3 lists steels for reinforcement and specified resistances R_s^s for each kind of steel, which represent:

a) the factory test minimum for yield point of mild steels shown in items 1-9;

b) the same for ultimate strength of hard steels shown in items 10-14.

The most economical are hot-rolled intermittently deformed bars, and also the cold-worked steels.

Due to their low resistance, plain round St-0 and St-3 bars that have not undergone cold manipulation are not economical and are only employed when better quality is not available, or for installation or auxiliary rods.

To ensure the concrete bond of cold-drawn wire it is used only in prefabricated mats and blocks. High-strength bars (Nos. 5, 9, 11, 13, 14 in Table 3) are only for prestressed members.

The specified modulus of elasticity for steel is $E_s^s = 2,100,000$ kg/cm². Cold manipulation does not alter this constant.

Reference numbers for various steels entering into working drawings of reinforced concrete construction in the U.S.S.R. are given in Supplement II.

5. Welded (Prefabricated) Mats and Blocks

Welded mats and blocks are furnished either in rolls or in flat units (Fig. 17). In the rolls, effective and spacing (installation) bars are arranged at right angles to each other. The maximum bar diameter that can be bent in rolling is 5.5 mm. In flat mats effective reinforcement can run either in one or two directions. State Standard 8478-57 lists available stocks of St-25Г2C roll and flat prefabricated mats (see Supplement III).

Table 3

Kinds, Diameters, and Specified Resistance Values of Steel Bars R_s^s in kg/cm²

Item No.	Kinds and grades of steel bars	Dia. in mm.*	R_s^s in kg/cm²
1	Hot-rolled plain, St-0 ** steel . . .	5-100	1,900
2	Ditto, St-3 steel	5-40	2,400
		42-100	Special specifications
3	Hot-rolled, intermittently deformed, St-5 steel	10-40	2,800
		45-90	2,700
4	Ditto, St-25Г2С steel	6-40	4,000
5	Ditto, St-30ХГ2С steel	10-32	6,000
6	Hot-rolled plain St-0 steel with subsequent cold stretch-strengthening	5-22	2,400
7	Ditto, St-3 steel	5-22	2,800
8	Hot-rolled St-5 steel intermittently deformed, with subsequent cold stretch-strengthening to a stress of 4,500 kg/cm² or an elongation of 5.5%	10-40	4,500
9	Ditto, St-25Г2С steel, with subsequent cold stretch-strengthening to a stress of 5,500 kg/cm² or an elongation of 3.5%	6-40	5,500
10	Cold-notched (intermittently deformed) St-0 and St-3 steel	6-32	4,500
11	Ditto, St-5 steel	10-32	6,000
12	Cold-drawn low-carbon wire for welded mats and blocks (State Standard 6727-53)	3-5.5	5,500
		6-10	4,500
13	Cold-drawn high-strength intermittently deformed bars (State Standard 8480-57)	2.5-8	18,000-12,000
14	Round high-carbon high-strength wire (State Standard 7348-55)	2.5-8	20,000-14,000

* See Supplement II.
** St-0 stands for steel-0, denoting grade of steel.— *Tr.*

Mats with other bar diameters than those given in the stock list may be incorporated into the design of reinforced concrete, but both bars and their spacings must conform to clearances of the welding machine.

Welded blocks consist of longitudinal and upright bars and are made in either two- or three-dimensional form (Fig. 18). Longitudi-

nal bars in a two-dimensional block may run in one row (Fig. 18, *a*, *b*, *c*) or in two (Fig. 18, *d*, *e*); they may be placed on one side (Fig. 18, *c*, *d*) or on both (Fig. 18, *a*, *b*, *e*). As a rule, it is recommended to arrange the bars of the block on one side to facilitate prefabrication and assure a better bond. Sometimes coupled blocks are used (Fig. 18, *f*, *g*) or one with additional longitudinal bars arc-welded to the effective reinforcement (Fig. 18, *h*).

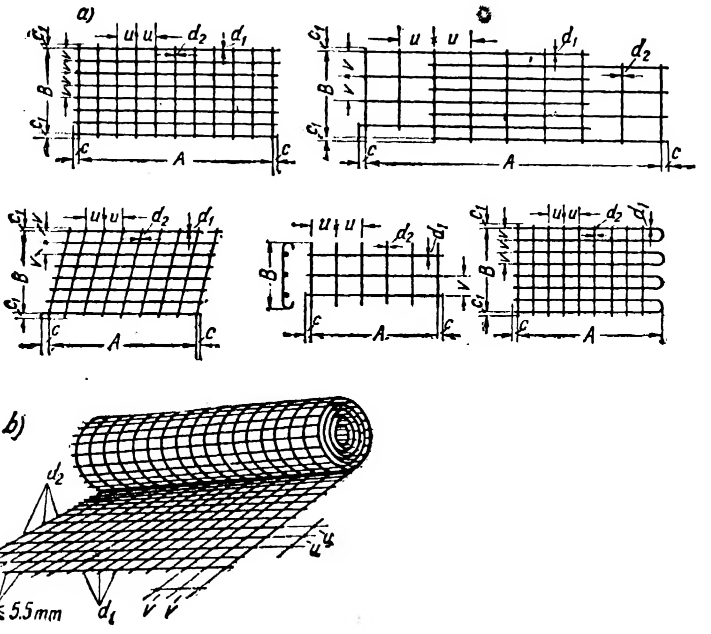

Fig. 17. Types of prefabricated (welded) mats
a—flat; *b*—in rolls

Box-type blocks (Fig. 18, *i*) can be fabricated on special machines such as the MK-251, or made up of prefabricated flat blocks, welded by means of a suspended welder, clamp welder, or by arc welding. They may also be assembled from bent mats.

Dependability of joints in mats and blocks is relevant to the mutual sizes of the assembled bars, a production factor that must not be overlooked when designing bar diameters. Data to aid in such design is given in Table 4.

<table>
<tr><td colspan="5">d_1 denotes either the diameter in mm of effective round bars, or the gauge of intermittently deformed bars for the mats and blocks</td><td>3-4</td><td>5-7</td></tr>
<tr><td rowspan="5">d_2 denotes minimum diameters (either in mm for round bars or in gauge for intermittently deformed bars) of the spacing steel of mats and the upright bars of blocks</td><td rowspan="4">Effective reinforcement arranged on one side only</td><td colspan="3">Intermittently deformed bars</td><td>—</td><td>3.5</td></tr>
<tr><td rowspan="3">Plain bars</td><td colspan="2">Non-lapped joints</td><td>3</td><td>3.5</td></tr>
<tr><td rowspan="2">Lapped joints</td><td>See Fig 20,*a*</td><td>3</td><td>3.5</td></tr>
<tr><td>See Figs. 20,*b* and *c*</td><td>3</td><td>3.5</td></tr>
<tr><td>Effective reinforcement arranged on both sides</td><td colspan="3">Intermittently deformed bars</td><td>--</td><td>6</td></tr>
<tr><td rowspan="3">v_{min} and u_{min} in mm are minimum centerings of bars</td><td colspan="4">Effective and spacing bars of mats, v_{min} and u_{min}, and lateral bars of blocks, u_{min}, arranged only on one side</td><td>50</td><td>50</td></tr>
<tr><td colspan="4">Upright bars of blocks, u_{min}, when the effective reinforcement is arranged on both sides</td><td>50</td><td>75</td></tr>
<tr><td colspan="4">Longitudinal bars of blocks, when the bars c_2 mm are arranged in two rows</td><td>30</td><td>30</td></tr>
<tr><td rowspan="2">u_{max} in mm is the maximum centering of spacing bars and of upright bars of blocks</td><td colspan="4">When effective reinforcement is either cold-drawn wire or stretch-strengthened bars</td><td>250</td><td>250</td></tr>
<tr><td colspan="4">When the effective bars are either intermittently deformed of any grade steel, or are St-3 or St-0 steel round bars</td><td colspan="2"></td></tr>
</table>

Mats and Blocks

8-9	10	12	14	16	18	20	22	25	28	32	36	40
4	4.5	5	5	6	6	8	8	8	10	12	12	14
4	4.5	5	5	6	6	8	8	8	10	12	12	14
4	4.5	5	6	8	8	10	10	12	14	18	20	22
4	4.5	5	6	8	10	12	14	16	18	20	22	25
6	8	8	8	8	8	8	10	10	12	12	14	16
75	75	75	75	75	100	100	100	150	150	150	200	200
75	100	100	150	150	200	200	250	250	300	300	400	400
30	30	40	40	40	40	50	50	50	60	70	80	80
300	300	300	300	300	400	400	400	—	—	—	—	—
Not limited												

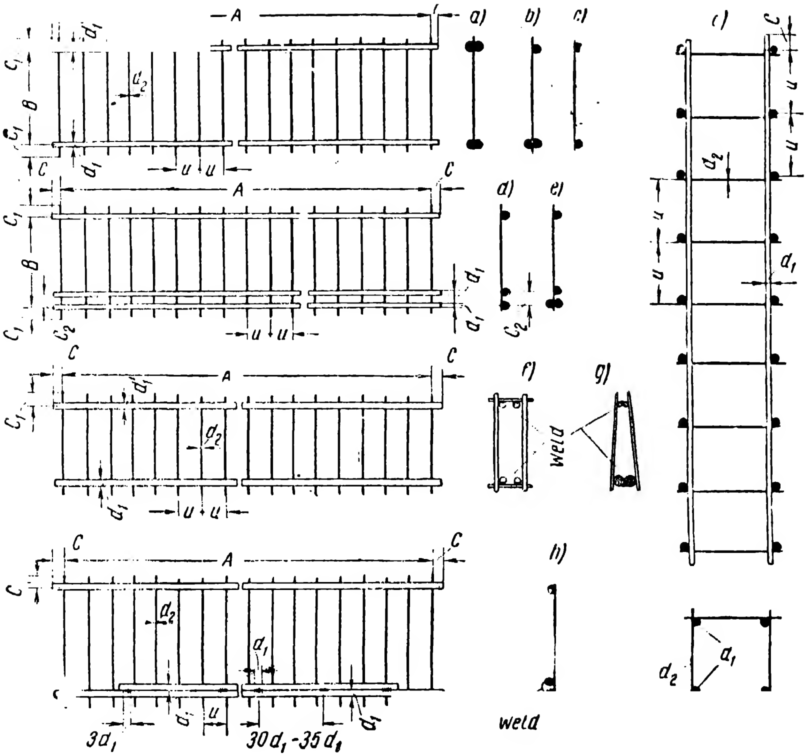

Fig. 18. Types of prefabricated blocks

6 Anchoring, Bending and Splicing of Bars

To avoid slipping, some of the ends of the tension-loaded plain round bars are hooked to anchor them in the concrete (Fig. 19, *a*). In lightweight concretes, short bars are placed under the hooks (Fig. 19, *b*) to spread the latter's bearing area.

Hooks are not needed in prefabricated mats and blocks, since bar crossings serve the purpose of additional anchorage. Nor are hooks used on deformed bars.

In prestressed construction the bars are provided with special anchors (see Chapter VI). Bars are always bent in parts of circles (Fig. 19, *c*). If lightweight concrete is used additional short bars are fixed at the places of bending.

Butting joints are usually welded. There are a number of such types:

1) *End butts are contact-welded* (Fig. 19, *d*). In this operation 10 mm is the minimum diameter if the bars are hot-rolled, and 14 mm

if they are cold-worked. Contact welding can only be done at the factory as it requires special equipment.

2) *Lapped butts are arc-welded* (Fig. 19, *e*) and used only for hot-rolled bars with a minimum diameter of 6 mm. Bar ends must be arranged so that a longitudinal force will not bend the lap. In one-face welds, the lap length l_1 must be 8 d for plain bars and 10d for deformed bars. If the weld is double-faced, the lap length must be 4 d and 5 d for plain and deformed bars respectively.

3) *Strapped and backed butts are arc-welded* (Fig. 19, *f*) and applicable to both hot-rolled or cold-notched bars with a minimum diam-

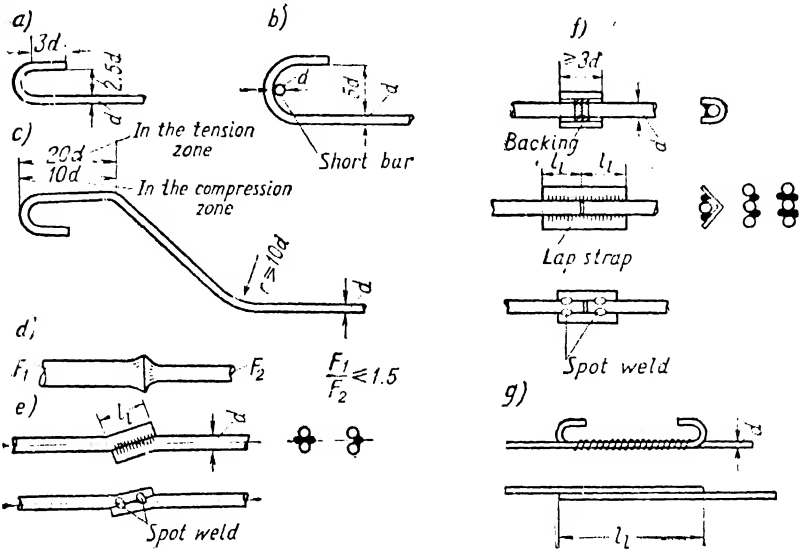

Fig. 19. Hooks, bent bars and joints

eter of 6 mm. In the backed joint the bar ends are welded together, the backing serving as the weld grip. In the strapped joint the bars are welded to the strap either by flank or electric-arc spot welding. The strap must equal the jointed bars in strength. The length of each half of the strap must be at least the same as the lapped joint.

When butting large bars (over 25 mm) either in the shop or on the site, arc welding in forms is used, executed within a standard copper form.

Welding methods are regulated by special welding specifications.

Another method of splicing, used mainly on the site in the assembly of blocks, is to overlap the joints and tie them with mild binding wire. Such joints are acceptable for all jobs, but the ends of tension-

loaded plain bars must be hooked (intermittently deformed bars need not be) as shown in Fig. 19, *g*.

To assure reliable end anchorage, the lap l_1 must be 35 *d* for tension bars in grade-150 concrete, and 25 *d* if the bars are in compression; 30 *d* for tension bars in 200-grade (or higher) concrete, and 20 *d* for compression bars. Smooth bars may be without hooks if the lap l_1 is increased to 30*d*. In lightweight concretes, the overlap of 16 mm (or larger) bars must be increased to 40 *d*.

If the bars are cold-notched, the stated overlaps must be increased by 5 *d*, or by 10 *d* for deformed bars of St-25Г 2C.

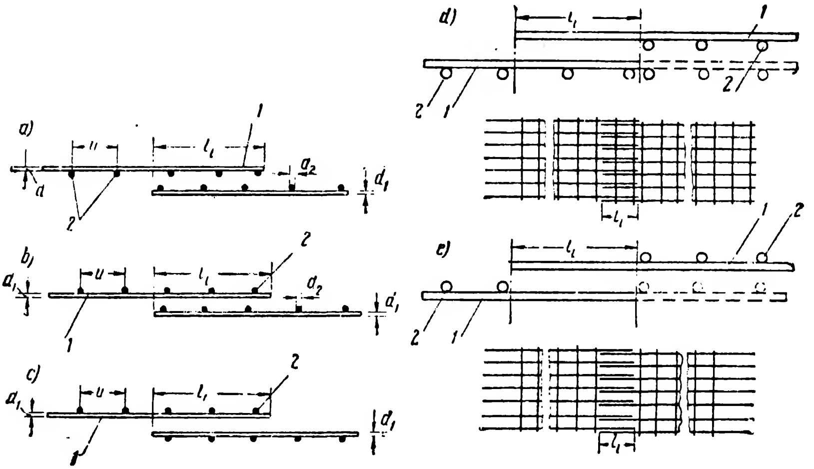

Fig. 20. Splicing of effective bars in mats and blocks
1—effective bars; *2*—installation bars

When longitudinal bars run only on one side of prefabricated mats and blocks, they are usually overlapped without welding, with the effective bars placed either on different planes (Fig. 20, *a*, *b*) or in one plane (Fig. 20, *c*). If the effective bars of coupled mats are intermittently deformed, it is recommended to splice them in one plane, for which purpose one of the mats (Fig. 20, *d*) or both (Fig. 20, *e*) must not have welded lateral bars.

Required lap l_1 for good anchorage of mats and blocks is given in Table 5; if the joint is in the compression area, the values are decreased by 10 *d*; but for lightweight concrete, the figures must be increased by 10 *d*.

Table 5

Dimensions of Overlaps Assembled without Welding in Mats and Blocks Placed in the Tension Zone of Heavy Concretes

Effective reinforcement of welded blocks and mats	Type of joint	Minimum overlap	
		Concrete of grade 100 and 150	Concrete of grade 200 and more
St-5 steel, hot-rolled intermittently deformed	Fig. 20, *a*, *b*, *c*	$30d_1$	$25d_1$
	Fig. 20, *d*, *e*	$35d_1$	$30d_1$
St-3 or St-0 steel, round, hot-rolled	Fig. 20, *a*, *b*, *c*		
Cold-notched	Fig. 20, *a*, *b*, *c*		
	Fig. 20, *d*, *e*	$40d_1$	$35d_1$
Cold-drawn wire and stretch-strengthened bars	Fig. 20, *a*, *b*, *c*		
St-25Г2С steel, hot-rolled intermittently deformed	Fig. 20, *a*, *b*, *c*		
	Fig. 20, *d*, *e*	$45d_1$	$40d_1$

The effective bars of coupled mats and blocks must overlap at least 250 mm; if the round bars are plain, each overlapped end must have at least 3 lateral ties along the joint.

When non-effective, or spacing, bars are being joined, they are overlapped 50 mm if their diameters d_2 are $\leqslant$ 4 mm, and 100 mm if $d_2 > 4$ mm (Fig. 21, *a*), or they are joined by an additional small mat (Fig. 21, *b*), overlapped $15d_2$ on each side, but not less than 100 mm. Such a mat is recommended if the effective bars are larger than 16 mm.

Lapped jointing of blocks is prohibited if the longitudinal bars are arranged on both sides.

Hot-rolled effective bars of prefabricated mats and blocks with minimum bar diameters of 22-25 mm may be joined by arc welding (either by a backed or strapped joint).

Mats or blocks that are coupled must be stagger-jointed so that the lateral area of the spliced bars in any one cross-section is not over 50% of the total bar area

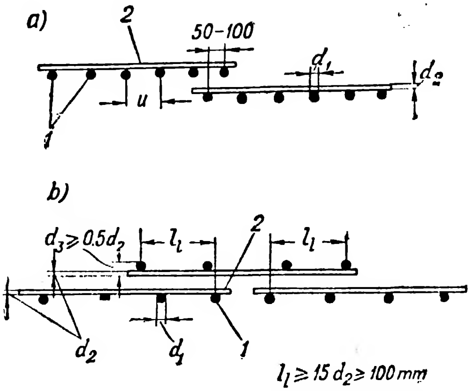

Fig. 21. Joining auxiliary reinforcement of welded mats and blocks
1—effective bars; *2*—installation bars

Sec. 3. REINFORCED CONCRETE

1. General Principles. Contraction and Creep

The properties of reinforced concrete depend upon those of concrete and steel but in a number of cases they do not coincide. Bond of the bars hinders free contraction of the concrete and leads to incipient stresses in both components: tension in the concrete and compression in the bars. Such contraction stresses are minimised by shortening uninterrupted structural lengths through the use of joints. Concrete creep, which is caused by constant loads, redistributes stresses between the concrete and embedded bars and offers the possibility of making full use of the strength of the two components in axially compressed members. Creep will eventually increase beam deflection. In prestressed members, creep and contraction will, in time, result in partial loss of the prestress, both of the bars and concrete. The physico-mechanical properties which control the conduct of reinforced concrete under working loads (resistance to bending, compression, tension, etc.) are treated under corresponding headings.

2. Effect of Temperature Changes on Reinforced Concrete

During a change of temperature the small differences in rates of thermal expansion between the neat cement, aggregate, and steel bars will cause one component to tie with another, thus hindering

free temperature deformation between them and resulting in internal counteracting strains.

Internal strains incited by temperatures up to 100° C are negligible and will not lower the efficiency of the reinforced concrete. But higher temperatures will have an adverse effect; 25% of the strength is lost at 200°-250° C, and if heated to 500°-600° C and then cooled, the concrete will fail completely.

The principal reasons for this breakdown under high temperatures are: 1) great internal stresses from difference of temperature deformation between the neat cement and aggregate; 2) chemical dehydration of the cement minerals which leads to segregation of free lime and its slaking upon cooling (through the action of either locked-in or free moisture) and is followed by the rupture of the concrete.

The effect of heat upon concrete-steel bond is qualified by the kind of bar surface and the amount of heat. If the bar is smooth, bond is greatly weakened beginning with 250° C, and almost entirely lost at 500° C. But if the bars are intermittently deformed, their grip is hardly effected even at 500° C.

Since the mechanical properties of ordinary reinforced concrete are vulnerable to high temperatures, structures subjected to such action (furnaces, flues, etc.) are built of heatproof concretes that retain their main physico-mechanical characteristics under protracted thermal regimes of 1,000°-1,400° C. Special specifications have been compiled for designing of heat-proof concrete structures.

Indeterminate structures of reinforced concrete (portal frames, arches, etc.) are not only subject to internal temperature strains, but also to additional forces caused by temperature changes. If the structure is large, these additional forces will acquire great scale even at normal temperature variations within a range of 30° C. To minimise these stresses, the structure is divided by compensation joints, which are usually combined with settling joints.

3. Weight of Reinforced Concrete

The weight of reinforced concrete consists of the respective weights of the concrete and embedded bars. When heavy reinforced concrete is compacted by vibrators, its weight will come to 2,500 kg/m³ (2,400 kg if placed without vibrators). If the percentage of reinforcement is high (over 3%), the weight will be the sum of the concrete and bars in a volume unit of concrete.

4. Protective Covering

In reinforced concrete the bars are embedded somewhat below the surface so as to form a *protective covering*. This embedment improves the concrete-steel bond and allows neither corrosion of the bars

nor their quick reaction to destructive heat from outside sources. The thicknesses of protective coverings are established by building codes based on service experience.

These thicknesses must be:

10 mm in slabs and walls having a thickness $h \leqslant 10$ cm;
15 mm " " " " " " " $h > 10$ cm;
20 mm in beams and columns with effective bars $d \leqslant 20$ mm;
25 mm " " " " " " " $d > 20$ mm;
30 mm " " " " " " " $d > 35$ mm;
50 mm for all rigid-type reinforcement (rolled steel section).

In structures subjected to aggressive environment (smoke, vapours, acids, and heavy moisture) the protective covering must be increased 10 mm. In factory-made precast construction of 200-grade (and higher) concrete, to which efficient vibrators have imparted great density, the protective covering may be reduced by 5 mm, but it must be at least 10 mm in slabs, 20 mm in beams and columns, and 15 mm for stirrups and upright bars of blocks.

CHAPTER II

MAIN PRINCIPLES IN REINFORCED CONCRETE DESIGN

Sec. 4. PREREQUISITES

1. Design Methods

In the design of structures the aim is to achieve the most economical member sizes that guarantee their safety under service loads. Such design in reinforced concrete must consider:

a) carrying capacity (mechanical strength and stability),

b) deformations due to deflection, vibration, etc.,

c) formation and widening of fissures.

Computations of carrying capacity are executed for all types of construction to avoid failure.

Deformation calculations are required where members, otherwise sound, may suffer in their structural properties from prohibitive deformation. For example, long-span slabs or beams of floors that are sufficiently strong, may prove unfit because of extreme deflection.

Computations to anticipate fissure formation are carried out only in individual structures that demand impermeability (reservoirs, piping, etc.), and also in prestressed reinforced concrete.

Computations to estimate fissure widths are required when fissures are permissible but their widths must be limited, as in reinforced concrete silos, smokestacks, members subjected to repeated dynamic loads, and in structures not protected against moisture or other atmospheric action that would corrode the reinforcing bars.

2. Computation Factors and Their Variability

Safe construction is gained by adopting allowable stresses that are less than those which will cause failure, i. e., the construction is given a margin of safety. The need of employing stresses less than those that would cause destruction is due to variability of computation factors (loads and strength of materials), for repeated tests

have shown that concrete and steel each display variations (scattered results) in resistance. Accepted norms for permissible external forces, known as *specified loads*, also vary. Such variability of computation factors is characterised by distribution curves illustrated in Fig. 22, which show that the greatest number of tests (observations) give a certain definite value, the so-called mean. The rest of the data is scattered from the mean, either towards a minus- or plus-result, and the greater the deviation, the rarer is it registered. The figure shows that if values possess little variability, the bell-shaped figure limited by the horizontal axis and distribution curve is high and narrow (curve *a*, Fig. 22), whereas great variability will result in a squat bell (curve *b*).

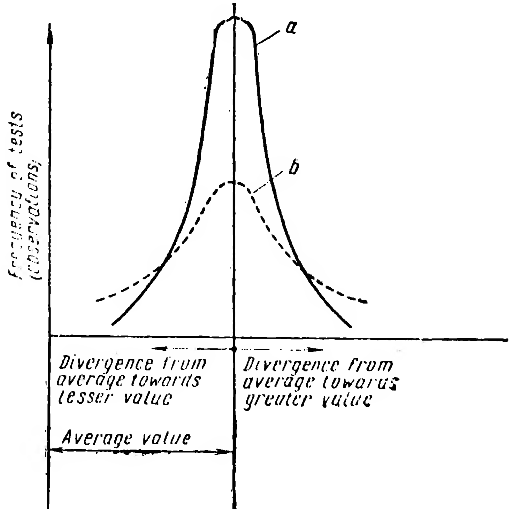

Fig. 22. Distribution curves showing variability in computation factors

Such distribution curves are used to chart external loads, such as wind, snow, etc., from statistical material of long-term observations. Similar charts of strength properties of concrete and reinforcement are made on the basis of numerous tests carried out by building and metallurgical laboratories. These static distribution curves serve to determine possible plus-variations for specified loads and minus-variations for specified strength of materials.

3. Load Coefficients. Uniformity Coefficients. Design Strength of Concrete and Reinforcement

Probable increases of load due to variability in specified loads are embraced by the *load coefficient*, $n > 1$.*

The product of the specified load q^s and the load coefficient n is known as the *design load*:

$$q = q^s n.$$

* In individual cases where diminishing an assumed load might prove dangerous, the $n < 1$ load coefficient is used. For instance, when verifying a structure against wind thrust or seismic overturning, $n < 1$ is adopted for the dead load that reacts against overturning.

Specified loads, load coefficients, and design loads are presented in Supplement IV.

Design loads for buildings and industrial structures must be chosen in their most unfavourable combinations, both for individual members and for the structure as a whole.

Main load combinations will consist of the dead weight of construction, live loads, snow loads, and loads from crane equipment. Additional combinations will consist of the above, plus wind loads, loads from erection cranes, or from the effects of temperature changes. Special combinations of loads will include extra forces (such as seismic action), dead weight of the structure, and live and wind loads. When additional or specific combinations enter load computations, the design loads (aside from dead weights) must be multiplied by the following coefficients:

a) a coefficient of 0.9 for additional combinations;

b) a coefficient of 0.8 for special combinations.

In designing columns, walls, and foundations of apartment houses and civic buildings (except educational institutions and theatres), the live loads must be assumed as follows:

for the first and second storeys, 100% of all the superimposed live loads counting from the top down;

for the third and fourth storeys, 85% of all the superimposed live loads counting from the top down;

for the fifth and sixth storeys, 70% of all the superimposed live loads counting from the top down;

for all other storeys 60% of all superimposed live loads.*

Probable decreases in strength values of materials due to variability in specified stresses are embraced in the *coefficient of uniformity* $k<1$.

Design resistance is the product of specified resistance of materials R^s and the coefficient of uniformity k:

$$R=R^s k.$$

The coefficient of uniformity for concrete k_c will depend on its preparation, kind of stress in the member, and the grade of concrete.

If the concrete is factory-made or otherwise mechanically prepared with the aid of automatic or semi-automatic batching equipment and is regularly strength-tested (fabrication method A), there is a lesser possibility of a minus-deviation in the resistance value of the concrete mix. In all other cases (fabricating method B) a lower concrete resistance is likely. Therefore a higher coefficient of uniformity is incorporated when the fabricating method is A than when it is B.

Distribution curves show a greater uniformity of concrete under compression than tension. Hence the coefficient of uniformity k_c

* Live loads for libraries, archives, and floors containing equipment are assumed as 100% in all cases.

must be grearer for compression than tension. Coefficients of uniformity run from 0.4 to 0.65 under varying conditions.

Design resistance for concrete $R = R^s k_c$ is given in Table 6.

Table 6

Design Resistance of Concrete in kg/cm²

Kind of stress	Symbol	Fabricating method	Design resistance for various grades of concrete								
			50	75	100	150	200	300	400	500	600
Axial compression (prism strength)	R_{pr}	A	24	36	48	70	90	140	190	230	270
		B	22	33	44	65	80	130	170	210	250
Compression in bending	R_{bc}	A	30	45	60	85	110	170	230	280	330
		B	27	41	55	80	100	160	210	260	310
Tension	R_t	A	2.7	3.6	4.5	5.8	7.2	10.5	12.5	14	15
		B	2.4	3.2	4	5.2	6.4	9.5	11	12.5	13.5

Note. A coefficient of 0.7 is used for alumina-cement concrete.

The working moduli of elasticity for concrete in compression are presented in Table 7.

Table 7

Working Moduli of Elasticity for Concrete in Compression E_c in kg/cm²

Grades of concrete	50		100	150	200	300	400	500	600
Heavy concrete	65,000	90,000	120,000	165,000	200,000	270,000	310,000	340,000	360,000
Lightweight concrete	50,000	60 000	75,000	100,000	115,000				

Since steel is less variable in strength than concrete, and also because the specified resistance of the reinforcement is taken at its factory-test minimums (σ_y or σ_r), its coefficient of uniformity is greater than that of concrete and is embraced within the limits of 0.75-0.9.

Design resistance of steel bars $R_s = R_s^s k_s$ is given in Table 8 in round numbers.

Due to the weak grip of steel in lightweight concretes (grades less than 100), the design resistance of bars in such cases is considered as 1,700 kg/cm² for all grades of steel, i.e., just as for St-0 steel.

The working modulus of elasticity for bars E_s is taken as 2,100,000 kg/cm²

Design Resistance of Bars R_s in kg/cm²

Item No.	Bars	Design resistance of bars R_s	
		In tension	In compression
1	Hot-rolled, plain round bars (also flat steel and rolled shapes) of St-0 steel	1,700	1,700
2	Ditto, St-3 steel	2,100	2,100
3	Hot-rolled intermittently deformed, dia. 10-40 mm of St-5 steel	2,400	2,400
4	Ditto, over 40 mm in dia.	2,300	2,300
5	Hot-rolled intermittently deformed, dia. 6-40 mm of St-25Г 2C steel	3,400	3,400
6	Ditto, dia. 10-32 mm of St-30XГ 2C steel	5,100	3,600
7	Hot-rolled, plain, dia. 5-22 mm of St-0 steel with subsequent stretch-strengthening	2,100	1,700
8	Ditto, up to dia. 12 mm of St-3 steel for use in welded mats and blocks . .	2,500	2,100
9	Hot-rolled intermittently deformed of St-5, with subsequent stretch-strengthening to a verified stress of 4,500 kg/cm²	4,050	2,400
10	Ditto, stretch-strengthened to a verified elongation of 5.5%	3,600	2,400
11	Hot-rolled intermittently deformed of St-25Г2C steel with subsequent stretch-strengthening to a verified stress of 5,500 kg/cm²	4,950	3,400
12	Ditto, stretch-strengthened to a verified elongation of 3.5%	4,400	3,400
13	Cold-notched, St-0 and St-3 steel . . .	3,600	3,600
14	Ditto, St-5 steel	4,500	4,500
15	Cold-drawn low-carbon wire, up to 5.5 mm dia. for welded mats and blocks (State Standard 6727-53)	4,500	4,500
16	Ditto, dia. 6-10 mm	3,600	3,600
17	Cold-drawn high-carbon wire, dia. 2.5-8 mm, intermittently deformed (State Standard 8480-57)	14,500-9,600	—
18	High-strength carbon wire, dia. 2.5-8 mm (State Standard 7348-55)	16,000-11,200	—

4. Coefficients of Service

Insufficiencies in the design diagram, unknowns in loads, aggreslove mediums, high-quality fabrication and other factors that either siwer or increase carrying capacity but which are not embraced in the

calculations, are all entered into the computations through a special *coefficient of service m.*

A case in point is where $m=1.1$ for precast bending members that are made in plants or casting yards and regularly tested as to strength, both of the whole and their component steel and concrete.* For in-situ axially compressed members of less than 30×30 cm cross-section and eccentrically compressed members whose larger cross-sectional dimension is less than 30 cm, $m=0.8$; for slabs having monolithic beams about their entire perimeters $m=1.25$, and so forth. However, in most instances $m=1.0$.

Sometimes bar resistance cannot be fully taken advantage of. In certain kinds of bars a partial loss in uniformity or partial slackening of the internal toughened steel structure must be considered. Therefore the *coefficient of service* $m_s \leqslant 1$ must be applied in computing the design resistance of the reinforcement R_s.

For 100-grade concrete, $m_s=0.9$ because of insufficient concrete-steel bond and a lower guarantee against corrosion of any type of tension-loaded bars.

For cold-notched bars possessing low plasticity either in tension or compression, and cold-drawn wire in fabricated mats and blocks, $m_s=0.65$, since there is a danger of slackening of internal structural toughness during welding work. For intermittently deformed St-5 and St-25Г2C bars that have been stretch-strengthened without verification of results, $m_s=0.9$.

In designing reinforced concrete bending members by the diagonal plane method (see Chapter III), a different *diagonal-plane coefficient of service* $m_{di} < 1$ is introduced for lateral reinforcement (stirrups, inclined bars, and upright bars of blocks), inasmuch as during calculations the stresses will not correspond to the value of $m_s R_s$ for some of the lateral reinforcement. For stirrups and inclined and upright bars of blocks made from all types of hot-rolled steel, $m_{di}=0.8$, and 0.7 for the same of cold-drawn rods.

Sec. 5. DESIGN METHODS

1. Calculations Concerning Carrying Capacity

The carrying capacity (resistance) of members is considered sufficient if their stresses, due to loads and their plus-variations (load coefficients), do not exceed the stresses that the given members can withstand, with due regard to minus-variations of resistance (coefficients of uniformity) and service variations.

The applied forces are a certain function N of the specified loads q^s, their load coefficients n, the design diagram, and other factors expressed

* In such cases the design resistance of concrete must be chosen according to fabrication method B.

by the letter l. The stresses that can be withstood by a member are in their turn a certain function Φ depending on the shape and size of the cross-section S, concrete resistance R_c, the coefficient of concrete uniformity k_c, bar resistance R_s, the coefficient of bar uniformity k_s and the coefficients of service m and m_s and sometimes m_{di}.

Thus the condition for structural resistance may be expressed by the following inequality:

$$N(q^s, n, l) \leqslant \Phi(m, S, R_c^s, k_c, R_s^s, k_s, m_s). \tag{17}$$

If condition (17) is complied with, sufficient resistance of the member will be assured.

2. Deformation Computation

The purpose of deformation computations is to verify whether the specified load deformation Δ exceeds deformation limits f as prescribed by code for the given type of construction, i.e.,

$$\Delta \leqslant f.$$

3. Computations for Anticipating Fissures and Their Widening

Sufficient resistance is attained by members against *fissure formation* if the stresses from specified loads do not exceed those that the construction can withstand prior to fissure formation in the tension zone, with due regard to resistance variations of the concrete and the service coefficient. *Fissure widening* is determined by computing the action of specified loads upon fissure growth, the latter being limited by building codes, i.e., $a_{fis} = 0.2$ mm.

Calculations of deformation or of fissure formation and growth must be based on *specified loads* (without the load coefficients), because loss of service assets in a member is less dangerous than the loss of carrying capacity, the computation of which is based on design loads (including he load coefficients).

Sec. 6. DEVELOPMENT OF DESIGN IN REINFORCED CONCRETE. THE FACTOR OF SAFETY

Design of structures is continually improving as science advances and building experience accumulates.

Formerly, cross-sections of reinforced concrete elements were determined by the *method of safe stresses*: reinforced concrete was considered as an elastic material under loads, and stresses were limited to so-called *allowable* ranges for the concrete and the reinforcement.

The next step in reinforced concrete design in the U.S.S.R. was the adoption in 1938 of the *failure-stage method* in cross-sectional analysis, which recognised the elasto-plastic properties in the material and therefore came closer to actual conditions under load. This method determined the failure value in the cross-section, and the relation between failure and allowable stresses fixed its reserve of resistance or *margin (factor) of safety*. This factor of safety, as a definite constant established by building codes, considered the variabilities of all computing factors jointly. But actually in some members certain factors (such as loads) are subject to greater change than others, and variability in the strength of materials, as well as service conditions, are likewise not always the same.

In the U.S.S.R. *ultimate stress* design was initiated in 1955. Ultimate stress is that stage in construction beyond which its normal exploitation is impossible; it may be reached either by loss of resistance (carrying capacity), by prohibitive deformation, or by the formation or overgrowth of fissures. In this method, just as in the previous failure-stage method, stresses in reinforced concrete are computed according to the failure stage with proper consideration for elasto-plasticity. But instead of a constant factor of safety, three types of variability coefficients are introduced: a load coefficient as a margin of load reserve, a uniformity coefficient as a safety margin for strength of materials, and a service coefficient as a factor of safety for service conditions.

The ultimate stress method of design is a forward step, for it allows a separate estimation for each computation factor. This assures greater reliability of construction under loads and sometimes affords economy of materials. Improvements in both building materials and workmanship, and closer assumptions of loads are sure to effect calculations in the future by corresponding alterations of computation coefficients.

CHAPTER III

BENDING MEMBERS

Sec. 7. DESIGN OF BENDING MEMBERS

1. General Principles

Slabs and beams are the most typical kinds of bending members. Slabs are elements whose thickness h is small in comparison to the span l or the width b. Beams are members whose sectional dimensions $b \times h$ are considerably less than their span l. A beam's height h is usually from $\frac{1}{8}$ to $\frac{1}{20} l$, while its width b is 0.25-0.5 h.

Reinforced concrete very often consists of beams and slabs. A ribbed floor panel, for instance (Fig. 23), is composed of beams (ribs) and their slab. Slabs and beams are built either in single- or multi-spanned (continuous) construction.

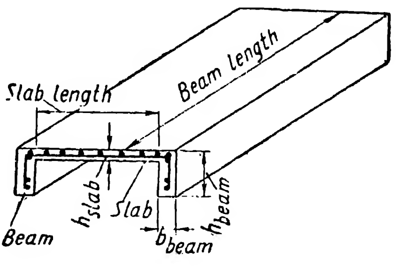

Fig. 23. Dimension nomenclature of beams and slabs

2. Slabs

Thicknesses of flat slabs are commonly in multiples of 1 cm and range from 6 to 10 cm or more. Slabs in two-way ribbed panels (T-beams) and cavity-slab construction are much less in thickness, i.e., 25-40 mm. Slabs are reinforced with mats the bars of which run in two perpendicular rows. Most often the mats are welded (prefabricated), but sometimes they are tied. Fig. 24 illustrates the bars of reinforced concrete slabs: the longitudinal elements are called effective bars, the others are spacing (installation) bars. Effective bars take the tensile stresses in the slab caused by bending action under load. Spacing bars, which make up the mat together with the effective reinforcement, keep the latter in place during concreting, bear uncomputed

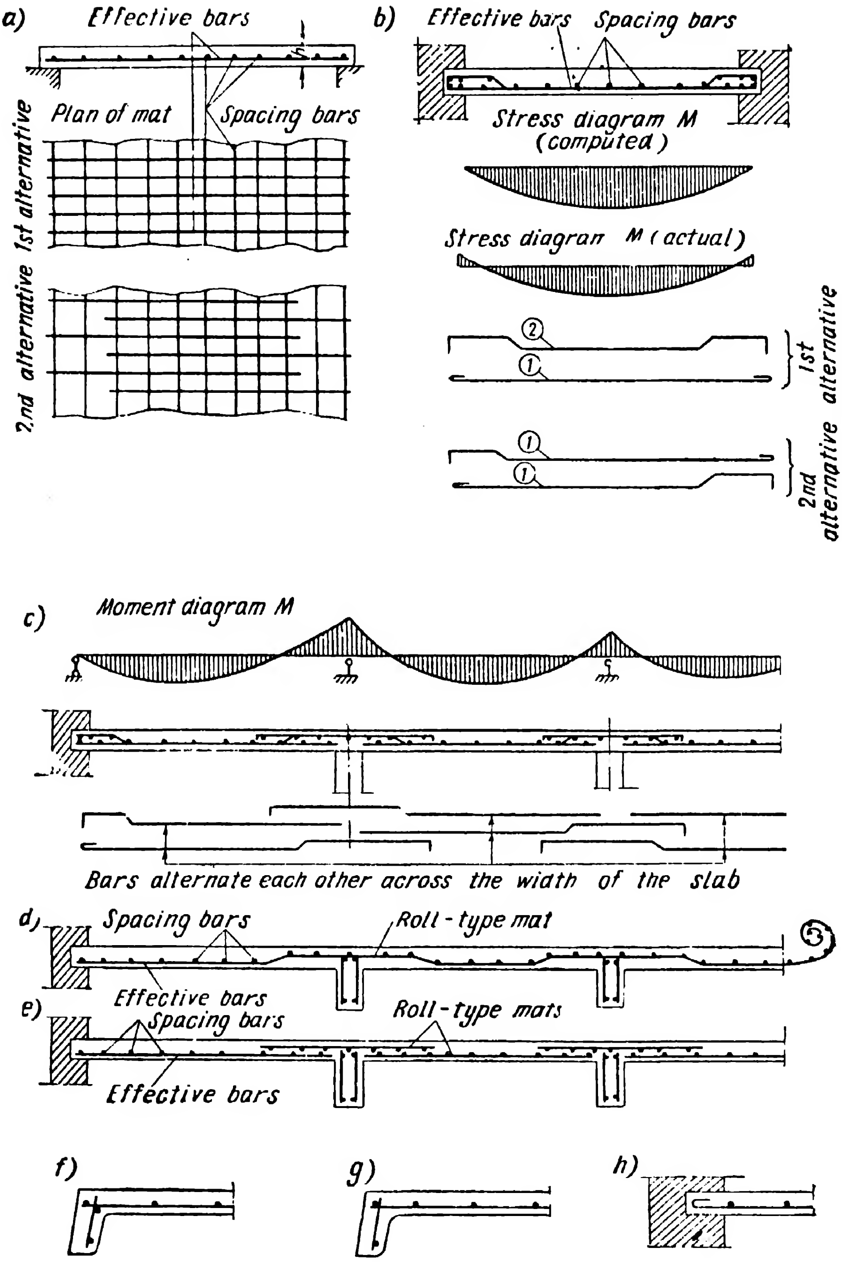

Fig. 24. Bars of reinforced concrete slabs

stresses of contraction and temperature alterations, and also distribute local loads over the whole slab. Sometimes both the longitudinal and lateral steel are made up of effective bars.

Effective bars of slabs are usually 3-10 mm dia. (12-16 mm in thick slabs) and spaced 100-200 mm along the slab width. Spacing bars are smaller and laid at intervals of 250-300 mm; moreover, their cross-sectional area must not be less than 10% of that of the effective bars per linear metre.

Not all effective bars need be carried to the supports (Fig. 24, *a*, alternative 2), but at least $\frac{1}{3}$ of all the lower effective bars, or a minimum of three bars per linear metre, must be taken beyond free-end supports. If the mats are the prefabricated type, usually all the lower longitudinal effective steel is brought to the end of the slab.

There may be various forms of effective bars in tied mats to satisfy the bending moment diagram of the slab. The most common are shown in Fig. 24, *b* and *c*. Varied bar forms are, as a rule, spaced alternately and evenly along the slab width.

To satisfy stress moments in continuous slab panels when roll-type prefabricated mats are used, their effective bars are either smoothly shunted from the lower part of the interspan to the upper part over the supports (Fig. 24, *d*), or separate mats are installed at the bottom in the middle of the slab and at the top over the supports (Fig. 24, *e*).

To secure sufficient anchorage at end supports, effective bars in tied mats are end-hooked, while in prefabricated mats at least one lateral bar is brought beyond the bearings (Fig. 24, *f*). If this cannot be done, then an extra lateral bar is welded on (Fig. 24, *g*), or the effective bars are hooked as shown in Fig. 24, *h*.

3. Beams

There are many types of cross-sections for reinforced concrete beams: T- and I-beams, rectangular and trapeziform, etc. (Fig. 25, *a*). The most common are rectangular and I-beams. Beams up to 50-60 cm in height h are usually designed in multiples of 5 cm, and in multiples of 10 cm for greater heights. As already said, the beam width b is usually 0.25-0.5 h. Precast beams are often made narrower to lessen their weight, their width being dictated by longitudinal bar placement.

Beam reinforcement consists of longitudinal, upright, and supplementary bars assembled into a block, generally welded together but sometimes spliced.

The *effective longitudinal bars* take tension stresses provoked by the bending of the beam under load. The percentage of reinforcement is determined by computation, and bar diameters range from 12 to 28 mm with bars running in single or double tiers.

Upright bars resist diagonal tension at the supports and are made in the form of stirrups and bent-up (inclined) bars in spliced blocks (Fig. 25, *b*). In prefabricated blocks the same purpose is served by upright (and sometimes inclined) bars welded to the effective and spacing bars (Fig. 25, *c*). The stirrups also unite the compression and tension zones of the beam and connect all the bars of the block. They may be either of the closed or open type (Fig. 25, *d*), but only the former is used in rectangular beams.

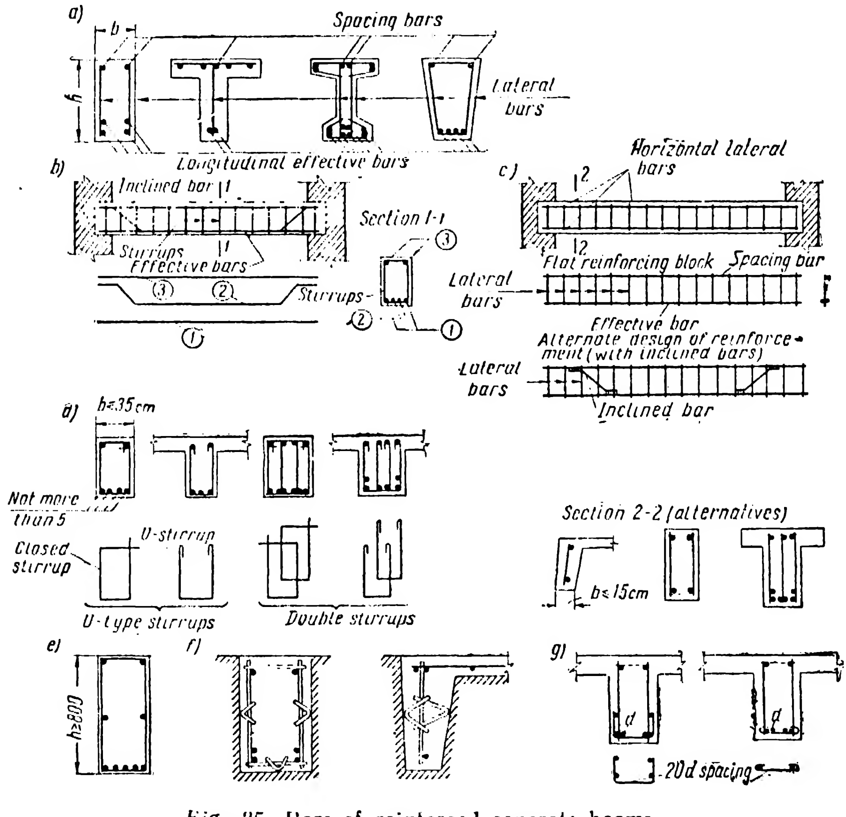

Fig. 25. Bars of reinforced concrete beams

Stirrups are either single or double U-shaped verticals, depending upon beam width and number of tension bars. Single U-stirrups are employed when the beam width $b \leqslant 35$ cm, or when there are no more than 5 tension bars in one tier. When $b > 35$ cm and there are more than 5 tension bars in one tier, double stirrups are used which consist of two adjacent single stirrups.

In beams up to 60 cm in height, stirrup diameters are usually 6 mm, and 8 mm for higher beams. Upright bar diameters of blocks

are determined by calculation, minimum sizes being limited by welding requirements and the values given in Table 4. Spacing of stirrups and upright bars along the block length is dictated by computation or construction requirements, as further described.

Installation bars connect the reinforcement block in the beam and bear uncomputed stresses of concrete contraction and temperature changes. They may sometimes function as effective reinforcement during hoisting operations. Their diameters must be 2-4 mm more than the upright bars in prefabricated blocks. In tied (spliced) blocks their diameters are usually 10-12 mm.

Beams exceeding 80 cm in height must have additional longitudinal bars (minimum dia. 10 mm) along their sides at height intervals of 40-50 cm.

To assure good concrete bond and satisfactory placing of concrete without voids or pits, the minimum clear distance between bars must equal their diameters and be at least 25 mm for lower, and 30 mm for upper, bars.

Narrow beams (up to 150 mm) are, as a rule, reinforced with *flat prefabricated latticework*, one such frame to each beam (Fig. 25, *c*). In wide beams that carry big loads 2 or 3 frames are used, tied into a box by horizontal lateral bars welded at spacings of 1 to 1.5 m (shown dotted in Fig. 25, *c*, section *2-2*). Flanges of T-and I-beams are reinforced with mats capping the blocks of the web (Fig.25, *a*).

Side embedment of bars is achieved by small bent bars, welded so as to butt into the forms (Fig. 25, *f*); lower embedment is similarly assured by bars (chairs) protruding regularly from the cage and resting on the lower formwork.

To strengthen the compression zone, sometimes longitudinal compression bars are computed for the beam. Buckling of such bars is prevented in tied blocks by closed stirrups spaced at maximum intervals of 15*d* of the compression bars. In prefabricated blocks, buckling is baffled either by a pan-shaped mat, or a horizontal one with hooked upright bars (Fig. 25, *g*).

Anchorage of longitudinal effective reinforcement at free-end supports is assured by either carrying such bars beyond the bearing, or by installing special mats (sometimes anchors) connected to the block (see Sec. 10).

To satisfy decrease of the bending moment when beams are reinforced with prefabricated blocks, the tension bars may either be cut off individually, or a whole block interrupted if there are several of them in the beam width. The percentage of longitudinal reinforcement, either continued to the supports or bent up into the compression zone, must not be more than half of the bar percentage in the cross-section having the greatest bending moment. If it is a spliced block, at least two tension bars must reach the beam support.

In continuous beams (Fig. 26), longitudinal tension reinforcement is placed at the bottom of interspans and at the top over interme-

diate supports, in accordance with the bending moment diagram (Fig. 26, *a*). The tension bars at the supports can be either in block form with effective steel at the top (Fig. 26, *a*), or in the form of a mat (Fig. 26, *c*).

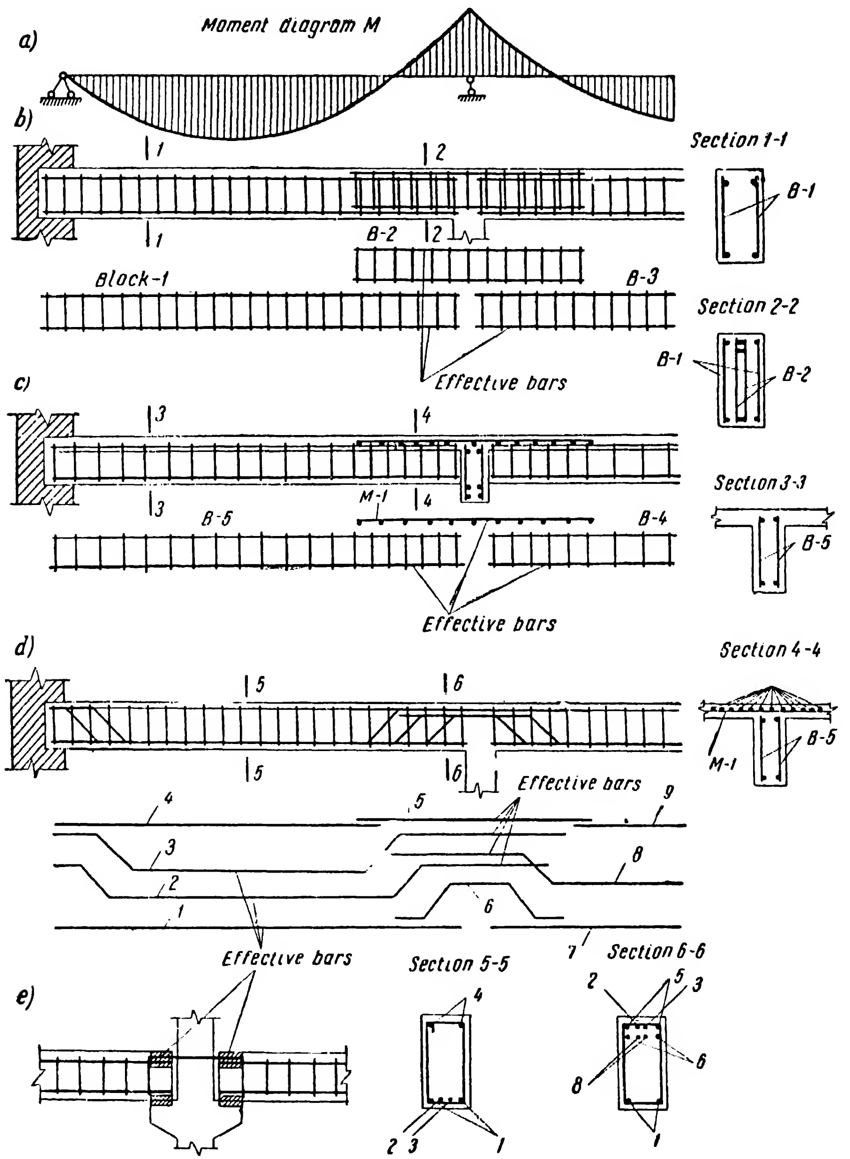

Fig. 26. Bars in continuous reinforced concrete beams

When the reinforcement blocks for continuous beam are of the tied type (Fig. 26, *d*), the upper tension bars over the supports may consist of either some of the bars from the interspan bent up at an angle of 45°, or of additional straight upper bars. In the first instance, the inclined part of the steel will bear the diagonal stresses jointly with the stirrups.

In a continuous-beam system made up of separate precast beams the upper tension reinforcement, already enclosed and protruding from each member, is furnished with welded-on steel fittings. When the connecting (jointing) bars are welded to these fittings, continuity of reinforcement is achieved at the supports.

Sec. 8. BEHAVIOUR OF A LOADED BENDING MEMBER. PHASES IN TENSION DEFORMATION

A study in the sequences of changes of stress and deformation in the cross-sections of members subject to bending (bending members) reveals the initial data needed for their design. Tests of beams have shown that they fail either along a vertical (normal), or a diagonal plane (Fig. 27). Rupture may even occur at once along both indicated planes, depending upon placement and percentage of reinforcement. The aim in the design of a member is to ensure its resistance at all planes.

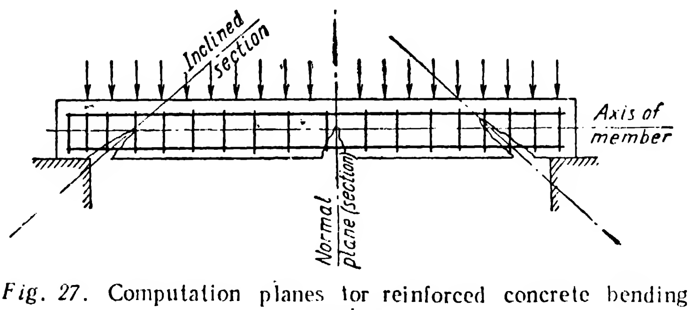

Fig. 27. Computation planes for reinforced concrete bending members

A reinforced concrete beam subject to a gradually increased load leading to failure experiences *three typical stages of stress deformation* (Fig. 28):

1 st Stage. When the beam is first loaded, the stresses in the compression and tension zones of the concrete are not intense and deformation in the section is mainly elastic. Plastic deformation is negligible and the stress-deformation diagram is a straight line. The normal stress diagram is triangular and the bars work jointly with the concrete and bear part of the tension. This condition of stress deformation is called Stage I.

As the load is further applied, stresses in the section grow. Elastic deformation develops rapidly in the tension zone, where the stress diagram acquires a definite curve and ultimate resistance is reached in the extreme fibres. Plastic deformation also becomes noticeable in the compression zone, where the diagram of normal stresses becomes somewhat curved.

This condition of stress deformation, known as Stage Ia, serves for computing fissure anticipation; its diagram of normal stresses is considered rectangular for the tension zone and triangular for the compression area.

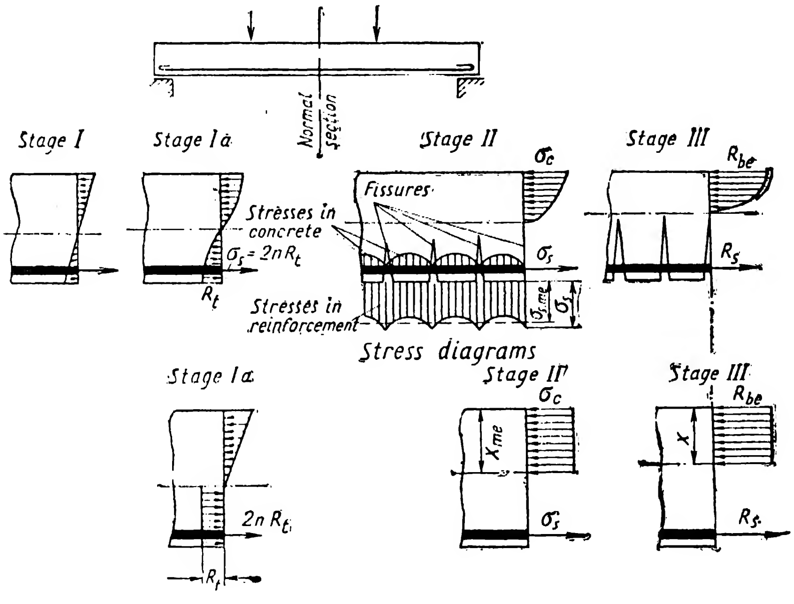

Fig. 28. Stages of stress deformation in a bending member and its stress diagrams

With a small addition to the load the concrete in the tension zone ruptures and ceases completely to resist in the immediate vicinity of the cracks, all the tension being transferred to the bars. Just prior to crack formation in the concrete, the stress in the tension reinforcement is comparatively low—about 200-300 kg/cm²—and may be computed on the basis of equal deformation of both bars and concrete. The ultimate deformation of the concrete in tension

$$\varepsilon_t = \frac{2R_t}{E_c}.$$

Hence, deformation of the bars is also

$$\varepsilon_s = \varepsilon_t = \frac{2R_t}{E_c}.$$

The stresses in the bars will be

$$\sigma_s = E_s \frac{2R_t}{E_c} = 2nR_t, \tag{18}$$

where

$$n = \frac{E_s}{E_c}.$$

2nd Stage. Subsequent stresses in the tension zone of the beam are more complicated: the steel grips the concrete between the cracks and continues to bear tension; in the concrete, tension grows with the distance from the cracks; in the bars the greatest stresses are at the cracks, diminishing with the distance from the latter. The mean stresses in the bars may be expressed in terms of σ_s (the stresses within the cracks) and a coefficient $\psi \leqslant 1$, which latter allows for tension in the concrete between the cracks:

$$\sigma_{s.me} = \psi\sigma_s.$$

Stresses increase in the compression zone, plastic deformation becomes apparent and the diagram of normal stresses becomes curved. Under continued action of the load, plastic deformation in the compression concrete increases and the curve of the diagram becomes sharper.

All this is Stage 2 and serves for computation of bending deformation and fissure widening in bending members. A rectangular diagram of normal stresses is adopted for the compression zone of the concrete.

3rd Stage. With a further increase of the load, the cracks in the tension zone open and tension in the bars reaches its limit R_s. Plastic deformation in the compression-zone concrete advances considerably, the stress diagram becomes sharply curved, and ultimate compression in bending R_s is attained. Complete failure commences.

The character of the transition from the 2nd to the 3rd (failure) Stage depends upon the amount of reinforcement. If the section is properly reinforced, failure will start in the tension zone: mild steel bars will begin to flow at yield-point stresses until the compression concrete cracks. If the bars are of hard steel, they will snap together with the crumbling of the compression-zone concrete. If the section is over-reinforced, the compression-zone concrete will fail before the bars reach their stress limit. As a rule, over-reinforced beams are neither economical nor used, inasmuch as the steel does not receive its full share of stresses.

Stage 3 serves as the basis for computing carrying capacity; its stress diagram is adopted as rectangular for the compression zone, giving only a negligib e error in calculations but simplifying the formulae.

Carrying-capacity computations of such bending members, based on the failure-stage method with due regard to the elasto-plasticity of the materials, reflect the actual behaviour of the loaded concrete.

Sec. 9. CALCULATING THE CARRYING CAPACITY OF BENDING MEMBERS ALONG NORMAL PLANES

1. Symmetrical Singly Reinforced Sections of Any Shape

We will now analyse a bending member of symmetrical cross-section (Fig. 29, *a*) affected by a bending moment M from a design load.

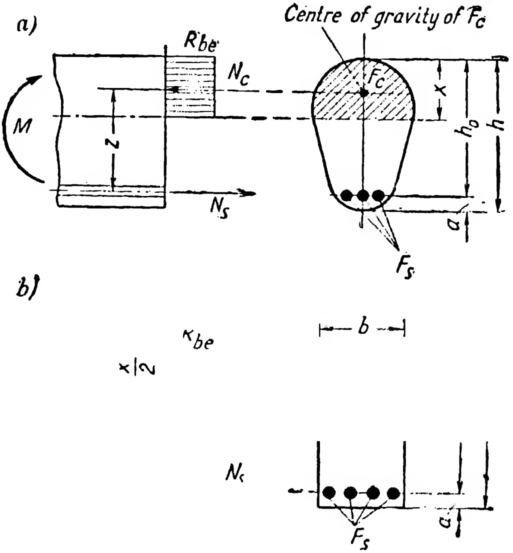

Fig. 29. Investigation of resistance in singly-reinforced members

a—any symmetrical cross section, *b*—rectangular cross-section

Notation for the section, normal to its axis, will be

h—height of section;

a—distance from the centre of gravity of tension bars to extreme fibre of tension zone. If the bars are in one tier, a will be equal to the protective covering plus ½ bar diameter. In beams, a=3 to 4 cm, and 1.5 to 2 cm in slabs;

$h_0=h-a$—effective height of section (from the compression zone's extreme fibre to the centre of gravity of tension bars);

F_c—area of concrete in compression zone;

F_s—area of tension bars;

x—height of compression zone, i.e., from extreme fibre of compression zone to neutral axis.

As already noted, carrying-capacity computations in a cross-section are executed on the basis of Stage 3; the stress diagram in the compression zone is considered as rectangular, and in-

ternal forces in the cross-section are based on design strength of the bars and concrete.

The resultant of tensile stresses in the bars N_s is the product of stresses R_s, area F_s, and the service coefficient of the bars m_s:

$$N_s = m_s R_s F_s.$$

The resultant of compressive stresses in the concrete N_c is the product of stresses R_{bc} and area F_c:

$$N_c = R_{be} F_c.$$

In accordance with the law of equilibrium, the sum of all the forces projected upon the axis of a member is equal to zero:

$$N_s - N_c = 0,$$

i.e.,

$$N_s = N_c, \quad \text{or} \quad m_s R_s F_s = R_{bc} F_c. \tag{19}$$

N_s and N_c, as two equally opposing forces, create the internal resisting couple and cross-sectional moment $M_{b.cs}$. The distance z between these forces is the arm of the resisting couple.

The value $M_{b.cs}$, borne by the bars and concrete may be expressed in two ways:

$$M_{b.cs} = mN_c z = mR_{be} F_c z, \tag{20}$$

or

$$M_{b.cs} = mN_s z = mm_s R_s F_s z, \tag{21}$$

where m—the service coefficient of the member.

$F_c z$ is the static moment of the concrete area of the compression zone in relation to the centre of gravity of the tension bars and is expressed by S_c. By introducing $S_c = F_c z$ into formula (20), we obtain

$$M_{b.cs} = mR_{bc} S_c. \tag{22}$$

Equations (19) (20), (21), and (22) are valid in all cases where ultimate resistance of both bars and concrete is reached in the failure stage. Tests have proved that this is possible when

$$S_c \leqslant 0.8 S_0, \tag{23}$$

where S_0 equals the static moment of the entire effective concrete area in the cross-section (within the range of h_0) in relation to the centre of gravity of the tension bars.

State Specifications 123-55 have adopted

$$S_c \leqslant 0.7 S_0 \tag{23a}$$

for members reinforced with prefabricated mats and blocks made of cold-drawn rods up to 5.5 mm dia., and

$$S_c \leqslant 0.6 S_0 \tag{23b}$$

for precast members computed with an m-coefficient of 1.1.

Safe carrying capacity is assured if the external bending moment M from the design load and load coefficient does not exceed the internal stress moment $M_{b.cs}$, which the cross-section is capable of bear-

ing with its given dimensions, design characteristics of material strength, and the coefficients of uniformity and service. Consequently, the conditions for safe carrying capacity, or strength, may be expressed as follows:

$$M \leqslant M_{\text{b.cs}}; \tag{24}$$

$$M \leqslant mR_{\text{be}}S_{\text{c}}; \tag{24a}$$

$$M \leqslant mm_{\text{s}}R_{\text{s}}F_{\text{s}}z. \tag{24b}$$

2. Rectangular Singly Reinforced Cross-Sections

For a rectangular cross-section with b width (Fig. 29, b), the area of the concrete's compression zone

$$F_{\text{c}}=bx$$

and the resultant of compression stresses in the concrete

$$N_{\text{c}}=R_{\text{be}}bx.$$

The resultant of tensile stresses in the bars will be, as before,

$$N_{\text{s}}=m_{\text{s}}R_{\text{s}}F_{\text{s}}.$$

Inasmuch as the resultant of compressive stresses is applied to the centre of gravity of the rectangular stress diagram, the arm of the internal resisting couple

$$z=h_0-\frac{x}{2}.$$

The internal forces N_{c} and N_{s} will form the following equation:

$$m_{\text{s}}R_{\text{s}}F_{\text{s}}=R_{\text{be}}bx. \tag{25}$$

From equation (25) we may determine the height of the compression zone x:

$$x=\frac{m_{\text{s}}R_{\text{s}}F_{\text{s}}}{R_{\text{be}}b}. \tag{26}$$

For a rectangular section we likewise derive

$$S_{\text{c}}=F_{\text{c}}z=bx\left(h_0-\frac{x}{2}\right)$$

wherefore conditions (24a) and (24b) for strength take the form of

$$M \leqslant mR_{\text{be}}\,bx\left(h_0-\frac{x}{2}\right) \tag{27}$$

or

$$M \leqslant mm_{\text{s}}R_{\text{s}}F_{\text{s}}\left(h_0-\frac{x}{2}\right). \tag{27a}$$

The static moment for the entire cross-section

$$S_0=bh_0\frac{h_0}{2}=0.5bh_0^2,$$

whence condition (23) takes the form of

$$bx\,(h_0 - 0.5x) \leqslant 0.4bh_0^2.$$

By dividing both members of the inequality by bh_0^2 and designating

$$\alpha = \frac{x}{h_0}, \tag{28}$$

we get $\alpha - 0.5\,\alpha^2 \leqslant 0.4$, from which $\alpha \leqslant 0.55$.

The coefficient α defines the height of the compression zone x, hence also the quantity of reinforcement, inasmuch as according to formula (26) the value x increases with an increase of the value F_s.

Thus, for a rectangular cross-section, bar and concrete stresses in the failure stage reach their ultimate resistance only when the height of the compression zone satisfies the following conditions:

$$\text{when } S_c \leqslant 0.6S_0 \quad x \leqslant 0.37h_0 \quad (\alpha_{ult} = 0.37) \tag{29}$$

$$\text{,, } S_c \leqslant 0.8S_0 \quad x \leqslant 0.55h_0 \quad (\alpha_{ult} = 0.55) \tag{29a}$$

$$\text{,, } S_c \leqslant 0.7S_0 \quad x \leqslant 0.45h_0 \quad (\alpha_{ult} = 0.45) \tag{29b}$$

Formulae (25), (27) and (27a) are valid if the height of the compression zone x is limited in accordance with condition (29).

The relation of the reinforcement area F_s to the whole effective area of the cross-section bh_0 is known as the coefficient of reinforcement μ:

$$\mu = \frac{F_s}{bh_0}. \tag{30}$$

The coefficient of reinforcement multiplied by 100 is called the percentage of reinforcement:

$$\mu\% = \frac{100F_s}{bh_0}. \tag{31}$$

By dividing both members of equation (26) by h_0, we get

$$\frac{x}{h_0} = \frac{F_s}{bh_0} \times \frac{m_s R_s}{R_{be}} \tag{32}$$

or

$$\alpha = \mu \frac{m_s R_s}{R_{be}}. \tag{32a}$$

The formula for the coefficient of reinforcement may be derived from expression (32a):

$$\mu = \alpha \frac{R_{be}}{m_s R_s}, \tag{33}$$

and the formula for percentage of reinforcement will then be

$$\mu\% = 100\alpha \frac{R_{be}}{m_s R_s}. \tag{34}$$

The ultimate percentage of reinforcement is derived from formula (34) when α_{ult} is determined by conditions (29)-(29b). Thus, for a rectangular cross-section

$$\mu_{ult}\% = 55 \frac{R_{be}}{m_s R_s}.$$

As already stated, at a percentage of reinforcement exceeding the ultimate, failure will be due to crumbling of the compression-zone concrete before the bars are stressed to their full capacity.

In such a case the carrying capacity of the section is determined by condition (22):

when $$S_c > 0.8S_0 = 0.4bh_0^2$$

$$M_{b.cs} = 0.4mR_{be}bh_0^2; \tag{35}$$

when $$S_c > 0.7S_0 = 0.35bh_0^2$$

$$M_{b.cs} = 0.35mR_{be}bh_0^2; \tag{35a}$$

when $$S_c > 0.6S_0 = 0.3bh_0^2$$

$$M_{b.cs} = 0.3mR_{be}bh_0^2. \tag{35b}$$

Required carrying capacity of the beam M may be achieved, as seen from equation (27a), through various values of F_s and h_0: if F_s increases, h_0 will be lessened, and vice versa. When F_s and h_0 alter, there is also a change in reinforcement percentage. Other things being equal, the cost of reinforced concrete is determined by the cost of the concrete and bars and therefore depends upon the percentage of reinforcement or the coefficient α, which latter is directly proportional to the reinforcement percentage in accordance with equation (32a).

An economic percentage of reinforcement with ordinary grades of concrete and steel comes to 1-2% for beams, and 0.3-0.6% for slabs. Economic values of α are usually 0.3-0.4%. Hence an economic percentage is, as a rule, less than the ultimate. In individual cases, when structural space limits the dimensions of the cross-section, either the ultimate percentage of steel is used (ultimate α), or double reinforcement is incorporated.

Structural considerations will dictate what the minimum reinforcement percentage can be, since the bars must take a number of uncomputed forces (concrete shrinkage, temperature changes, etc). The level of this minimum (from 0.1 to 0.25%) depends upon concrete and steel grades and is established by building code (Table 9).

Table 9

Minimum Percentage of Reinforcement

Kinds of bars	μ% for the following grades of concrete			
	100-150	200	300-400	500-600
St-0 and St-3 steel	0.1	0.15	0.2	0.25
Hot-rolled intermittently deformed bars of St-5 and 25Г2C steel; cold-notched bars; prefabricated mats and blocks of cold-drawn rods . .	0.1	0.1	0.15	0.2

3. Use of Tables in the Computation of Rectangular Sections

Formulae (25) and (27) are for direct use only when checking the carrying capacity of a given sectional plane. In most instances the cross-section of the member and its bars must be computed from a known design moment and given grades of concrete and steel. This operation is called *determining the cross-section* and is done with the aid of tables.

For this purpose let us transform the principal formulae.

From condition (28) the compression-zone height

$$x=\alpha h_0.$$

Solving for x under strength condition (27) we have

$$M \leqslant mR_{be}\, b\alpha h_0 \left(h_0 - \frac{\alpha h_0}{2}\right).$$

Removing h_0 from the brackets, we have

$$M \leqslant mR_{be} b h_0^2 \alpha \left(1 - \frac{\alpha}{2}\right).$$

Designating $\alpha\left(1-\frac{\alpha}{2}\right)=A_0$, the final formula takes the form of

$$M \leqslant mR_{be} b h_0^2 A_0, \tag{36}$$

whence

$$A_0=\frac{M}{mR_{be} b h_0^2}. \tag{37}$$

The arm of the internal resisting couple z may be also expressed through α:

$$z=h_0-\frac{x}{2}=h_0-\frac{\alpha h_0}{2}=h_0\left(1-\frac{\alpha}{2}\right).$$

After designating

$$1-\frac{\alpha}{2}=\gamma_0,$$

we get

$$z=\gamma_0 h_0. \tag{38}$$

From formula (36), the effective cross-sectional height becomes

$$h_0=\sqrt{\frac{M}{mR_{be} b A_0}}.$$

After designating

$$\sqrt{\frac{1}{A_0}}=r_0,$$

we finally get

$$h_0=r_0\sqrt{\frac{M}{mR_{be} b}}. \tag{39}$$

The values of the coefficients A_0, γ_0 and r_0, computed for various ratings of α, are presented in Table 10.

Table 10

Table for Computation of Rectangular and T- and I-Sections for All Grades of Concrete and Steel

α	r_0	γ_0	A_0	α	r_0	γ_0	A_0
0.01	10	0.995	0.01	0.29	2.01	0.855	0.248
0.02	7.12	0.99	0.02	0.3	1.98	0.85	0.255
0.03	5.82	0.985	0.03	0.31	1.95	0.845	0.262
0.04	5.05	0.98	0.039	0.32	1.93	0.84	0.269
0.05	4.53	0.975	0.048	0.33	1.9	0.835	0.275
0.06	4.15	0.97	0.058	0.34	1.88	0.83	0.282
0.07	3.85	0.965	0.067	0.35	1.86	0.825	0.289
0.08	3.61	0.96	0.077	0.36	1.84	0.82	0.295
0.09	3.41	0.955	0.085	0.37	1.82	0.815	0.301
0.1	3.24	0.95	0.095				
0.11	3.11	0.945	0.104	0.38	1.8	0.81	0.309
0.12	2.98	0.94	0.113	0.39	1.78	0.805	0.314
0.13	2.88	0.935	0.121	0.4	1.77	0.8	0.32
0.14	2.77	0.93	0.13	0.41	1.75	0.795	0.326
0.15	2.68	0.925	0.139	0.42	1.74	0.79	0.332
0.16	2.61	0.92	0.147	0.43	1.72	0.785	0.337
0.17	2.53	0.915	0.155	0.44	1.71	0.78	0.343
0.18	2.47	0.91	0.164	0.45	1.69	0.775	0.349
0.19	2.41	0.905	0.172				
0.2	2.36	0.9	0.18	0.46	1.68	0.77	0.354
0.21	2.21	0.895	0.188	0.47	1.67	0.765	0.359
0.22	2.26	0.89	0.196	0.48	1.66	0.76	0.365
0.23	2.22	0.885	0.203	0.49	1.64	0.755	0.37
0.24	2.18	0.88	0.211	0.5	1.63	0.75	0.375
0.25	2.14	0.875	0.219	0.51	1.62	0.745	0.38
0.26	2.1	0.87	0.226	0.52	1.61	0.74	0.385
0.27	2.07	0.865	0.234	0.53	1.6	0.735	0.39
0.28	2.04	0.86	0.241	0.54	1.59	0.73	0.394
				0.55	1.58	0.724	0.4

The area of reinforcement F_s is determined by condition (24b) or (27a), but instead of the expression for the arm of the internal resisting couple,

$$z=h_0-\frac{x}{2},$$

we take the value

$$z=\gamma_0 h_0,$$

in other words,

$$F_s=\frac{M}{mm_s R_s \gamma_0 h_0}. \tag{40}$$

Another expression for F_s may be obtained from equation (32) by considering that $\frac{x}{h_0}=\alpha$:

$$F_s=\alpha b h_0 \frac{R_{bc}}{m_s R_s}. \tag{41}$$

If the design moment and grades of concrete and steel are given when determining the cross-section by table, two types of problems are met with:

1st type: b and h of the section are known, F_s must be found.

Sequence of procedure.

1) A_0 is computed by formula (37).
2) With A_0 known, then γ_0 or α is taken from Table 10.
3) F_s is computed by formula (40) or (41).

2nd type: neither b nor h are known, nor the area of the bars F_s; all must be determined.

Sequence of procedure.

1) The cross-sectional width b and the coefficient $\alpha=0.3$-0.4 are assumed.
2) α in Table 10 determines r_0.
3) h_0 is found by formula (39) and the section's dimensions corrected according to the recommended ratio $b:h$ and height gradations.
4) F_s is computed as in the 1st type.

If the carrying capacity of the section must be found when b, h and F_s are given, it is best to use the principal equations adopted when verifying the stresses in the section. When such is the case, the height of the compression zone x is first determined by formula (26), after which the moment of the given sectional plane is found by formula (27) or (27a).

The carrying capacity of a given plane may also be decided by table as follows:

1) Determine $\alpha=x/h_0$ by formula (32).
2) With α known, find A_0 by table 10.
3) Compute the moment M of the section by formula (36).

Illustrative problem 1. Given: $M=14.3$ t/m for the bending moment in a beam from the design load, 200-B grade of concrete,* bars of hot-rolled intermittently deformed St-5 steel. $m=1$ as the coefficient of service, cross-section $b=20$ cm, $h=50$ cm. Determine F_s, the area of the bars.

Solution.

Gathering design data: Table 6 gives $R_{be}=100$ kg/cm², Table 8 gives $R_s=3{,}400$ kg/cm², $m_s=1$, $h_0=50-3.5=46.5$.

Formula (37) determines A_0:

$$A_0=\frac{1{,}430{,}000}{1\times100\times20\times46.5^2}=0.33.$$

With A_0 known, Table 10 gives $\gamma_0=0.792$** or $\alpha=0.416$.

Computing by formula (40) we find that

$$F_s=\frac{1{,}430{,}000}{1\times1\times2{,}400\times0.792\times46.5}=16.15\ \text{cm}^2$$

* Fabricating methods are represented throughout by the letters A or B together with the grade of concrete.

** Linear interpolation is used with all tables.

or by formula (41)

$$F_s=0.416\times 20\times 46.5\ \frac{100}{1\times 2{,}400}=16.15\ \text{cm}^2,$$

which is satisfied by $2\phi 25$ D $+2\phi 22$ D ($F_s=17.42$ cm²).*

Illustrative problem 2. Given: $M=300$ kg/m as the bending moment from the design load in a slab, 150-A grade of concrete, reinforcement of prefabricated mats of cold-drawn low-carbon rods with maximum diameter 5.5 mm, $m=1.25$ as the service coefficient; cross-section: $b=100$ cm, $h=8$ cm. Determine F_s, the area of reinforcement.

Solution.

Gathering design data: $R_{be}=85$ kg/cm², $R_s=4{,}500$ kg/cm², $m_s=0.65$, $h_0=8-$ $-1.5=6.5$ cm.

A_0 is found by formula (37):

$$A_0=\frac{30{,}000}{1.25\times 85\times 100\times 6.5^2}=0.067.$$

With A_0 known, Table 10 gives $\gamma_0=0.965$, and from formula (40) we calculate

$$F_s=\frac{30{,}000}{1.25\times 0.65\times 4{,}500\times 0.965\times 6.5}=1.32\ \text{cm}^2,$$

which is satisfied by $7\phi 5$ C ($F_s=1.37$ cm²).

Illustrative problem 3. Given: $M=8{,}000$ kg/m as the bending moment from the beam's design load, 200-B grade of concrete, bars of St-5 steel, hot-rolled intermittently deformed, $m=1$ as the service coefficient. Determine: cross-section $b\times h$ and bar area F_s.

Solution.

Gathering design data: $R_{be}=100$ kg/cm², $R_s=2{,}400$ kg/cm², $m_s=1$.

Assume $b=20$ cm (width of the beam) and coefficient $\alpha=0.3$; then with α known, Table 10 gives $r_0=1.98$.

Formula (39) determines that

$$h_0=1.98\sqrt{\frac{800{,}000}{1\times 100\times 20}}=40\ \text{cm}.$$

Height $h=h_0+a=40+3=43$ cm. We finally fix $h=45$ cm, in which case $h_0=45-3=42$ cm. Furthermore, formula (37) gives

$$A_0=\frac{800{,}000}{1\times 100\times 20\times 42^2}=0.227.$$

With A_0 known, Table 10 gives $\gamma_0=0.87$, or $\alpha=0.261$. From formula (40) we compute

$$F_s=\frac{800{,}000}{1\times 1\times 2{,}400\times 0.87\times 42}=9.15\ \text{cm}^2,$$

or from formula (41)

$$F_s=0.261\times 20\times 42\ \frac{100}{1\times 2{,}400}=9.15\ \text{cm}^2,$$

which is satisfied by $2\phi 18$ D $+2\phi 16$ D ($F_s=9.11$ cm²).

Illustrative problem 4. Determine: carrying capacity of a reinforced concrete beam. Given: a rectangular cross-section: $b=20$ cm, $h=40$ cm, grade of concrete 150-B; bars of round hot-rolled St-3 steel, $F_s=8.04$ cm² ($4\phi 16$), $m=1$ as the service coefficient.

* See Supplement 11.

Solution.

Gathering design data: $R_{bc}=80$ kg/cm², $R_s=2{,}100$ kg/cm², $m_s=1$, $h_0=40-$ $-3=37$ cm.

Formula (26) decides the height of compression zone:

$$\frac{1\times 2{,}100\times 8.04}{80\times 20}=10.6 \text{ cm}.$$

The area of the compression zone $F_c=20\times 10.6=212$ cm².

Formula (20) determines carrying capacity of the cross-section:

$$M_{b\cdot cs}=1\times 80\times 212\left(37-\frac{10.6}{2}\right)=536{,}000 \text{ kg/cm}=5.36 \text{ t/m},$$

or by formula (21),

$$M_{b\cdot cs}=1\times 1\times 2{,}100\times 8.04\left(37-\frac{10.6}{2}\right)=536{,}000 \text{ kg/cm}=5.36 \text{ t/m}.$$

The problem may also be solved through the use of Table 10: formula (32) determines that

$$\alpha\frac{x}{h_0}=\frac{8.04\times 1\times 2{,}400}{20\times 37\times 80}=0.285;$$

Table 10 shows that if $\alpha=0.285$, then $A_0=0.244$, and calculation by formula (36) gives:

$$M_{cs}=1\times 80\times 20\times 37^2\times 0.244=536{,}000 \text{ kg/cm}=5.36 \text{ t/m}.$$

4. Rectangular Beams with Double Reinforcement

If the compression zone exceeds the ultimate height when the cross-section, grade of concrete, and bending moment are all given, i.e., when $\alpha=\frac{x}{h_0}>0.55$ (or correspondingly when $\alpha>0.45$ or $\alpha>0.37$), and if it is not possible nor practical to increase the sectional dimensions or raise the concrete's grade, then the compression zone must be strengthened with compression bars (Fig. 30). This use of double reinforcement, i.e., F_s and F'_s, as a rule, is not economical since it means a high steel expenditure, therefore its use must be well founded. For instance, a greater cross-section in a precast member may be restricted by the hoisting capacity of the crane; or it may be necessary to impose varying loads on beams having similar shapes. Bars may also be required in the compression zone either because of inverse bending-moment signs, or because of structural needs (such as over intermediate supports in continuous beams).

Fig. 30,*a* illustrates stress conditions in a rectangular cross-section.

The resultant of compressive stresses in the concrete will be

$$N_c=R_{bc}bx,$$

the resultant of compressive stresses in the F'_s bars will amount to

$$N'_s=m_sR_sF'_s,$$

and the resultant of tensile stresses in the regular bars will be

$$N'_s = m_s R_s F_s.$$

Two equations can be formulated to compute the carrying capacity of the section:

1. If all forces are projected on the axis of the member so that the sum is zero (to satisfy the state of equilibrium), an equation will

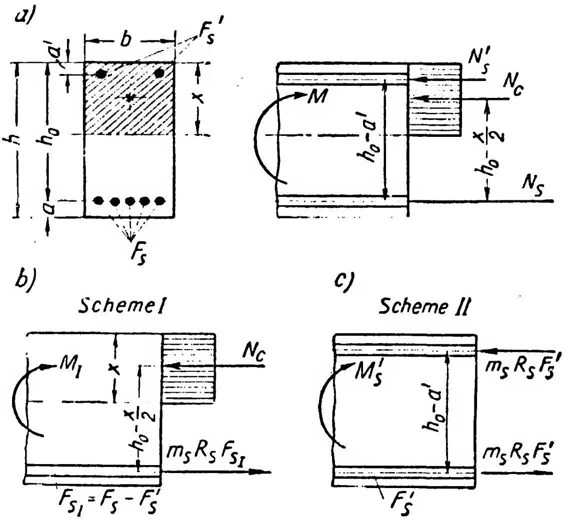

Fig. 30. Investigation of resistance in members having double reinforcement

be derived to determine the position of the neutral axis, i.e., to find x:

$$m_s R_s F_s - m_s R_s F'_s - R_{be} bx = 0. \tag{42}$$

2. The condition for resistance of the cross-section can be obtained by summing up the moments of all forces in relation to the centre of gravity of the tension bars; this means that the external bending moment from the design load does not exceed the moment of internal forces:

$$M \leqslant m \left[R_{be} bx \left(h_0 - \frac{x}{2}\right) + m_s R_s F'_s (h_0 - a')\right]. \tag{43}$$

In order that ultimate resistance be attained in the compression-zone concrete and in the tension bars during the failure stage, the height of the compression zone—just as with single reinforcement—is limited, i.e.

$$x \leqslant 0.55 h_0 \text{ (or } 0.45 h_0, \text{ or } 0.37 h_0).$$

In order that the stresses in the compression bars F'_s reach their ultimate value in the failure stage, the minimum height of the compression zone must also be limited:

$$x \geqslant 2a'. \tag{43a}$$

For convenience in tabular computation for singly reinforced sections, the conditions of stress shown in Fig. 30, *a* are divided into two schemes (Fig. 30, *b* and *c*).

Scheme I. Compression is borne by the concrete, and tension by part of the tension bars whose area $F_{sI}=F_s - F'_s$. This scheme corresponds to stress conditions of a singly reinforced section and embraces already stated height limits of the compression zone.

The moment of the section in Scheme I is expressed as M_I and can be computed by formulae and tables for rectangular sections with single reinforcement, as follows:

$$M_I=mR_{be}bx\left(h_0-\frac{x}{2}\right) \text{ or } M_I=mR_{be}bh_0^2A_0.$$

Scheme II. Compression is borne by the bars F'_s and tension by the rest of the steel whose area is equal to that of F'_s (inasmuch as $F_s - F_{sI}=F'_s$).

The internal resisting couple created in Scheme II is capable of taking the external bending moment M'_s. The magnitude of stresses in the bars equals $m_sR_sF'_s$, and the arm of the resisting couple amounts to h^0-a'. Hence if the coefficient of service is included, the bending moment will be

$$M'_s=mm_sR_sF'_s\,(h_0-a'). \tag{44}$$

Thus the entire bending moment M in a doubly reinforced section can be presented as the sum of two moments:

$$M=M_I+M'_s. \tag{45}$$

With the moment and the grades of concrete and steel known, two types of problems present themselves in determining a doubly-reinforced cross-section.

1st type. Given: $b \times h$. To be determined: the area of reinforcement F_s and F'_s.

Sequence of procedure.

1) A_0 is found by formula (37) to decide whether compression bars are needed. If $A_0>A_{0\,ult}$, compression bars F'_s are required.

2) Calculations are made of the cross-sectional moment according to Scheme I (Fig. 30) with the full use of the compression concrete

$$M_I=R_{be}bh_0^2A_{0\,ult} \tag{46}$$

and of the corresponding area of tension bars F_{sI} by formula (40) or (41) and at an ultimate value of γ_0 or α.

3) The cross-sectional moment M'_s is determined according to Scheme II (Fig. 30):

$$M'_s = M - M_I. \tag{47}$$

4) Bar area F'_s is found by equation (44):

$$F'_s = \frac{M'_s}{mm_s R_s (h_0 - a')}. \tag{48}$$

5) Full tension-bar area F_s is calculated:

$$F_s = F_{sI} + F'_s. \tag{49}$$

2nd type. Given: cross-section $b \times h$, and compression bars F''_s. To be found: area of bars F_s.

Sequence of procedure.

1) Formula (44) solves M'_s.

2) The term $M_I = M - M'_s$ is computed.

3) The magnitude of $A_0 = \frac{M_I}{mR_{be} bh_0^2}$ in Table 10 will give a corresponding value of γ_0 or α, after which the bar area F_{sI} is determined by formula (40) or (41).

4) Formula (49) will reveal the full tension-bar area.

If the known quantities are b, h, F''_s, and grades of material, then to find the section's carrying capacity use is made of the principal equations; x is found by equation (42), and the value of M_{cs} is given by equation (43).

Illustrative problem 5.

Given: $b=25$ cm, $h=50$ cm, $M=26$ t/m, concrete is grade 200-B, bars are intermittently deformed of St-5 steel, $m=1$. To be determined: F_s and F'_s.

Solution.

Gathering design data: $R_{be}=100$ kg/cm², $R_s=2{,}400$ kg/cm², $m_s=1$.
It is assumed that $a=4$ cm and $a'=3$ cm. Then $h_0=50-4=46$ cm.
According to formula (37) it is found that

$$A_0 = \frac{2{,}600{,}000}{1 \times 100 \times 25 \times 46^2} = 0.49 > A_{0\ \mathrm{ult}} = 0.4,$$

hence a singly reinforced cross-section is insufficient and reinforcement in the compression zone is necessary.

According to formula (46),

$$M_I = 1 \times 100 \times 25 \times 46^2 \times 0.4 = 2{,}120{,}000 \text{ kg/cm} = 21.2 \text{ t/m}.$$

When $A_{0\ \mathrm{ult}}=0.4$, it is found by Table 10 that $\gamma_{0\ \mathrm{ult}}=0.725$. Then formula (40) finds that

$$F_{sI} = \frac{2{,}120{,}000}{1 \times 1 \times 2{,}400 \times 0.725 \times 46} = 26.5 \text{ cm}^2,$$

formula (47) gives

$$M'_s = 26 - 21.2 = 4.8 \text{ t/m},$$

formula (48) gives

$$F'_s=\frac{480{,}000}{1\times 1\times 2{,}400\,(46-3)}=4.65\ \text{cm}^2,$$

and from formula (49) we derive

$$F_s=26.5+4.65=31.15\ \text{cm}^2,$$

which is satisfied by 3ϕ14 D for the compression bars (F'_s=4.62 cm²) and 5ϕ28 D for tension bars (F_s=30.79 cm²).

Illustrative problem 6.

Given: the same as in illustrative problem 5, but with the following added data: compression bars, F'_s=9.41 cm² (3ϕ20 D). To be determined: bar area F_s.

Solution.

From formula (44) we derive

$$M'_s=1\times 1\times 2{,}400\times 9.41\,(46-3)=970{,}000\ \text{kg/cm}=9.7\ \text{t/m}.$$

$$M_1=M-M'_s=26-9.7=16.3\ \text{t/m}.$$

$$A_0=\frac{1{,}630{,}000}{1\times 100\times 25\times 46^2}=0.308.$$

It is seen from Table 10 that if A_0=0.308, then γ_0=0.81 and α=0.38.

$$F_s=F_{s1}+F'_s=\frac{1{,}630{,}000}{1\times 1\times 2{,}400\times 0.81\times 46}+9.41=27.71\ \text{cm}^2,$$

which is satisfied by 6ϕ25 D (F_s=29.45 cm²).

Condition (43a) is then verified:

$$x=\alpha h_0=0.38\times 46=17.4>2a'=6\ \text{cm}.$$

5. T-Beams

T-beams are widely used in reinforced concrete and usually adopted for overhead-crane girders, ribs of precast slabs, beams of in-situ floors, etc. Fig. 31,*a* shows the flange and rib of such a shape, the flange most often occupying the compression zone but sometimes taking tensile stresses as in section 2-2, Fig. 31.

Calculations of external forces and carrying capacities of T-beams whose flanges occupy only the compression zone will now be considered. Tension flanges add no strength in comparison to rectangular $b\times h$ shapes, inasmuch as concrete in tension takes no load in Stage III.

If the compression flange is very wide, its parts furthest from the web will be less stressed than those that are closer. For this reason the effective width of the flange is limited by building code. For instance, the effective width of the flange b_{fl} in self-contained beams, precast slabs, floors, etc., must not exceed $\frac{1}{3}$ of the span nor be more than 12 $h_{fl}+b$. If the slab portion (the flange) of the member has an

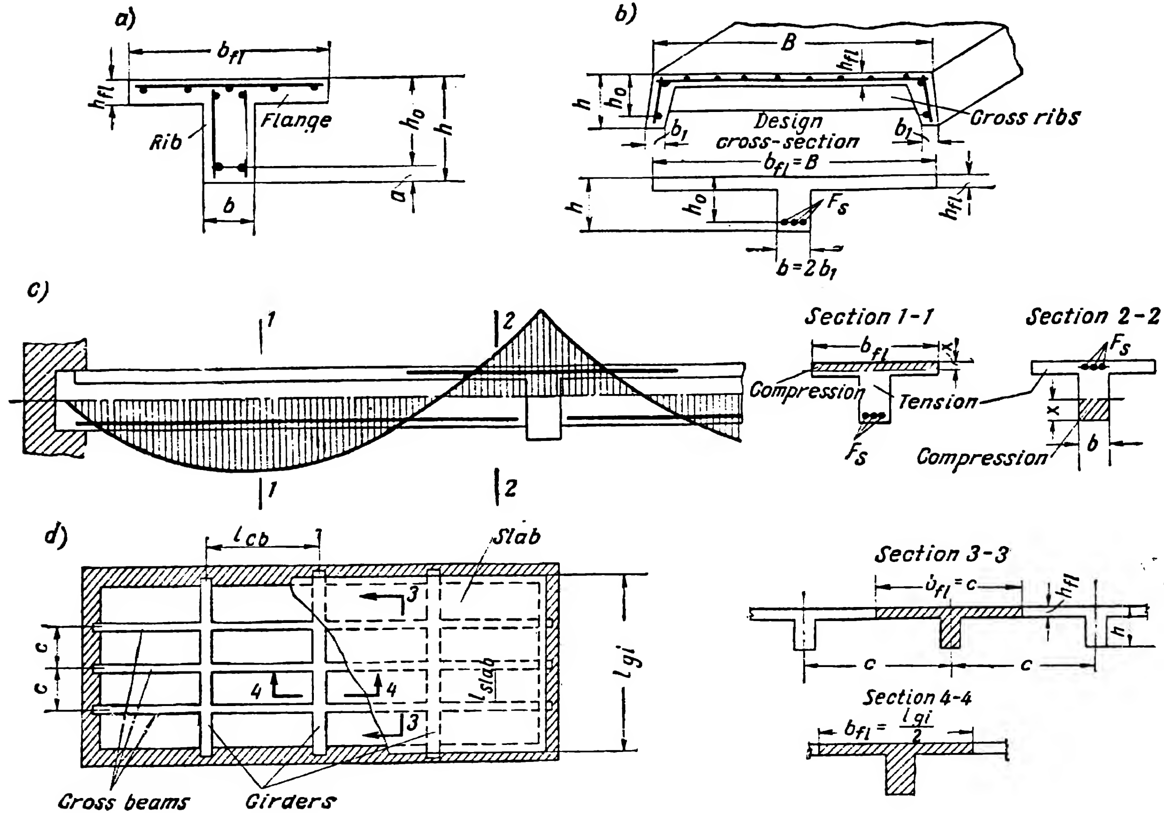

Fig. 31. Reinforced concrete T-beams

intermediate lateral rib (Fig. 31,*b*), then its actual width may be considered as the design width

When $\frac{h_{fl}}{h} \geqslant 0.1$ in in-situ ribbed floors (Fig. 31,*d*), centre-to-centre spacings are taken as the actual flange widths for cross beams, and the distance midway between centres taken for girders ($\frac{l}{4}$ on each side of the web centre) If $\frac{h_{fl}}{h} < 0.1$ in a thin slab, then the effective width must be limited to $12h + b$.

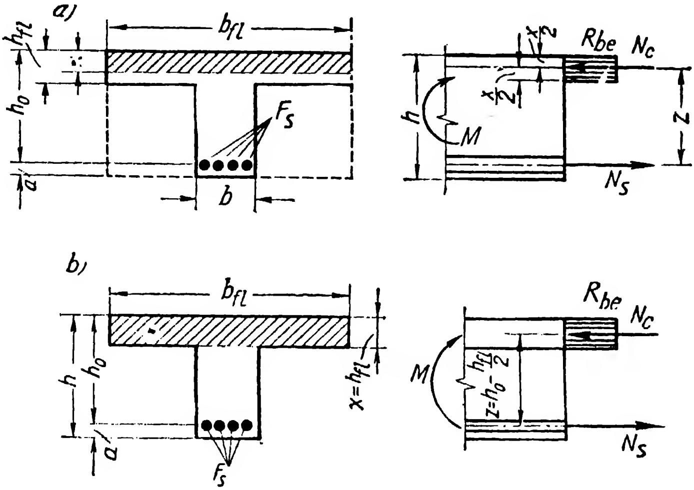

Fig. 32. Investigation of resistance in T-beams (Case I)

Two cases are met with in T-beam computations. In the 1st case the neutral axis is within the flange, in the 2nd case it bisects the web below the flange.

Case I. When the neutral axis is within the flange (i.e., when $x \leqslant h_{fl}$); stress conditions are shown in Fig. 32,*a*.

The resultant of compressive stresses in the concrete will be

$$N_c = R_{be} b_{fl} x,$$

and the resultant of tensile stresses in the bars will amount to

$$N_s = m_s R_s F_s.$$

The first condition for the state of equilibrium is

$$N_c = N_s \text{ or } R_{be} b_{fl} x = m_s R_s F_s. \tag{50}$$

The height of the compression zone x is found through equation (50). The arm of the internal resisting couple

$$z = h_0 - \frac{x}{2}.$$

The condition for resistance of the cross-section (taking a service coefficient into account) may be expressed as

$$M \leqslant mR_{bc}b_{fl}x\left(h_0 - \frac{x}{2}\right), \tag{51}$$

or as

$$M \leqslant mm_sR_sF_s\left(h_0 - \frac{x}{2}\right). \tag{51a}$$

Condition (51) differs from the analogous condition (27) for a rectangular section in that b is substituted by b_{fl} in the right-hand member, since concrete in tension is no aid to carrying capacity.

Hence, a T-beam whose neutral axis lies within the flange is computed as an ordinary rectangular section with a width of b_{fl} and height h (shown as a dotted line in Fig. 32,*a*). Calculations for such beams are carried out with the aid of tables and formulae already presented for rectangular beams.

A close approximation can be had of the reinforcement area F_s if the relationship in the T-section is $\frac{h_{fl}}{h} \leqslant 0.2$ (since the arm of the internal resisting couple, $z = h_0 - \frac{x}{2}$, is little effected whether the neutral axis is at the lower fibre of the flange or somewhat higher within the flange), for which reason it may be assumed that $x = h_{fl}$ (Fig. 32,*b*).

Then

$$z = h_0 - \frac{h_{fl}}{2},$$

and the condition for resistance (51a) can be expressed as

$$M \leqslant mm_sR_sF_s\left(h_0 - \frac{h_{fl}}{2}\right)$$

whence

$$F_s = \frac{M}{mm_sR_s\left(h_0 - \frac{h_{fl}}{2}\right)}. \tag{52}$$

Formula (52) gives a somewhat excessive value of F_s (about 5-10%).

If the neutral axis in a T-beam aligns with the lower edge of the flange, an examination of the stresses will reveal to which case it belongs.

The resultant of compression stresses in the concrete will be

$$N_c = R_{bc}b_{fl}h_{fl},$$

the arm of the internal resisting couple will be

$$z = h_0 - \frac{h_{fl}}{2},$$

and carrying capacity, expressed by the force N_c, can be written as follows:

$$M_{b.cs} = mN_cz = mR_{bc}b_{fl}h_{fl}\left(h_0 - \frac{h_{fl}}{2}\right). \tag{53}$$

If the bending moment from the design load is $M \leqslant M_{b\cdot cs}$, then $x \leqslant h_{fl}$ and the problem applies to Case 1.

But if $M > M_{b.cs}$, then $x > h_{fl}$ and the computations will concern Case 2.

Experience has shown that, as a rule, with a relative increase of the flange it will embrace the neutral axis, whereas a narrow flange and large bending moment will drop the neutral axis so that it will cross the web.

Case 2. When the neutral axis is below the flange (crossing the web), i.e., when $x > h_{fl}$. Here the stress diagram is shown in Fig. 33,*a*.

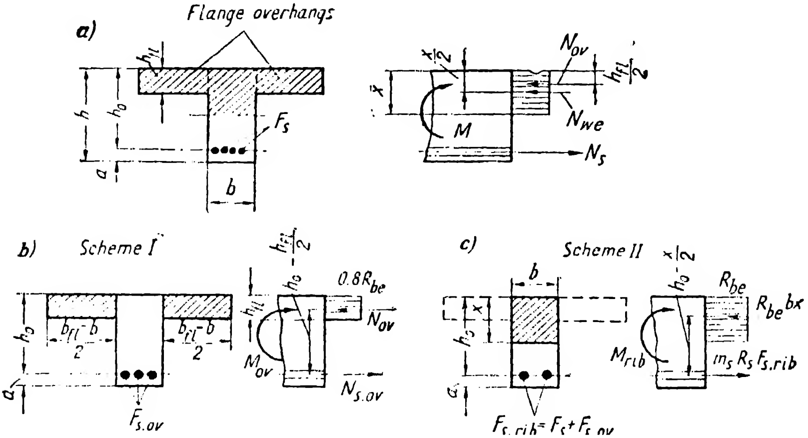

Fig. 33. Investigation of resistance in T-beams (Case 2)

Failure stage will find the stresses in the compression zone equal to R_{be}. But in the overhangs of the flange, compressed throughout their plane, the stresses will be somewhat less and can be assumed as $R_{pr} = 0.8\ R_{be}$, just as in an axially compressed member.

The resultant of compression stresses in the flange overhangs will be

$$N_{ov} = 0.8 R_{be} (b_{fl} - b)\ h_{fl}, \tag{54}$$

the resultant of compressive stresses in the web will equal

$$N_{we} = R_{be} bx,$$

and the resultant of tensile stresses in the bars will be

$$N_s = m_s R_s F_s.$$

By projecting all forces upon the axis and equating their sum to satisfy zero equilibrium, we obtain

$$N_s - N_{ov} - N_{we} = 0$$

or

$$R_{bc}[0.8(b_{fl}-b)h_{fl}+bx]=m_sR_sF_s. \tag{55}$$

The height of the compression zone x is derived from equation (55).

If we formulate the moment of internal forces in relation to the centre of gravity of tension-bar area and equate it with the design load's bending moment, the condition for resistance in the cross-section will be

$$M \leq m\left[0.8R_{bc}(b_{fl}-b)h_{fl}\left(h_0-\frac{h_{fl}}{2}\right)+R_{bc}bx\left(h_0-\frac{x}{2}\right)\right]. \tag{56}$$

Equations (55) and (56) allow verification of the resistance of T-sections when $x>h_{fl}$.

In order that concrete and bar stresses attain their ultimate in the failure stage just as in the case of any other symmetrical cross-section, condition (23) must be observed.

In determining the cross-section, it is convenient to divide the scheme into two parts, as in Fig. 33,*b* and *c*.

In Scheme I, compression is borne by the flange overhangs, and tension by part of the reinforcement which may be denoted as $F_{s.ov}$. The section in this scheme bears only part of the bending moment, to be known as M_{ov}.

In Scheme II the rib acts as a rectangular beam whose compression-zone height is x and whose bars $F_{s.rib}=F_s-F_{s.ov}$; this rib takes the rest of the bending moment, to be known as M_{rib}.

Then,

$$M=M_{ov}+M_{rib}. \tag{57}$$

Computing by Scheme I. The resultant of compressive forces in the concrete of the flange overhang N_{ov} is found by equation (54).

The arm of the internal resisting couple

$$z=h_0-\frac{h_{fl}}{2}.$$

The moment borne by the section in Scheme I will be

$$M_{ov}=m\left[0.8R_{bc}(b_{fl}-b)h_{fl}\left(h_0-\frac{h_{fl}}{2}\right)\right], \tag{58}$$

and the corresponding bar area will be

$$F_{s.ov}=\frac{M_{ov}}{mm_sR_s\left(h_0-\frac{h_{fl}}{2}\right)}. \tag{59}$$

Computing by Scheme II. From condition (57)

$$M_{rib}=M-M_{ov}. \tag{60}$$

The moment M_{rib} is taken by the rib just as in a rectangular singly-reinforced beam and the corresponding bar area is determined in the usual manner:

A_0 is solved by formula (37), corresponding values of γ_0 or α are given by Table 10, after which $F_{s.rib}$ is derived through formula (40) or (41).

The full amount of tensile reinforcement will be

$$F_s = F_{s.ov} + F_{s.rib}. \tag{61}$$

Furthermore, condition (23) must be observed for the section as a whole.

The most frequent problem met with in T-beams is to find the bar area F_s when the section dimensions, grades of material, and bending moment M are known.

Sequence of procedure.

1) The effective flange width b_{fl} is calculated.

2) Formula (53) decides which case the problem belongs to.

3) Bar area F_s is found: if Case 1, it is computed either as a rectangular beam with b_{fl} width or by the approximate formula (52); if it is Case II, first M_{ov} is determined by formula (58) and $F_{s.ov}$ by formula (59). Then M_{rib} is derived through formula (60) and $F_{s.rib}$ is found as for a rectangular beam whose width is b. Formula (61) gives the full amount of tension bars F_s.

4) When F_s is known, condition (23) is to be checked.

If section dimensions, bar area, and steel and concrete grades are all given in order to find the carrying capacity, the method will be as follows:

from equation (50) we have

$$x = \frac{m_s R_s F_s}{b_{fl} R_{be}}. \tag{62}$$

If $x \leq h_{fl}$, then Case 1 is observed, and the carrying capacity determined by formula (51) or (51a).

But if $x > h_{fl}$, it will be Case 2, then:

1) the value of x is corrected by equation (55),

2) the magnitudes of S_0 and S_c are computed to check condition (23),

3) the carrying capacity of the section $M_{b.cs}$ is found by formula (22) or (56).

Illustrative problem 7. Given: a self-contained T-beam, length $l=4.8$ m, height $h=50$ cm, web width $b=25$ cm, flange width $b_{fl}=130$ cm, flange thickness $h_{fl}=8$ cm, grade of concrete 150-B, bars of St-3 steel, the service coefficient $m=1.1$, $M=12$ t/m. To find: the area of tension bars F_s.

Solution.

Gathering design data: $R_{be}=80$ kg/cm², $R_s=2{,}100$ kg/cm², $m_s=1$, $h_0=h-a=$ $=50-3.5=46.5$ cm.

The effective width of the flange will be

$$b_{fl} = \frac{l}{3} = \frac{480}{3} = 160 > 12h_{fl} + b = 12\times 8 + 25 = 121 \text{ cm}.$$

This means that the effective width $b_{fl}=121$ cm, instead of the actual vidthof 130 cm.

Condition (53) decides the case to be followed:

$$mR_{be}b_{fl}\ h_{fl}\left(h_0-\frac{h_{fl}}{2}\right)=1.1\times80\times121\times8\left(46.5-\frac{8}{2}\right)=3{,}580{,}000\ \text{kg/cm}=$$

$$=35.8\ \text{t/m}>M=12\ \text{t/m}.$$

Hence, $x<h_{fl}$, i.e., we must deal with Case 1.*

We compute

$$A_0=\frac{M}{mR_{be}b_{fl}\ h_0^2}=\frac{1{,}200{,}000}{1.1\times80\times121\times46.5^2}=0.053.$$

Table 10 shows that $\gamma_0=0.972$, and from formula (40) we derive

$$F_s=\frac{1{,}200{,}000}{1.1\times2{,}100\times0.972\times46.5}=11.5\ \text{cm}^2,$$

which is satisfied by $2\phi20+2\phi18$ ($F_s=11.3$ cm²).

From the approximate formula (52)

$$F_s=\frac{1{,}200{,}000}{1.1\times1\times2{,}100\left(46.5-\frac{8}{2}\right)}=12.2\ \text{cm}^2,$$

i.e., there is an excess of 6%.

Illustrative problem 8. Given: a cross beam in a floor, height $h=60$ cm, width $b=25$ cm, 8-cm slab thickness, 2-metre beam spacing, concrete grade 150-A, cold-notched grade St-3 bars, $F_s=20$ cm², $m=1$. To be determined: carrying capacity.

Solution.

Design data gathered: $R_{be}=85$ kg/cm², $R_s=3{,}600$ kg/cm², $m_s=0.65$, $h_{fl}=8$ cm, $h_0=56$ cm, effective width of flange $b_{fl}=200$ cm.

Compression-zone height is obtained from formula (62):

$$x=\frac{0.65\times3{,}600\times20}{200\times85}=2.8\ \text{cm}<h_{fl}=8\ \text{cm};$$

hence, it is Case 1.

Carrying capacity is determined by condition (51):

$$M_{b.cs}=1\times85\times200\times2.8\left(56-\frac{2.8}{2}\right)=2{,}600{,}000\ \text{kg/cm}=26\ \text{t/m}$$

or by condition (51a):

$$M_{b.cs}=1\times0.65\times3{,}600\times20\left(56-\frac{2.8}{2}\right)=2{,}600{,}000\ \text{kg/cm}=26\ \text{t/m}.$$

Illustrative problem 9. Given: $M=17$ t/m, $h=50$ cm, $b=16$ cm, $b_{fl}=40$ cm, $h_{fl}=10$ cm, concrete grade 200-B, bars of intermittently deformed St-5 steel, $m=1$ Determine: bar area F_s.

Solution.

Design data gathered: $h_0=46$ cm, $R_{be}=100$ kg/cm², $R_s=2{,}400$ kg/cm², $m_s=1$.

Determine which case is concerned:

$$mR_{be}b_{fl}\ h_{fl}\left(h_0-\frac{h_{fl}}{2}\right)=1\times100\times40\times10\left(46-\frac{10}{2}\right)=1{,}635{,}000\ \text{kg/cm}=$$

$$=16.35\ \text{t/m}<M=17\ \text{t/m},$$

which shows that Case 2 is to be dealt with.

* Inasmuch as the section has a developed flange, it was clear that $x<h_{fl}$.

Computing by formula (58)

$$M_{ov}=1\left[0.8\times100\,(40-16)\times10\left(46-\frac{10}{2}\right)\right]=785{,}000\ \text{kg/cm}=7.85\ \text{t m}.$$

From formula (59)

$$F_{s.ov}=\frac{785{,}000}{1\times1\times2{,}400\left(46-\frac{10}{2}\right)}=8\ \text{cm}^2.$$

The moment borne by the rib is found by formula (60):

$$M_{rib}=17-7.85=9.15\ \text{t/m}.$$

From formula (37)

$$A_0=\frac{915{,}000}{1\times100\times16\times46^2}=0.27.$$

Table 10 indicates that $\gamma_0=0.84$, and from formula (40)

$$F_{s.rib}=\frac{915{,}000}{1\times1\times2{,}400\times0.84\times46}=9.85\ \text{cm}^2.$$

Full tension-bar area is found by formula (61):

$$F_s=8+9.85=17.85\ \text{cm}^2.$$

Condition (23) is verified: to find S_c, the height of the rib's compression zone is determined by formula (55):

$$x=\frac{\frac{m_sR_sF_s}{R_{bc}}-0.8\,(b_{fl}-b)\,h_{fl}}{b}=\frac{\frac{1\times2{,}400\times17.85}{100}-0.8\,(40-16)\,10}{16}=$$

$$=14.8\ \text{cm}>h_{fl}=10\ \text{cm}.$$

The static moment of the area of the entire effective cross-section in relation to the centre of gravity of the bar area F_s is

$$S_0=16\times16\times\frac{46}{2}+(40-16)\,10\times41=16{,}900+9{,}840=26{,}740\ \text{cm}^3.$$

The static moment of the compression zone in relation to the centre of gravity of the bar area F_s is

$$S_c=16\times14.8\left(46-\frac{14.8}{2}\right)+(40-16)\,10\times41=9{,}150+9{,}840=18{,}990\ \text{cm}^3.$$

$0.8S_0=0.8\times26{,}740=21{,}400\ \text{cm}^3>S_c=18{,}990\ \text{cm}^3$, which indicates that condition (23) has been observed.

Sec. 10. THE DIAGONAL PLANE (CROSS-SECTION) AS USED IN COMPUTING THE CARRYING CAPACITY OF BENDING MEMBERS

1. Test Results

Tests have shown that diagonal cracks form near the supports of bending members because of mutual action of the shearing force and the bending moment (see Fig. 27), the angle of the fissure depending upon the kind and amount of reinforcement, sectional dimensions, and grades of concrete and steel.

It is known from Strength of Materials that normal stresses σ are caused in a beam by the bending moment M and that tangent (shearing) stresses τ are due to shearing forces Q. The diagonal planes are acted upon by diagonal tensile and diagonal compressive stresses which are determined by the formula

$$\sigma_{di}=\frac{\sigma}{2}\pm\sqrt{\frac{\sigma^2}{4}+\tau^2}\,.$$

When the diagonal tensile stresses reach a value equal to ultimate tensile resistance R_t of the concrete (Fig. 34, *a*), the latter fails, accompanied by the formation of diagonal fissures (Stage Ia). The parts

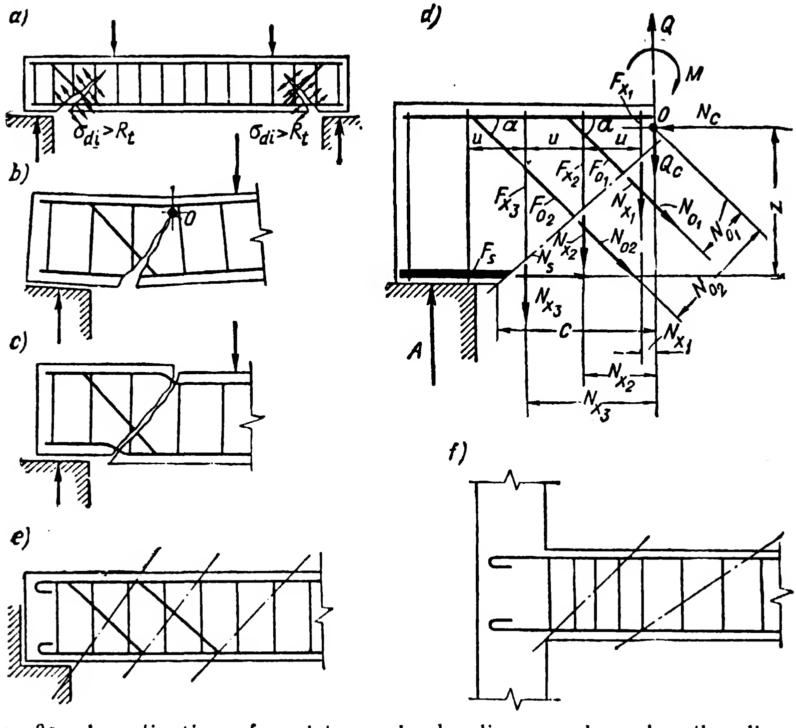

Fig. 34. Investigation of resistance in bending members by the diagonal plane method

a—sketch showing formation of diagonal fissures; *b* and *c*—failure along inclined cracks; *d*—design diagram at a diagonal plane; *e* and *f*—computation diagonals

situated at the right and left of these inclined fissures will tend to revolve mutually about a point lying in the compression zone above. This tendency is hindered by the tension steel (longitudinal and bent-up bars and stirrups) that crosses the fissures. After fissuring, stress deformation of the member will correspond to Stage II. As the load further increases, the member will fail from either one of two causes:

Cause 1. Resistance of the steel that bisects the inclined crack is overcome; the resulting yield of tension-stressed bars opens the crack (Fig. 34, *b*); the two parts twist mutually about the momentary centre O located above the inclined crack in the centre of gravity of the compression zone and the concrete in the latter fails.

Cause 2. The compression-zone concrete either shears or is broken by compressive forces (Fig. 34, *c*), often before the yield point is reached in the stirrups (upright bars of the reinforcement block) and bent-up bars that cross the crack. This type of failure occurs with a large percentage of longitudinal bars, well anchored at the supports.

2. Checking the Need of Diagonal Reinforcement

As already noted, diagonal cracks appear when diagonal tension in the concrete exceeds R_t. The magnitude of this tension depends upon the shearing force Q, an approximation of which is given by the formula

$$\sigma_{di} = \frac{Q}{mbh_0}. \tag{63}$$

Resistance at the diagonal plane, conditioned by the tensile resistance of the concrete, will be sufficient if

$$\sigma_{di} = \frac{Q}{mbh_0} \leqslant R_t$$

which is the same as if

$$Q \leqslant mR_t bh_0. \tag{64}$$

By satisfying condition (64), stirrups, bent-up bars, and upright bars of blocks may be installed without theoretical computation, i.e., only to satisfy structural requirements. But when condition (64) is violated, reinforcement must be computed for the diagonal plane to avoid inclined cracks.

3. Provisions for Resistance at Diagonal Planes

The diagram in Fig. 34, *d* will serve in considering stress conditions at a diagonal plane.

Notation:

c—projected length of diagonal plane along axis of the member;

u—spacing of stirrups (or lateral bars of blocks);

α—angle of bent-up bars;

F_s F_{incl}, F_{st}—area of longitudinal and inclined bars and stirrups (upright bars), respectively;

N_s, N_{incl}, N_{st}—resultant of tensile stresses of longitudinal and inclined bars and stirrups, respectively;

N_c—projection on axis of member of resultant stresses in the concrete compression zone in the diagonal plane;

Q_c—projection, on the normal of the axis, of resultant stresses in the concrete compression zone of the diagonal plane;

Q—shearing force from the design load at the apex of the diagonal plane;

M—design bending moment from the design load at the apex of the diagonal plane.

Just as in computations at a normal plane, the internal forces of bars and concrete are dependent upon design resistance and service coefficients:

$$N_s = m_s R_s F_s;$$
$$N_{incl} = m_s R_s F_{incl};$$
$$N_{st} = m_s R_s F_{st}.$$

Tests have shown that

$$Q_c = \frac{0,15 R_{be} b h_0^2}{c}. \qquad (65)$$

Resistance at the diagonal plane of a bending member is assured when its design forces Q and M do not exceed its carrying capacity, the diagonal plane being computed on the basis of its normal plane, reinforcement, design resistance of concrete and bars, and service coefficients.

Thus, failure will not occur along a diagonal plane from Cause 1 (i.e., from the moment factor) if the effective bending moment M is not more than the sum of all the moments of all internal forces in relation to O, the centre of the compression zone. Inasmuch as a failure crack usually crosses several stirrups and inclined bars, the forces in these components are expressed as a sum of the term Σ as follows:

$$M \leqslant m\,(N_s z + \Sigma N_{incl} z_{incl} + \Sigma N_{st} z_{st}),$$

where z, z_{incl}, z_{st} are the distances from longitudinal and bent-up bars and stirrups, respectively, to the moment point.

If we substitute N_s, N_{incl}, and N_{st} by their values and remove the brackets from $m_s R_s$, the result is

$$M \leqslant m m_s R_s\,(F_s z + \Sigma F_{incl} z_{incl} + \Sigma F_{st} z_{st}). \qquad (66)$$

Failure will not occur along a diagonal plane from Cause 2 (the shearing force) if the design shearing force Q does not exceed the sum of all the internal forces projected on the normal of the axis. As already stated, since the stresses in the inclined bars and stirrups might not attain design resistance values of these components, the stresses in the steel F_{incl} and F_{st} are computed with a service coefficient m_{di}:

$$Q \leqslant m\,(\Sigma m_{di} z_{incl} \sin\alpha + \Sigma m_{di} z_{st} + Q_c).$$

If we replace N_{incl} and N_{st} by their values and remove the brackets from $m_s m_{di}$, the result is

$$Q \leqslant m\,[m_s m_{di} R_s\,(\Sigma F_{incl} \sin\alpha + \Sigma F_{st}) + Q_c]. \tag{67}$$

Condition (67) for resistance must always be checked if there is any possibility of diagonal failure cracks. Diagonal planes that cross the edges of supports and the starting points of bent-up bars must likewise be verified by condition (67) (Fig. 34, *e*), as also those places where there is a change of stirrup (or upright bar) spacing (Fig.34,*f*).

As regards condition (66) for moment resistance, tests have shown it to be satisfied by fulfilling certain structural requirements, as indicated below.

4. Computing Upright Bars of Blocks (Stirrups) and Inclined (Bent-Up) Bars

Let us assume a member reinforced only with upright bars (stirrups) capable of taking a vertical force q_{st} per unit length of member. Then the shearing force borne by the upright bars in a beam length equal to the projection of the diagonal plane c will be

$$Q_{st} = q_{st} c.$$

The magnitude of the shearing force $Q_{st.c}$ borne mutually by the upright bars (stirrups) and the concrete of the compression zone, will be

$$Q_{st.c} = Q_{st} + Q_c,$$

$$Q_{st.c} = q_{st} c + \frac{0.15 R_{be} b h_0^2}{c}. \tag{68}$$

Diagonal planes may possess various inclines (Fig. 35, *a*), and the bars must assure resistance at any incline. To achieve this, resistance formulae must be based on a minimum value of $Q_{st.c}$. This minimum is found by reducing to zero the first derivative of $Q_{st.c}$ with respect to the variable c:

$$\frac{dQ_{st.c}}{dc} = q_{st} - \frac{0,15 R_{be} b h_0^2}{c^2} = 0,$$

whence

$$c = \sqrt{\frac{0.15 R_{be} b h_0^2}{q_{st}}}.$$

By entering the solved value of c into equation (68) and trans forming algebraically, we obtain

$$Q_{st.c} = \sqrt{0.6 R_{be} b h_0^2 q_{st}}. \tag{69}$$

When the knowns are the spacing u, area of one vertical f_{st}, and number of verticals (in the beam width) n (Fig. 35, b), the force borne by the upright bars (stirrups) in a unit of beam length can be found through the condition

$$q_{st}=\frac{N_{st}}{u},$$

which, when N_{st} is substituted by its value, becomes

$$q_{st}=\frac{m_s m_{di} R_s f_{st} n}{u}. \qquad (70)$$

To determine the magnitude of the shearing force, $Q_{s.c}$, that the member can bear with given upright bars (stirrups), the solved value of q_{st} enters into formula (69).

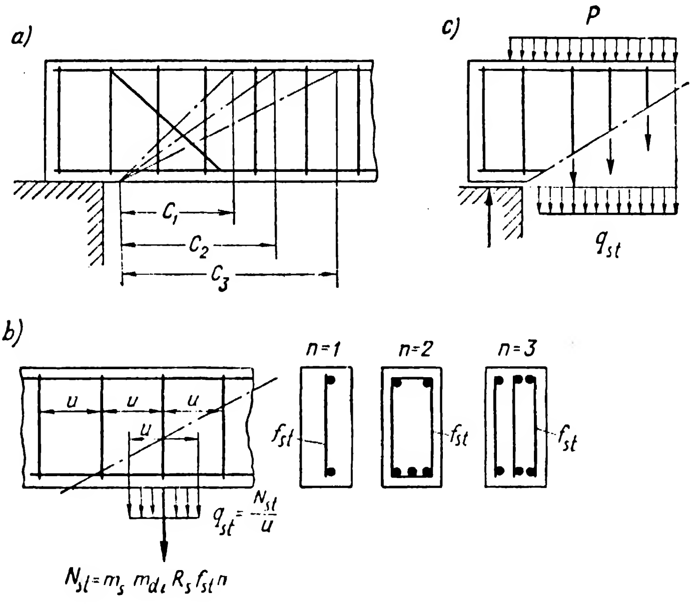

Fig. 35. Computation of upright bars in beams

A constant uniform load p (e.g., hydrostatic pressure, soil load, etc.) acting within the projected limits of the diagonal plane (Fig. 35, c) will sum up with the force q_{st}, hence,

$$Q_{s.c}=\sqrt{0.6R_{bc}bh_0^2\left(q_{st}+\frac{p}{m}\right)}.$$

If it is found that $Q>mQ_{st.c}$, then either the upright bars must be strengthened by enlarging their diameters or reducing their

spacing, or inclined bars must be installed to bear the surplus shearing force, so that

$$Q_{incl} = \frac{Q}{m} - Q_{st.c}. \tag{71}$$

Inasmuch as the resultant of the stresses in the inclined bars is directed at an angle of α, the vertical projection of these forces will be

$$m_s m_{di} R_s F_{incl} \sin \alpha. \tag{72}$$

Thus, the formula determining the area of bent-up bars will be the result of equating expressions (71) and (72):

$$F_{incl} = \frac{\frac{Q}{m} - Q_{st.c}}{m_s m_{di} R_s \sin \alpha}. \tag{73}$$

In calculating bent-up bars, the shearing force at the edge of the support enters formula (73) for the first bend transverse from

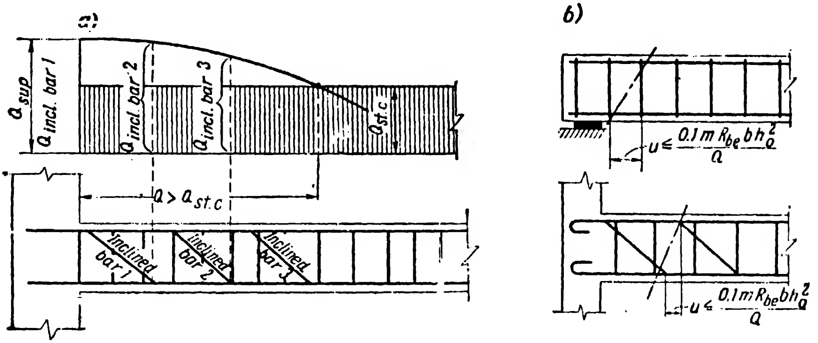

Fig. 36. Position of bent-up bars in beams, and maximum spacing between stirrups and bent-up bars

the support, and the shearing force at the lower point of each previous bend transverse is entered into the formula for each subsequent bend transverse (Fig. 36,*a*). The number of bend transverses depends upon the plotted length of the stress curve Q, where $Q > Q_{st.c}$ to allow for structural considerations as given below.

If only upright bars (stirrups) are used without bent-up bars, then the condition to be followed must be

$$\frac{Q}{m} \leqslant Q_{st.c}. \tag{74}$$

This dictates the following sequence of computation:

1) The shearing force q_{st} per unit of length of the member that is borne by the upright bars is determined by formula (69):

$$q_{st}=\frac{\left(\frac{Q}{m}\right)^2}{0.6R_{be}bh_0^2}. \tag{75}$$

2) q_{st} being found, an assumption is made of the diameters of the upright bars (i. e., their area f_{st}) and their spacing is obtained through formula (70):

$$u=\frac{m_s m_{di} R_s f_{st} n}{q_{st}}, \tag{76}$$

or vice versa: spacing u is assumed and the upright-bar area is computed:

$$f_{st}=\frac{q_{st}u}{m_s m_{di} R_s n}. \tag{77}$$

In this operation the mutual sizes of longitudinal and upright bars must satisfy welding requirements. Moreover, upright bar spacing u is to be limited by the value u_{max} because of the following factors: in the diagonal planes between two adjacent upright bars (stirrups) or bent-up bars (Fig. 36, *b*) resistance is assured only by the compression-zone concrete. Hence, here the condition to be observed is

$$Q \leqslant Q_c.$$

The magnitude of Q_c, as obtained by formula (65), depends upon the projected length c of the diagonal plane. In the given instance $c=u$, hence,

$$Q=\frac{0.15R_{le}bh_0^2}{u}. \tag{78}$$

Since there is a possibility of faulty work in spacing the stirrups longitudinally, the assumed carrying capacity of the compression-zone concrete is slightly lowered, a coefficient of 0.1 being introduced into the right-hand member of formula (78) instead of 0.15, so that the final computation, together with the service coefficient will be

$$u_{max}\leqslant\frac{0.1mR_{be}bh_0^2}{Q}. \tag{79}$$

5. Distinguishing Features of Safe Design by the Diagonal Plane Method

The paramount feature in safe design of spanning members by the diagonal plane method is anticipation of diagonal crack formation. The criterion in such anticipation of cracks is condition (64).

1. Where condition (64) is observed, i.e., $Q\leqslant mR_t bh_0$.

In this case the smallest diameters are chosen for the upright bars in prefabricated blocks, in accordance with Table 4. Stirrups of tied blocks, as already said, will be 6-8 mm but not less than 0.25 d of longitudinal bars. If stirrups are of cold-drawn rods, they may have a minimum diameter of 5 mm. Longitudinal spacing of stirrups or upright bars of blocks must not exceed $\frac{3}{4}$ of the beam

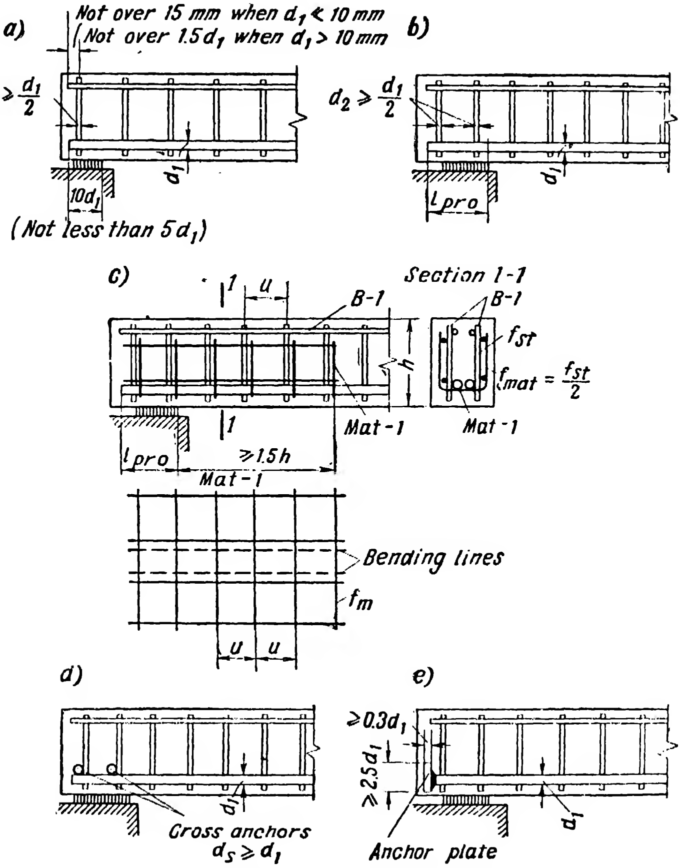

Fig. 37. Anchoring longitudinal bars of blocks at free-end beam supports

height and not be over 50 cm. A spacing of 20 cm is usual for beams up to 40 cm in height; if higher, spacing is made half the height.

In free-end supports, the blocks are brought to the end of the beam with longitudinal tension bars well anchored beyond the supports for a distance of at least 5 d. If the longitudinal tension bars are plain, then additional anchors should consist of at least one upright bar beyond the supports (Fig. 37, *a*).

2. Where condition (64) is not observed, i.e., $Q > mR_1bh_0$.

In this case, resistance of the diagonal cross-section against the shearing force is assured, i.e., condition (67) is observed, by computing the diameters and spacings of upright bars, such spacing not to exceed u_{max} in formula (79). Furthermore, the value of u is to be limited by the following conditions:

$$\left.\begin{array}{l} u \leqslant 20 \text{ cm for beams up to 40 cm in height;} \\ u \leqslant \frac{h}{2} \text{ " " " above 40 cm " " ;} \\ u \leqslant 50 \text{ cm} \end{array}\right\} \quad (79a)$$

To ensure bending-moment resistance in the diagonal plane, condition (66) may be ignored by following the structural methods given below. Practically, this means guarding against both pulling out, and overcoming tensile resistance of, the longitudinal reinforcement.

When prefabricated blocks are used for reinforcement. All longitudinal rods must run to, and 15 d* beyond, the supports. If the bars are plain, at least 2 uprights must cross the protruded length l_{pro} of the longitudinal bars (Fig. 37, *b*). If the upright bars are strengthened for a length of $l_{pro}+1.5h$ at the protrusion, the latter may be 5 d less. The cross-section of this length of upright reinforcement is increased 50% more than computed, either by the addition of a pan-shaped mat (Fig. 37, *c*), lessening spacing, or by still further increasing bar diameter (in narrow beams having one block).

If structural obstacles hinder the protrusion l_{pro}, then the tension steel is additionally anchored by welded-on bars or plates (Fig. 37, *d* and *e*). Part of the tension steel can be interrupted (cut off) where it is allowed by the decreased bending moment, the termination to be executed beyond this plane (Fig. 38, *a*) at a distance of

$$w = \frac{\frac{Q}{m}}{2q_{st}} + 5d, \quad (80)$$

where Q—design shearing force at the theoretical point of bar interruption;

q_{st}—vertical force per unit of beam length, borne by the upright bars and determined by formula (70).

The length w, derived from formula (80), must be at least 20 d, and include not less than 2 upright bars if they are of plain steel.

When spliced blocks are used for the reinforcement. Longitudinal tension bars must go 15 d beyond free-end supports if plain hooked bars are used, 15 d if they are intermittently-deformed without hooks and the concrete is of grade 150, and 10 d if the concrete is of grade 200 or higher.

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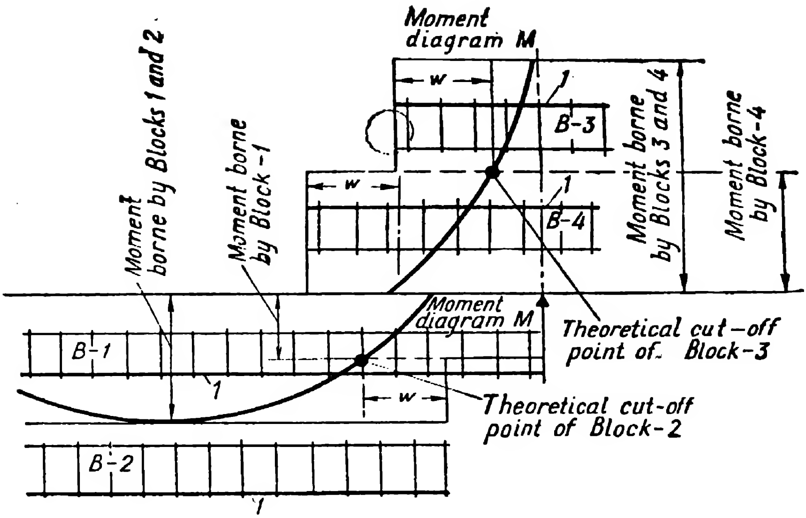
* 10 d if the bars are of intermittently deformed St-5 steel or of cold-notched St-0 and St-3 steel in 200-, or higher, grade of concrete.



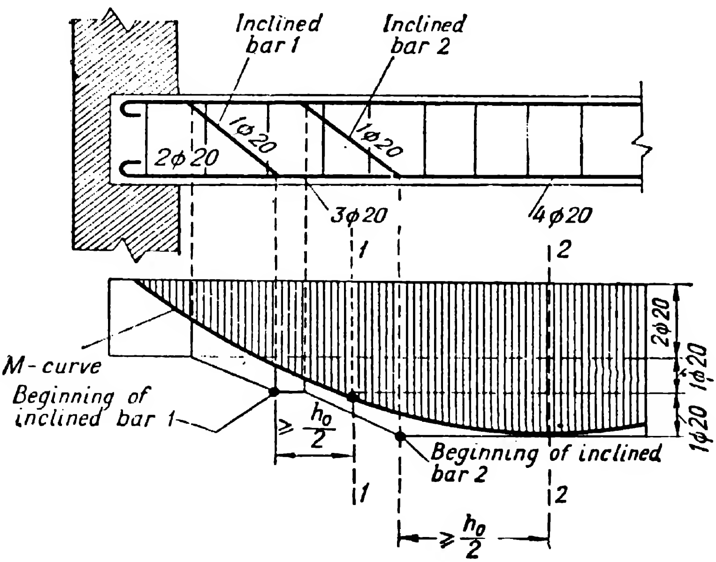


Fig. 38. Diagrams of components (materials): determining cut-off points of bars and blocks

1—effective bars of blocks; section *1-1* is where inclined bar *1* is fully used in bending (in a normal cross-section); section *2-2* is where bent-up bar *2* is fully used in bending (in a normal cross-section)

Inclined bars are usually bent 45° to the horizontal and 60° in very high beams, their area being determined by computation. They are placed in that part of the beam where $Q > Q_{st.c}$ (Fig. 36, *a*); the first bend transverse must align with the edge of the support (or not more than 5 cm away), and the last with the point where $Q = Q_{st.c}$.* The maximum distance between the end of one bend transverse and the beginning of the next (Fig. 36, *b*) is given in formula (79). Inclined bars are usually continuations of mid-span reinforcement. Bends in tension-zone bars must begin not closer than $\frac{h_0}{2}$ from the point where they lend their full strength in the normal cross-section (Fig. 38,*b*).

Sec. 11. CALCULATIONS OF DEFORMATION AND FISSURE OPENING IN BENDING MEMBERS

1. Relationship between Deformation and Stresses. Rigidity of Reinforced Concrete as Effected by Fissures in the Tension Zone

Reinforced concrete bending members are, as a rule, fissured under working stresses in the tension zone. Hence, deformation computations concerning deflection, angle of rotation, and widening of fissures are based on Stage II of stress deformation as caused by specified loads.

The reader already knows that in Stage II fissures are formed in the tension zone of the concrete and spaced at intervals of l_{fis}, and that stresses in the compression concrete are equal to $\sigma_c < R_{be}$. The height of the compression zone in a plane lying between the fissures is more than at the fissures proper and results in a wavelike neutral axis (Fig. 39).

Stresses will vary along the length of the tension bars: the maximum at the fissures, equal to σ_s, will decrease with the distance from fissure edges (Fig. 28) because the concrete-steel bond will still exist between fissures and the concrete will continue to bear some of the tension.

Mean bar stresses $\sigma_{s.me}$ between fissures can be expressed in terms of σ_s within the fissures proper and the coefficient $\psi \leq 1$, which latter allows for the concrete's tensile resistance between the fissures:

$$\sigma_{s.me} = \psi\sigma_s. \tag{81}$$

Such mean stresses can also be expressed in terms corresponding to mean deformation $\varepsilon_{s.me}$:

$$\sigma_{s.me} = \varepsilon_{s.me} E_s. \tag{82}$$

* The last bend transverse may be one stirrup interval u closer to the supports than the point where $Q = Q_{st.c}$ if the loads are concentrated.

The bar stresses σ_s in the fissure plane, expressed as a term ε_s, are equal to

$$\sigma_s = \varepsilon_s E_s. \tag{83}$$

By entering the value of σ_s from (81) into formula (83), we obtain

$$\frac{\sigma_{s.me}}{\psi} = \varepsilon_s E_s$$

or

$$\sigma_{s.me} = \psi \varepsilon_s E_s. \tag{84}$$

By equating formulae (82) and (84), we obtain

$$\varepsilon_{s.me} = \psi \varepsilon_s. \tag{85}$$

It therefore follows from equations (81) and (85) that

$$\psi = \frac{\sigma_{s.me}}{-} = \frac{\varepsilon_{s.me}}{-} \leqslant 1.$$

The coefficient ψ depends upon reinforcement percentage, amount of σ_s stresses, and concrete grade, and varies from a minimum of

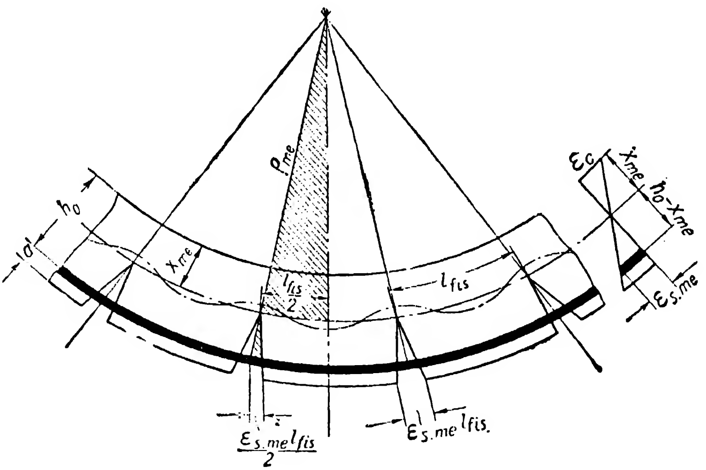


Fig. 39. Sketch of deformations in a reinforced concrete beam

03-0.5 (formation of fissures) to a value close to 1.0. Eventually, as plastic deformation progresses in the tension zone between fissures, the coefficient increases to $\psi = 1$ under repeated moving loads (such as in a crane girder). The stresses in the plane of the fissure σ_s may be expressed as a term of deformation ε_s as in formula (83), and also as one of mean deformation $\varepsilon_{s.me}$ according to formula (85):

$$\sigma_s = \varepsilon_s E_s = \frac{\varepsilon_{s.me} E_s}{\psi} \tag{86}$$

Stress-deformation diagrams $\sigma_s-\varepsilon_s$ and $\sigma_s-\varepsilon_{s \cdot me}$ are given in Fig.40, from which the reader may see that the tangent of the angle of the straight incline $\sigma_s-\varepsilon_s$ is the modulus of elasticity of the free bars within the fissures:

$$\tan\alpha = \frac{\sigma_s}{\varepsilon_s} = E_s,$$

and the tangent of the angle of the secant intersecting the curve σ_s-$\varepsilon_{s \cdot me}$ is the mean modulus of elasticity of the tension bars:

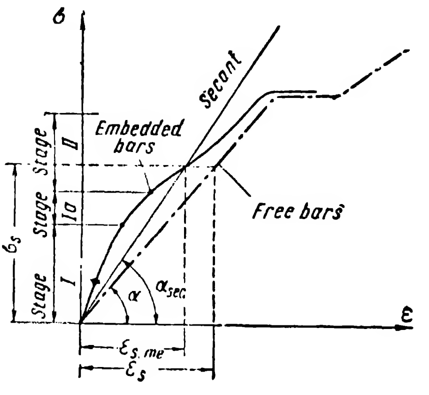


Fig. 40. Stress-deformation diagram σ-ε for tension bars in reinforced concrete

$$\tan\alpha_s = \frac{\sigma_s}{\varepsilon_{s.n}} = E_{s\,me}.$$

Hence,

$$E_{s\,me} = \frac{\sigma_s}{\varepsilon_{s.me}} = \frac{\sigma_s}{\psi\varepsilon_s} = \frac{E_s}{\psi} \qquad (87)$$

As already noted, the identity between stress and deformation in the compression zone of the concrete is

$$\sigma_c = E_c'\varepsilon_c = (1-\lambda)\,E_c\varepsilon_c = \nu E_c\varepsilon_c.$$

The coefficient λ, obtained as the ratio between plastic and absolute deformation, depends upon stress value and concrete grade. The progress of plastic deformation in the compression zone will eventually change the coefficient λ from its minimum (approaching zero when loading begins) to a value close to 0.6 (due to the constant service load).

In Stage II, concrete fracture in the tension zone and subsequent internal displacement will distort the section. Nevertheless the Hypothesis of Plane Sections (Navier's Hypothesis) remains valid for mean deformation in the planes between fissures, i.e., the plane stays normal in bending because deformation is linear in height (Fig. 39).

An identity may be established between mean deformation and bending forces on the basis of this hypothesis.

Let us examine the deformed area between fissures (Fig.39). The mean height of the compression zone is x_{me}, the mean radius of deflection is ϱ_{me} and the mean deformation of tension bars is $\varepsilon_{s.me}$. The stretching of tension bars within the length l_{fis} will be $\varepsilon_{s.me}l_{fis}$, or 0.5 $\varepsilon_{s.me}$ l_{fis} per each end of this length. Assuming a similarity to a triangle (shown hatched in Fig. 39), we obtain

$$\frac{l_{fis}}{2} : \varrho_{me} = \varepsilon_{s.me}\frac{l_{fis}}{2} : (h_0 - x_{me}).$$

After reducing both members of the equation by $\frac{l_{ns}}{2}$, we obtain

$$\frac{1}{\varrho_{ne}} = \frac{\varepsilon_{s.me}}{h_0 - x_{n.e}}. \tag{88}$$

By substituting $\varepsilon_{s.me} = \frac{\psi\sigma_s}{E_s}$ from (86), we obtain

$$\frac{1}{\varrho_{me}} = \frac{\psi\sigma_s}{E_s(h_0 - x_{me})}. \tag{88a}$$

The stresses in the reinforcement σ_s are determined by analysing the state of stress in Stage II (Fig. 41). Development of plastic deformation in the concrete of the compression zone bends the

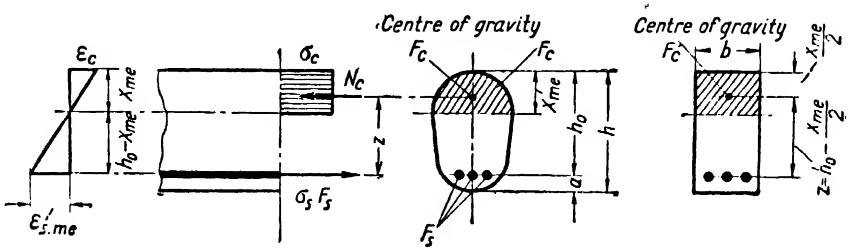


Fig. 41. Computing deformation in reinforced concrete bending members

stress diagram of normal stresses so sharply that in order to simplify calculations it may be considered rectangular. The force moment in the tension bars, in relation to the centre of gravity of the concrete in the compression zone of the cross-section is

$$M = \sigma_s F_s z,$$

from which

$$\sigma_s = \frac{M}{F_s z},$$

or

$$\sigma_s = \frac{M}{W_s}, \tag{89}$$

whence

$$W_s = F_s z, \tag{90}$$

which is known as the elasto-plastic resisting moment of the reinforced concrete cross-section in Stage II. The magnitude of W_s depends upon the geometric shape of the cross-section and the coefficients ψ and λ (which effect the value of z).

By substituting σ_s in equation (88a) for its value in (89), we will have

$$\frac{1}{\varrho_{me}} = \frac{\psi M}{E_s W_s(h_0 - x_{me})} = \frac{M}{B}, \tag{91}$$

where

$$B = \frac{1}{\psi} E_s W_s (h_0 - x_{me}). \tag{92}$$

If formula (91) be compared with the well-known formula in Strength of Materials, i.e.,

$$\frac{1}{\varrho} = \frac{M}{EJ},$$

it will be seen that B represents rigidity of the concrete's cross-section in Stage II.

Tests have shown that rigidity B, calculated for the plane with the greatest bending moment, may be considered with sufficient exactness to be the same for the entire part whose bending moment is of the same sign (Fig. 42).

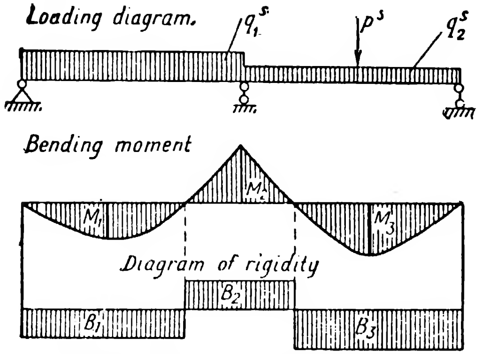


Fig. 42. Design rigidity in a bending member

2. Rigidity Computations

In computing rigidity B by formula (92) it is necessary to know the compression-zone height x_{mc}, which depends upon the section's shape, percentage of reinforcement and grade of concrete.

Let us consider a singly-reinforced rectangular beam (see Fig. 41): cross-section equilibrium dictates that counteracting tensile and compressive forces be equal:

$$\sigma_s F_s = \sigma_c b x_{mc}. \tag{93}$$

If both members of this equation are divided by bh_0 and we designate

$$\xi_{me} = \frac{x_{me}}{h_0},$$

then

$$\sigma_s \mu = \sigma_c \xi_{mc}. \tag{93a}$$

The conditions of linear variations in mean height deformation give

$$\frac{\varepsilon_c}{\varepsilon_{s.me}} = \frac{x_{me}}{h_0 - x_{me}}. \tag{94}$$

If the numerator and denominator of the right-hand member of equation (94) be divided by h_0, then

$$\frac{\varepsilon_c}{\varepsilon_{s.me}}=\frac{\xi_{me}}{1-\xi_{me}}. \qquad (94a)$$

Inasmuch as $\varepsilon_c=\frac{\sigma_c}{\nu E_c}$ and $\varepsilon_{s.me}=\frac{\psi\sigma_s}{E_s}$, equation (94a) becomes

$$\frac{E_s}{\psi\nu E_c}\times\frac{\sigma_c}{\sigma_s}=\frac{\xi_{me}}{1-\xi_{me}}. \qquad (95)$$

Stresses σ_s are derived from equation (95):

$$\sigma_s=\frac{n}{\psi\nu}\sigma_c\frac{1-\xi_{me}}{\xi_{me}}, \qquad (96)$$

in which

$$n=\frac{E_s}{E_c}.$$

By entering the found value of σ_s into equation (93a) and reducing by σ_c we obtain

$$\frac{\mu n}{\psi\nu}\times\frac{1-\xi_{me}}{\xi_{me}}=\xi_{me}.$$

Designating $\alpha=\frac{\mu n}{\psi\nu}$, we derive

$$\alpha\frac{1-\xi_{me}}{\xi_{me}}=\xi_{me},$$

whence

$$\xi_{me}^2+\alpha\xi_{me}-\alpha=0.$$

ξ_{me} and the compression-zone height are obtained by solving the quadratic equation

$$x_{me}=\xi_{me}h_0=\left(-\frac{\alpha}{2}+\sqrt{\frac{\alpha^2}{4}+\alpha}\right)h_0. \qquad (97)$$

The arm of the resisting couple, for the given rectangular beam, $z=h_0-\frac{x_{me}}{2}$ from which in accordance with the identity (90), the elasto-plastic resisting moment

$$W_s=F_s\left(h_0-\frac{x_{me}}{2}\right). \qquad (98)$$

From the identity (92), rigidity under momentary loads B_{mo} will be

$$B_{mo}=\frac{1}{\psi}E_sF_s\left(h_0-\frac{x_{me}}{2}\right)(h_0-x_{me}). \qquad (99)$$

Practical computations of rigidity are executed with the aid of tables compiled for various shapes: rectangular singly- and doubly-reinforced beams, T-beams with flanges either in the compression

or tension zone, and I- and box-beams (Fig. 43). Furthermore, on the basis of tests, momentary loads are taken as $\psi\nu = \frac{1}{3}$.

Reinforcement stresses and cross-section rigidity are evolved from the formulae

$$\sigma_s = \frac{M^s}{\eta h_0 F_s} \tag{100}$$

and

$$B_{mo} = \frac{1}{\psi} E_s c F_s h_0^2, \tag{101}$$

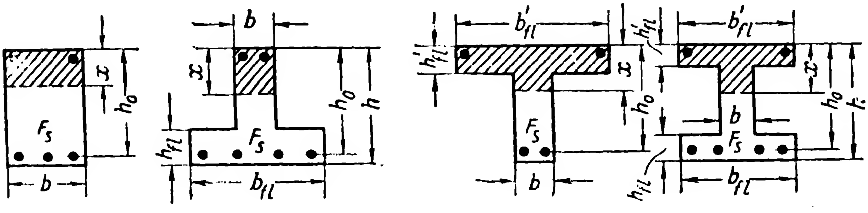


Fig. 43. Investigation of rigidity in bending members

in which M^s is the bending moment from specified loads; the coefficient ψ is taken from Tables 11 and 12* after solving α, $\gamma_1 = \frac{(b_{fl} - b)\, h_{fl}}{bh}$ and σ_s found through formula (100); the coefficients $\eta = 1 - 0.5\, \xi_{me}$ and $c = (1 - 0.5\, \xi_{me})(1 - \xi_{me})$ are derived from Table 13 after solving

$$\alpha = \frac{\mu n}{\psi \nu} = 3\mu n \text{ and } \gamma' = \frac{(b'_{fl} - b)\, h'_{fl}}{b h_0}.$$

Constant loads caused by plastic deformation increase the height of the compression zone x_{me} and lessen rigidity B. The magnitude of rigidity B under a constant load is fixed by the formula

$$B = B_{mo} \frac{q^s}{g^s \theta + p^s}, \tag{102}$$

in which g^s—the constant specified load;

p^s—the momentary specified load;

$q^s = g^s + p^s$—full specified load;

θ—coefficient of decreased rigidity under a constant load and which is equal to:

a) 1.5 for T-beams whose flanges are in the compression zone;

b) 2.0 for rectangular shapes, I-beams, box-shapes, and other such types;

---

* Tables 11 and 12 are for rectangular and T-shapes with flanges in their compression zone, and for I- and box-beams.

Table 11

Values for Coefficients ψ and k_1 in Rectangular Beams

α	Values of ψ for σ_s in kg/cm²						Values of k_1
	1,000	1,250	1,500	2,000	2,500	3,000	
0.05	—	—	—	—	—	0.39	22.8
0.06	—	—	—	—	0.4	0.57	19.2
0.07	—	—	—	—	0.49	0.68	16
0.08	—	—	—	0.4	0.63	0.75	14
0.1	—	—	0.4	0.58	0.76	0.86	11.8
0.125	—	—	0.47	0.73	0.85	0.91	9.6
0.15	—	0.44	0.63	0.82	0.9	0.94	8
0.2	0.47	0.65	0.78	0.9	0.94	0.97	6
0.3	0.71	0.82	0.89	0.94	0.97	0.98	4.3
0.4	0.81	0.88	0.92	0.96	0.98	0.99	3.4
0.5	0.85	0.9	0.93	0.96	0.98	0.99	2.8
0.8	0.9	0.93	0.95	0.97	0.98	0.99	2.2

Notes. 1) when $\alpha > 0.8$, then $\psi = 1$. 2) the values of ψ from Table 11 are also applicable to doubly-reinforced rectangular shapes, as well as to T-beams whose flanges are in the compression zone.

c) 2.5 for T-beams with flanges in the tension zone.

In deciding the sum of g^s, all dead loads are included in the constant load while live loads are considered in accordance with their nature.

Thus in dwellings and civic buildings, live loads are entirely excluded from g^s; in libraries, archives, etc., g^s embraces all live loads; while in industrial buildings all live loads are included in g^s but minus 150 kg/m².

When computing rigidity for giant-panel cavity slabs, a coefficient of 1.2, which allows for an increase of rigidity conditioned by this special shape, is included with B in formula (102).

In determining the n-ratio, the design modulus of concrete elasticity is used; but if the precast members come from a factory or well-equipped casting yard that renders their designated grade dependable, the specified modulus is adopted.

3. Calculations for Deflection

Floor and roof deflection, at its maximum, must not exceed from $\frac{1}{200}$ to $\frac{1}{400}\, l$, depending on the type of structure, shape of member and width of span.

Values for Coefficients ψ and k_1 in I and Box Shaped Bending Members

	Value of ψ and k_1																				
	$\gamma_1 = 0.36$							$\gamma_1 = 0.72$							$\gamma_1 = 1.08$						
	$\sigma_s = 1,000$	$\sigma_s = 1,250$	$\sigma_s = 1,500$	$\sigma_s = 2,000$	$\sigma_s = 2,500$	$\sigma_s = 3,000$	k_1	$\sigma_s = 1,000$	$\sigma_s = 1,250$	$\sigma_s = 1,500$	$\sigma_s = 2,000$	$\sigma_s = 2,500$	$\sigma_s = 3,000$	k_1	$\sigma_s = 1,000$	$\sigma_s = 1,250$	$\sigma_s = 1,500$	$\sigma_s = 2,000$	$\sigma_s = 2,500$	$\sigma_s = 3,000$	k_1
0.1	—	—	—	—	—	0.4	23	—	—	—	—	—	—	34	—	—	—	—	—	—	45
0.15	—	—	—	0.4	0.56	0.71	15	—	—	—	—	—	—	22.6	—	—	—	—	—	—	39
0.2	—	—	0.4	0.6	0.76	0.84	11.5	—	—	—	—	0.43	0.61	17.3	—	—	—	—	—	—	23
0.3	—	0.47	0.65	0.82	0.89	0.93	8.2	—	—	0.4	0.59	0.74	0.83	12.2	—	—	—	0.4	0.55	0.69	16
0.4	0.49	0.7	0.8	0.89	0.94	0.96	6.8	—	0.4	0.57	0.77	0.86	0.91	9.8	—	—	0.4	0.58	0.74	0.82	13
0.5	0.63	0.8	0.88	0.93	0.96	0.97	5.6	—	0.6	0.69	0.85	0.91	0.94	8	—	0.4	0.51	0.73	0.83	0.89	11.2
0.6	0.74	0.84	0.9	0.95	0.97	0.98	4.6	0.5	0.7	0.79	0.89	0.93	0.96	6.8	0.4	0.55	0.68	0.83	0.9	0.93	9.6
0.8	0.83	0.9	0.93	0.96	0.98	0.99	3.8	0.7	0.8	0.88	0.93	0.96	0.97	5.5	0.51	0.7	0.79	0.89	0.93	0.95	7.2

Note When $\alpha = 3\mu n$ is less than 0.1, then $\psi = 0.4$ and when α is greater than 0.8, $\psi = 1$

Values of Coefficients ξ_{me}, η and c for Bending Members: Singly-Reinforced Rectangular Shapes, T-Beams with Flanges in the Tension Zone, T-Beams with Flanges in the Compression Zone and I-(Box-) Beams

γ'		0	0.2	0.4	0.6	0.8	1	1.4
$\alpha = 0.1$	ξ_{me}	0.27	0.2	0.15	0.12	0.1	0.1	0.1
	η	0.86	0.92	0.94	0.95	0.95	0.95	0.95
	c	0.63	0.74	0.8	0.83	0.86	0.88	0.9
$\alpha = 0.2$	ξ_{me}	0.36	0.29	0.24	0.2	0.17	0.15	0.12
	η	0.82	0.9	0.92	0.93	0.94	0.95	0.95
	c	0.52	0.63	0.71	0.74	0.78	0.8	0.83
$\alpha = 0.3$	ξ_{me}	0.42	0.35	0.3	0.26	0.23	0.2	0.16
	η	0.79	0.87	0.91	0.93	0.94	0.94	0.95
	c	0.46	0.56	0.63	0.68	0.72	0.75	0.79
$\alpha = 0.4$	ξ_{me}	0.46	0.4	0.35	0.3	0.27	0.24	0.2
	η	0.77	0.85	0.89	0.92	0.93	0.94	0.95
	c	0.42	0.51	0.58	0.63	0.68	0.71	0.75
$\alpha = 0.5$	ξ_{me}	0.5	0.44	0.39	0.34	0.3	0.28	0.24
	η	0.75	0.83	0.88	0.9	0.92	0.93	0.94
	c	0.38	0.47	0.54	0.6	0.64	0.67	0.72
$\alpha = 0.6$	ξ_{me}	0.53	0.47	0.42	0.38	0.34	0.31	0.27
	η	0.73	0.82	0.87	0.9	0.91	0.93	0.94
	c	0.34	0.43	0.5	0.55	0.6	0.63	0.68
$\alpha = 0.8$	ξ_{me}	0.58	0.53	0.48	0.44	0.4	0.37	0.32
	η	0.71	0.8	0.85	0.88	0.9	0.92	0.93
	c	0.3	0.37	0.44	0.5	0.54	0.57	0.63
$\alpha = 1$	ξ_{me}	0.62	0.57	0.52	0.48	0.44	0.42	0.36
	η	0.69	0.78	0.83	0.86	0.89	0.9	0.92
	c	0.26	0.34	0.4	0.45	0.49	0.53	0.59
$\alpha = 1.2$	ξ_{me}	0.65	0.6	0.56	0.52	0.48	0.45	0.4
	η	0.68	0.76	0.82	0.85	0.88	0.89	0.92
	c	0.24	0.3	0.37	0.41	0.45	0.49	0.55
$\alpha = 1.6$	ξ_{me}	0.7	0.65	0.61	0.58	0.54	0.51	0.46
	η	0.65	0.74	0.79	0.83	0.86	0.88	0.9
	c	0.2	0.26	0.31	0.35	0.39	0.43	0.49
$\alpha = 2$	ξ_{me}	0.73	0.69	0.65	0.62	0.59	0.56	0.51
	η	0.53	0.72	0.78	0.82	0.85	0.87	0.89
	c	0.17	0.22	0.27	0.31	0.35	0.38	0.41
$\alpha = 2.4$	ξ_{me}	0.76	0.72	0.69	0.65	0.63	0.6	0.55
	η	0.62	0.71	0.76	0.8	0.83	0.86	0.88
	c	0.15	0.2	0.24	0.28	0.31	0.34	0.4

Note. For singly-reinforced rectangular shapes and T-beams with flanges in the tension zone, the values of ξ_{me}, η, and c are determined from Table 13 when $\gamma' = 0$.

Maximum deflections of bending members, as established by building code, are presented in Table 14.

Table 14

Maximum Deflections for Bending Members

Element considered	Maximum deflection (given as a ratio of the span)
1. Overhead-crane girder:	
hand operated	1/500
electrically operated.	1/600
2. Flat-ceiling floor construction:	
$l < 7$ m	1/200
$l \geqslant 7$ m	1/300
3. Ribbed-ceiling floor construction and ribbed stair flight:	
$l < 5$ m	1/200
5 m $\leqslant l < 7$ m	1/300
$l \geqslant 7$ m	1/400
4. Roof construction in industrial buildings	
$l < 7$ m	1/200
$l \geqslant 7$ m	1/300

Note. If ceilings are plastered, the deflection from the live load must not exceed $\frac{1}{350}l$.

Deflection is computed on the basis of rigidity B by the methods of Structural Mechanics. Thus, in a simple example of a freely supported bending member bearing a uniform load q^s the deflection in the middle of the span

$$f=\frac{5q^s l^4}{384B}.$$

The actual conditions at the supports are a very important factor in deflection calculations. If the supports of the beam are built in properly and well grouted, deflection is decreased in the middle of the span. The building code recommends that the bending moment at the supports for slabs, floor panels, etc., built into the wall, be taken as 15% of the moment of a freely supported member. In this case, with the element elastically fixed at both ends and assuming a uniform B rigidity for its entire length, the deflection in the middle of the span will be

$$f=0.0107\frac{q^s l^4}{B}. \tag{103}$$

The σ_s stresses are likewise determined through the M^s moment, decreased by 15%, i.e., it is considered that the moment in the span's centre

$$M^s=0.106\,q^s l^2. \tag{104}$$

4. Calculating Fissure Widths

Fissures will form in the concrete's tension zone when its stresses are equal to, or exceed, its ultimate tensile resistance, i.e., when $\sigma_{c.t}=\frac{M^s}{W_{fis}} \geqslant R_t$. Under these circumstances the stress lines will take the form shown in Fig. 28 (Stage Ia).

The elasto-plastic moment during fissure formation in Stage Ia for a rectangular beam will be

$$W_{fis}=(0.292+0.75\alpha_1)\,bh^2 \tag{105}$$

and for a T-beam whose flange is in the compression zone

$$W_{fis}=(0.292+0.75\alpha_1+0.0075\gamma_1')\,bh^2, \tag{106}$$

in which

$$\alpha_1=2n\frac{F_s}{bh};\quad \gamma_1'=\frac{2(b_{fl}'-b)\,h_{fl}}{bh}.$$

If the trivial deformation of the tension-stressed concrete be ignored, the width of the fissures a_{fis} can be computed through mean bar deformation $\varepsilon_{s.me}$ and fissure spacing l_{fis}:

$$a_{fis}=\varepsilon_{s.me}\,l_{fis}=\frac{\psi\sigma_s}{E_s}\,l_{fis} \tag{107}$$

where ψ is taken from Tables 11 and 12 just as for rigidity computations;

σ_s is determined by formula (100);

l_{fis}—fissure spacing as computed for smooth bars by the formula

$$l_{fis}=k_1nu, \tag{108}$$

where k_1 is found in Tables 11 and 12 under the knowns α and γ,

$$n=\frac{E_s}{E_c};\qquad u=\frac{F_s}{S};$$

S—the cross-sectional perimeter of the bars.

The values of l_{fis}, derived from formula (108), are multiplied by 0.5 for intermittently deformed bars, and by 1.25 for cold-drawn rods in prefabricated mats and blocks.

In silos, smokestacks, and in structures subjected to repeated dynamic loads or exposure to the weather or to very moist air (more than 60% humidity), fissure width a_{fis} must be verified and must not exceed 0.2 mm.

Sec. 12. ILLUSTRATIVE PROBLEM IN CALCULATING AND DESIGNING A REINFORCED CONCRETE BEAM

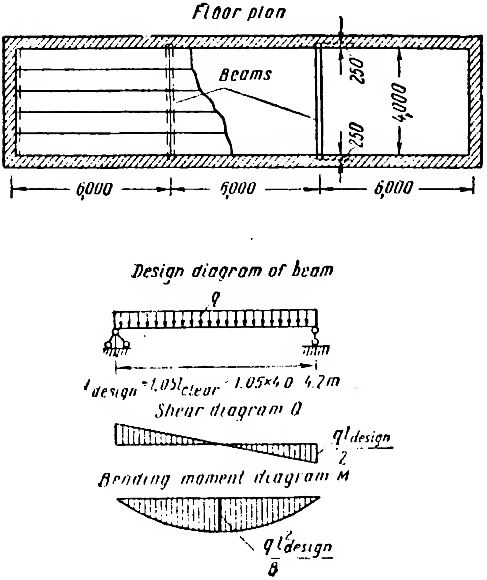


Fig. 44a. Illustrating problem 10: floor plan, and structural and moment diagrams; $l_{clear}=4$m

Illustrative problem 10. Given: a single-span floor beam in a factory building; clear span between walls $l_{cl}=4$ m (Fig. 44,*a*), centre spacing of beam is 6 m; depth of beam bearing is 0.25 m. Loading is uniform: specified dead load is 300 kg/m², specified live load is 500 kg/m². Live load coefficient $n_1=1.1$, dead load coefficient $n_2=1.2$. Grade of concrete 150-A. Intermittently deformed St-25Γ2C steel longitudinal bars, hot-rolled St-3 steel upright bars (stirrups). Service coefficient $m=1$, maximum allowable fissures $a_{fis}=0.2$ mm.

Compute the beam and design it in 2 alternatives: with prefabricated and with tied blocks (single bars).

Solution.

1) Computing the load for 1 metre of the beam's length. The specified load will be:

a) dead load of floor construction:
300×6=1,800 kg per metre length.

To find the dead load of the floor construction, the beam's cross-section is roughly assumed as

$$h\approx\frac{1}{10}l=40 \text{ cm},\qquad b=0.5h=20 \text{ cm};$$

$$\underline{0.4\times0.2\times2{,}500=200 \text{ kg per metre length.}}$$
$$\text{In all, } g^s=2{,}000 \text{ kg per metre length.}$$

b) live load: $p^s=500\times6=3{,}000$ kg per metre length.
Total design load,

$$q=g^sn_1+p^sn_2=2{,}000\times1.1+3{,}000\times1.2=2{,}200+3{,}600=5{,}800 \text{ kg per metre length.}$$

2) Determining the design forces M and Q. Effective span $l_{des}=1.05\times l_{cl}=1.05\times4=4.2$ *m.*

Maximum moment in the middle of the span

$$M=\frac{ql^2_{des}}{8}=\frac{5{,}800\times4.2^2}{8}=12{,}800 \text{ kg m.}$$

Maximum shearing force at the supports

$$Q=\frac{ql_{des}}{2}=\frac{5{,}800\times4.2}{2}=12{,}200 \text{ kg.}$$

3) Gathering design data for determining the cross-section:

$$R_{be}=85 \text{ kg/cm}^2,\ R_t=5.8 \text{ kg/cm}^2.$$

For longitudinal bars $R_s=3{,}400$ kg/cm²; $m_s=1$. For upright bars (stirrups), $R_s=2{,}100$ kg/cm²; $m_s=1$; $m_{di}=0.8$

4) Computing resistance at a normal plane.

Assuming the width $b=20$ cm and $\alpha=0.4$, the rough height of the beam is first determined:

From Table 10, $r_0=1.77$.

Formula (39) gives

$$h_0=1.77\sqrt{\frac{1{,}280{,}000}{1\times 85\times 20}}=48 \text{ cm};\ h=48+3=51 \text{ cm},$$

and the final decision is $h=50$ cm; $h_0=47$ cm, $b=20$ cm.

Then from formula (37) $A_0=\dfrac{1{,}280{,}000}{1\times 85\times 47^2\times 20}=0.342.$

From Table 10, $\gamma_0=0.78$.

From formula (40)

$$F_s=\frac{1{,}280{,}000}{1\times 1\times 3{,}400\times 0.78\times 47}=10.3 \text{ cm}^2,$$

which is satisfied by 4ø18DA ($F_s=10.18$ cm²) placed in one tier. If prefabricated blocks are used, two of them with effective bars on both sides of each are placed in the beam width.

5) Computing diagonal-plane resistance when the reinforcement consists of prefabricated blocks.

Checking whether condition (64) need be observed for the diagonal plane:

$$mR_tbh_0=1\times 5.8\times 20\times 47=5{,}450 \text{ kg}<Q=12{,}200 \text{ kg},$$

which shows that the condition must be observed.

Table 4 gives the minimum diameters of upright bars of blocks to satisfy welding requirements: $d_2=8$ mm; $f_{st}=0.5$ cm²; $n=2$.

From formula (75), $q_{st}=\dfrac{12{,}200^2}{0.6\times 85\times 20\times 47^2}=66$ kg/cm.

Spacing of upright bars: resistance requirements by formula (76) gives

$$u=\frac{1\times 0.8\times 2{,}100\times 0.5\times 2}{66}=25.5 \text{ cm};$$

maximum spacing is derived from formula (79)

$$u_{max}=\frac{0.1\times 85\times 20\times 47^2}{12{,}200}=30.8 \text{ cm};$$

and condition (79a) indicates that

$$u\leq\frac{h}{2}=\frac{50}{2}=25 \text{ cm}.$$

Therefore the final decision is $u=25$ cm.

Determine data for beam design as required for bending-moment resistance at the diagonal planes.

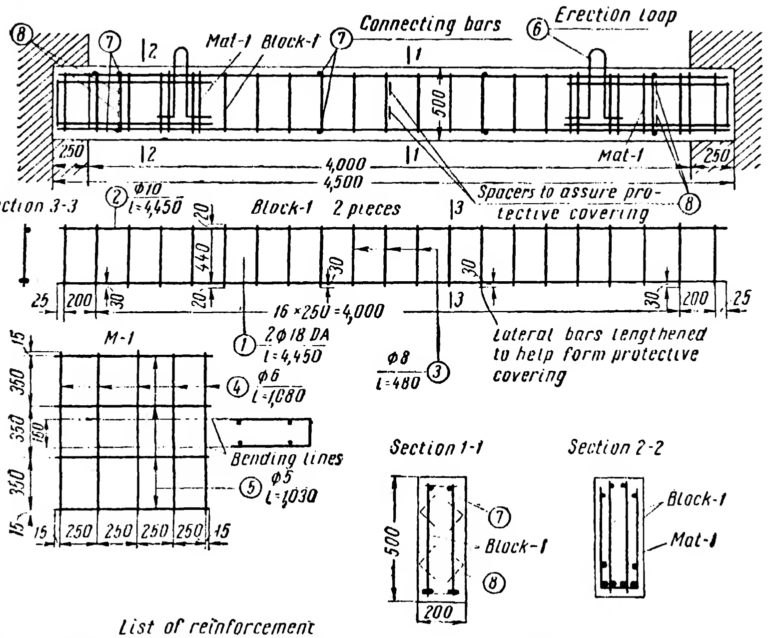
All longitudinal tension bars are carried to the supports. The length l_{pro} of bars protruding beyond the supports must equal $15d=15\times 1.8=27$ cm, which cannot be fulfilled with the given 25-cm depth of beam bearing. Protrusion l_{pro} can only be 23 cm, which, being more than $10d$, does not require special anchorage; the area of the upright bars need merely be increased 50% for a length of $l_{pro}+1.5h=23+1.5\times 50=98$ cm, or ≈ 1 m.

Pan-shaped mats are installed along this length. Their bent bars, spaced identically with the upright bars ($u=25$ cm), must have an area of

$$f_{mat}=\frac{f_{st}}{2}=\frac{0.5}{2}=0.25\ \text{cm}^2,$$

which is satisfied by ø6 ($f_{gri}=0.283$ cm²).

Fig. 44,*b* illustrates the working drawing of the beam.



List of reinforcement

Reinforcement	Bar sketch	Position	Diameter, mm	Length, mm	Quantity (n) 1 member	Quantity (n) Total	lin. m	Weight per m, kg	Total weight
Block-1 2 pcs	See drawing	1	Φ18 DA	4,450	2	4	17.8	2.0	35.6
		2	Φ10	4,450	1	2	8.9	0.62	5.5
		3	Φ8	480	19	38	18.2	0.39	7.1
Mat-1 2 pcs	—"—	4	Φ6	1,080	5	10	10.8	0.22	2.4
		5	Φ5	1,030	4	8	8.2	0.15	1.3
Separate bars	⊐	6	Φ12	1,000	18 DA	2	2.0	0.89	1.8
	—	7	Φ6	180	18 DA	8	1.5	0.22	0.3
	<	8	Φ6	100	6	12	1.2	0.22	0.3

Total $G=55$ kg

Volume of concrete, $V=0.45$ m³

Steel content:

$$K=\frac{G}{V}=\frac{55}{0.45}=122\ \text{kg/m}^3$$

Fig. 44b. Illustrating problem 10: the beam is reinforced with prefabricated (welded) blocks

6) Computing the diagonal plane resistance when the reinforcement consists of tied blocks.

U-shaped stirrups are used, ø6 mm, $f_{st}=0.283$ cm², $n=2$.

Their spacing is given by formula (79): $u=30.8$ cm.

Condition (79a) dictates that $u=25$ cm.

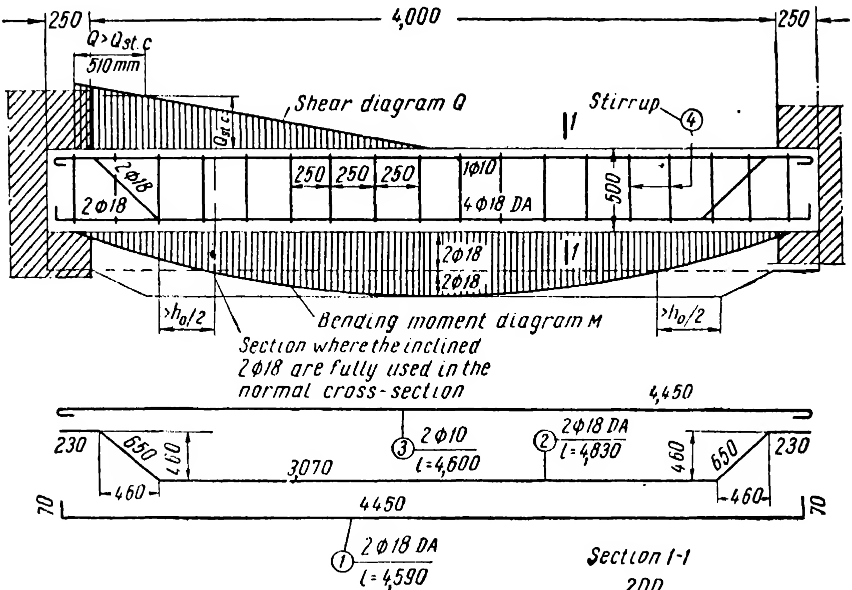
From formula (70)

$$q_{st}=\frac{1\times 0.8\times 2{,}100\times 0.283\times 2}{25}=38 \text{ kg/cm},$$

and from formula (69)

$$Q_{st.c}=\sqrt{0.6\times 85\times 20\times 47^2\times 38}\doteq 9{,}250 \text{ kg}<\frac{Q}{m}=12{,}200 \text{ kg},$$

hence, bent-up bars are required.



List of reinforcement

Bar No.	Bar sketch	Diameter, mm	l, mm	pcs n	l:n.m	Weight, lin m	Total weight, kg
1	See drawing	Ф18 DA	4,590	2	9.18	2.0	18.4
2	—"—	—"—	4,830	2	9.66	—"—	19.4
3	—"—	Ф10	4,600	2	9.20	0.62	5.7
4	—"—	Ф6	1,340	18	24.2	0.22	5.3

Total G=49 kg

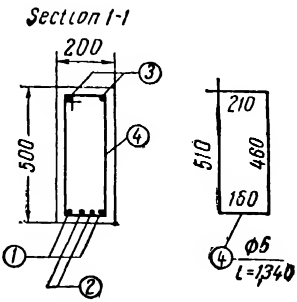


Fig. 44c. Illustrating problem 10: when the beam reinforcement consists of tied blocks

The length of the beam, upon which $\frac{Q}{m}>Q_{st.c}$, is 51 cm (Fig. 44,*c*); therefore only one bend transverse is required for the inclined bars (bent at 45°), with a bar area, from formula (73),

$$F_0=\frac{12{,}200-9{,}250}{1\times 0.8\times 3{,}400\times 0.707}=1.9 \text{ cm}^2,$$

which is satisfied by 2ø18DA (F_0=5.09 cm²).

The required two protrusions ($l_{pro}=15\times1.8=27$ cm) of the bars that are continued to the supports, are fulfilled by making suitable bends at the ends of these bars.

In the course of designing, an additional check must be made of the correct position of the bent-up bars.

7) Deflection computations.

$E_c=165{,}000$ kg/cm²; $E_s=2{,}100{,}000$ kg/cm²; $n=\frac{E_s}{E_c}=12.7$; $\mu=\frac{10.18}{20\times47}=0.0108$; $\alpha=3\mu n=3\times0.0108\times12.7=0.41$; $\gamma'=0$.

From Table 13 $\eta=0.77$; $c=0.42$.

The specified load

$q^s=g^s+p^s=2{,}000+3{,}000=5{,}000$ kg per metre of length.

The bending moment from the specified load

$$M^s=\frac{5{,}000\times4.2^2}{8}=11{,}000 \text{ kg m}.$$

From formula (100) $\sigma_s=\frac{1{,}100{,}000}{10.18\times0.77\times47}=3{,}000$ kg/cm².

When $\sigma_s=3{,}000$ kg/cm² and $\alpha=0.41$, Table 11 shows that $\psi=0.99$ and $k_1=3.4$.

From formula (101) beam rigidity from the momentary load will be

$$B_{mo}=\frac{2{,}100{,}000}{0.99}\times10.18\times0.42\times47^2=2\times10^{10}\text{ kg/cm}^2.$$

When both the constant g^s, and momentary acting p^s, loads have been ascertained, then rigidity is computed by formula (102) for protracted action of the loads:

$g^s=2{,}000+(500-150)6=4{,}100$ kg per metre of length;

$p^s=150\times6=900$ kg per metre length;

$q^s=4{,}100+900=5{,}000$ kg per metre of length; $\Theta=2$;

$$B=2\times10^{10}\frac{5{,}000}{4{,}100\times2+900}=1.1\times10^{10}\text{ kg/cm}^2.$$

The deflection in the middle of the span

$$f=\frac{5\times ql^3}{384\times B}l=\frac{5\times50\times420^3}{384\times1.1\times10^{10}}l=\frac{1}{226}l<\frac{1}{200}l.$$

8) Computation of fissure width.

The total cross-sectional perimeter of the bars

$$S=4\times3.14\times1.8=22.6\text{ cm};$$

$$u=\frac{F_s}{S}=\frac{10.18}{22.6}=0{,}45.$$

From formula (108), fissure spacing

$$l_{fs}=3.4\times12.7\times0.45=19.4\text{ cm},$$

but with an allowance for intermittently deformed bars

$$l_{fis}=0.5\times19.4=9.7\text{ cm}.$$

From formula (107) fissure width will be:

$$a_{fis}=0.99\frac{3{,}000}{2{,}100{,}000}9.7=0.0137\text{ cm}\approx0.14\text{ mm}<0.2\text{ mm}.$$

CHAPTER IV

AXIALLY AND ECCENTRICALLY COMPRESSED ELEMENTS

Sec. 13. AXIALLY COMPRESSED ELEMENTS

1. Their Design

When a longitudinal compressive force N acts along the geometric axis of a reinforced concrete element, the latter is subject to axial compression (Fig. 45, *a*). Columns are the most typical examples of axially compressed members (Fig. 45, *b*). Other members that undergo this type of compression are the top chord of trusses whose loads are transmitted to them via their joints (Fig. 45, *c*), truss posts and struts, and various other kinds of elements. Axially loaded members are usually rectangular (or square) in section and reinforced with reinforcement blocks made up of longitudinal effective bars whose area is derived through calculation, and of stirrups, the latter preventing bulging of the longitudinal bars under load.

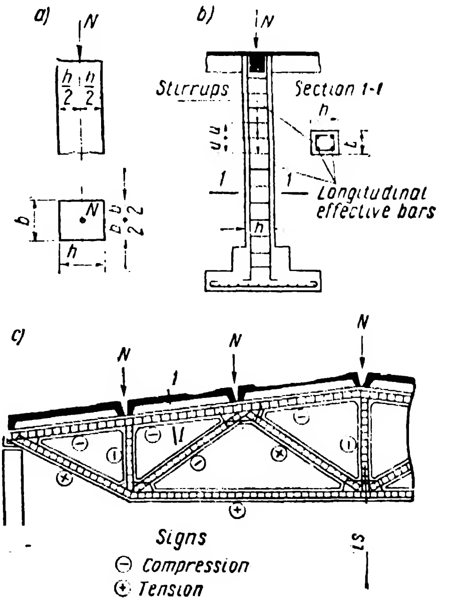


Fig. 45. Axially compressed reinforced concrete elements

Columns are made in multiples of 5 cm in their cross-section, and usually not less than 25×25 cm; longitudinal effective reinforcement is placed about the sectional perimeter with allowance for protective embedment. In order to pack the concrete properly, the least clear distance between bars is dependent upon the method

of concrete pouring; this minimum must be 5 cm for vertically poured columns (as in in-situ work), but if the element is prefabricated in a horizontal position (precast columns and trusses) the clear distance may be the same as for bending members.

12-40 mm bars are used for effective longitudinal reinforcement, whose area F_s in relation to the concrete area of the member F_c is known as the coefficient of reinforcement μ:

$$\mu = \frac{F_s}{F_c}. \tag{109}$$

This coefficient, multiplied by 100, is the percentage of reinforcement $\mu\%$:

$$\mu\% = \frac{F_s}{F_c} \times 100. \tag{110}$$

If $\mu \leqslant 3\%$, then the area F_c is assumed to equal the entire area of the member F. But when $\mu > 3\%$, then it is considered that $F_c = F - F_s$. Commonly, $\mu = 1$-2%; the minimum allowed by building code is 0.5%.

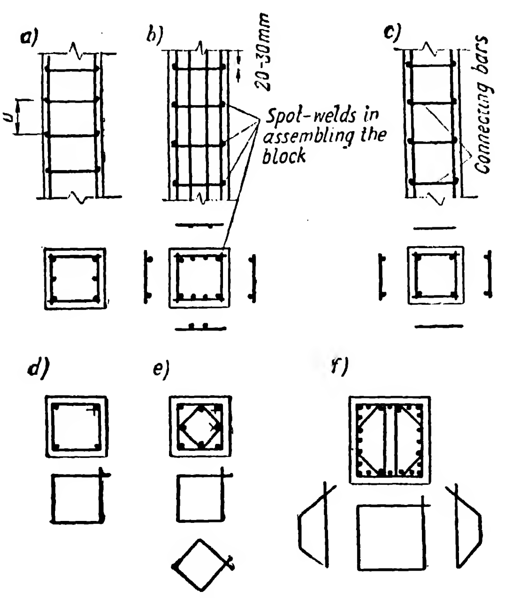


Fig. 46. Reinforcing axially compressed columns

Prefabricated blocks may be made either as a complete unit (Fig. 46, *a*) or welded from several flat frames (Fig. 46, *b* and *c*).

In tied blocks the stirrup bars are bent to close around the perimeter (Fig. 46, *d*). If the effective reinforcement is closely spaced, additional rhombus or trapezoidal stirrups are used together with the rectangular stirrups, so that at least every other longitudinal bar is embraced by a stirrup corner (Fig. 46, *e* and *f*).

The diameter d of the longitudinal compression reinforcement qualifies the diameters of the stirrups d_s. Thus, $d_{st} \geqslant 25d$ (but not less than 6 mm) for stirrups of hot-rolled St-0 and St-3 rods. For cold-drawn stirrups, $d_{st} \geqslant 0.2d$ (but not less than 5 mm). The minimum diameter of stirrups is also regulated by welding conditions (see Table 4). In order to hold, and prevent bulging of, the effective bars, their diameters d are also a factor in determining stirrup

spacing u: in prefabricated blocks $u \leqslant 20d$; in the tied type of blocks $u \leqslant 15d$. Furthermore, structural considerations dictate that stirrup spacing must not exceed the least sectional dimension of the given member b, nor be more than 40 cm. In tied blocks where the bars are overlapped without welding, and also where $\mu > 3\%$, stirrup spacing cannot be more than $10d$ of effective bars.

When a comparatively small section must resist a heavy longitudinal force, round or polygonal columns are sometimes adopted, reinforced with longitudinal bars bound either by a spiral or welded hoop-ties (perpendicular reinforcement). The carrying capacity of the member is considerably raised by such spirals or ties (Fig. 47), which create a casing around the concrete core and prevent transverse deformation of the latter.

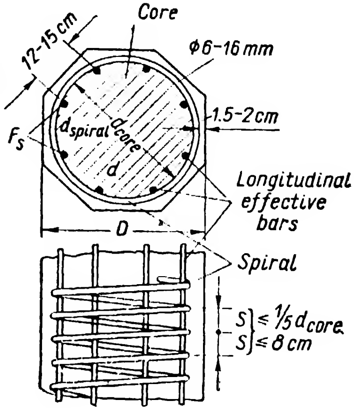


Fig. 47. Spiral reinforcement in a column

2. Computation Formulae for Rectangular Axially Compressed Elements

Let us examine the stress deformation in the section of an axially compressed reinforced concrete member subject to a force N (Fig. 48, *a*). The stress in the concrete area F_c is equal to σ_c, while that in the steel area F_s is equal to σ_s. The outer force N is at all times balanced by the internal resistance of the concrete and bars:

$$N = \sigma_c F_c + \sigma_s F_s. \quad (111)$$

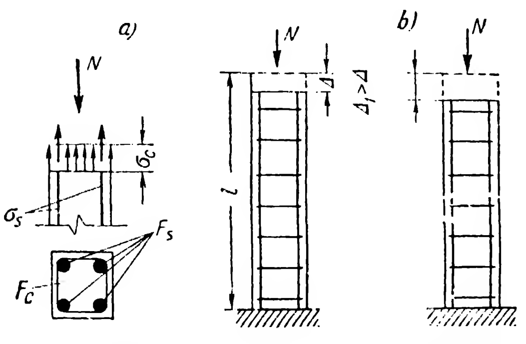


Fig. 48. Behaviour of an axially compressed member

a—when first loaded; *b*—after creep has set in: $\sigma_{s \cdot 1} > \sigma_s$; $\sigma_{c \cdot 1} < \sigma_c$

Concrete creep effects a redistribution of stresses between the concrete and steel (Fig. 48, *b*), and tests have proved that failure of an axially compressed member occurs when both the concrete and steel are subject to ultimate stresses. From this, condition (111) for strength takes the form of

$$N \leqslant m \left(R_{pr} F_c + m_s R_s F_s\right), \quad (112)$$

which, after F_c has been removed from the brackets and when it is considered that $\frac{F_s}{F_c}=\mu$, becomes

$$N \leqslant mF_c(R_{pr}+m_sR_s\mu). \tag{113}$$

The ultimate compression of concrete, i. e., its comparative deformation at failure $\varepsilon_{com}=0.002$. The bars undergo the same deformation at failure because of the concrete-steel bond, hence their stress cannot exceed $\sigma_s=0.002\times2,100,000=4,200$ kg/cm². If reinforcement is used that possesses either a higher yield point or greater ultimate stress (hard steels), then an R_s equal to 4,200 kg/cm² is introduced into formulae (112) and (113). This implies that high-strength bars are not rational for axially compressed elements, inasmuch as the steel's full resistance cannot be taken advantage of.

3. Buckling

Experiments have established that only if the buckling of an axially compressed member does not cause it to lose its general stability, failure will be due to the attainment of ultimate stresses in the concrete and steel. In flexible (very long) elements, destruction by buckling will take place even before the stresses in the concrete and reinforcement bars reach R_{pr} and R_s, respectively. The lowered carrying capacity of flexible axially compressed members is accounted for by the coefficient $\varphi<1$ introduced into formulae (112) and (113) and which depends upon the flexibility of the element:

$$N \leqslant \varphi m(R_{pr}F_c+m_sR_sF_s) \tag{114}$$

or

$$N \leqslant \varphi mF_c(R_{pr}+m_sR_s\mu). \tag{115}$$

Flexibility is judged by the relationship of the design length l_0 to the least radius of gyration r of the section. The design length depends upon how the ends of the member are fixed: if both ends are stationary-hinged, the unsupported length will equal the actual length, i. e., $l_0=l$; if one end is fixed while the other is stationary-hinged, then $l_0=0.7l$; with both ends fixed, $l_0=0.5l$, and, finally, with one end fixed and the other free (unrestrained support), $l_0=2l$. In all other cases the unsupported length will depend upon the actual condition of fixing.

Tests have proved that a decrease in carrying capacity due to buckling will occur at a flexibility of $\frac{l_0}{r}>50$, which corresponds to a relationship of $\frac{l_0}{b}>14$ for a rectangular section. Rectangular columns with a relationship of $\frac{l_0}{b}>30$ or $\frac{l_0}{h}>25$ are, as a rule, not used.

Values for buckling coefficient φ in computing axially loaded reinforced concrete members are presented in Table 15.

Table 15

Buckling Coefficients φ for Reinforced Concrete Members

l_0/r	50	55.4	62.2	69	76	83	90	97	104
l_0/b	14	16	18	20	22	24	26	28	30
φ	1	0.88	0.8	0.73	0.67	0.62	0.57	0.53	0.5

4. Determining the Cross-Section of Axially Loaded Members

Two types of problems are met with in determining the cross-section of an axially compressed member when the knowns are the longitudinal effective force, the member's effective length, and the grades of concrete and steel.

1st type. The sectional dimensions are known. The area of the bars F_s is to be found.

Sequence of procedure.

1) $\frac{l_0}{b}$ is determined, the coefficient φ is taken from Table 15 and, depending upon the sectional dimensions, the coefficient m* is found.

2) From formula (114)

$$\frac{\frac{N}{\varphi m} - R_{pr}F_c}{m_s R_s} \tag{116}$$

If $F_s<0$ or $\mu<0.5\%$, then either installation bars are adopted with $\mu_{min}=0.5\%$, or the cross-section is diminished (if possible), or the grade of concrete is lowered.

2nd type. Both the bar area and sectional dimensions are to be determined.

Sequence of procedure.

1) It is assumed that $\varphi=1$ and $\mu=1\%$ $(\mu=0.01)$.

2) From formula (115)

$$F_s=\frac{N}{\varphi m\,(R_{pr}+m_s R_s \mu)}\,. \tag{117}$$

* See Chapter II, Sec. 4, Item 4.

3) Sectional dimensions are established. With a square section $b=\sqrt{F_c}$. If necessary, a multiple of 5 cm is adopted and F_s is rectified. For an in-situ column less than 30×30 cm, $m=0.8$

4) φ and F_s are found as in the 1st type of problem.

5) Formula (110) gives the value of μ. If the percentage of reinforcement is too high or too low, the dimensions are altered and F_s is again evolved as in the 1st type of problem.

When the carrying capacity of a given cross-section is to be found, the procedure will be as follows:

1) $\frac{l_0}{b}$ is determined and φ is taken from Table 15.

2) N_{cs} is derived from formula (114) or (115).

Illustrative problem 11. Given: longitudinal design load $N=140$ t, cross-section of column is equal to 40×40 cm, grade 200-B concrete, bars to be hot-rolled St-5 intermittently deformed steel, member's design length $lo=6.4$ m. Find the bar area F_s.

Solution.

1) Design data gathered: $R_{pr}=80$ kg/cm², $R_s=2{,}400$ kg/cm², $m_s=1$, $m=1$ (column dimensions exceed 30×30 cm).

2) Solving: $\frac{l_0}{b}=\frac{640}{40}=16>14$, and Table 15 gives $\varphi=0.88$.

3) From formula (116)

$$F_s=\frac{\frac{140{,}000}{0.88}-80\times 40\times 40}{2{,}400}=12.9 \text{ cm}^2,$$

which is satisfied by 4ø20 D ($F_s=12.56$ cm²).

Stirrups are of 6 mm rolled wire ($d_{st}>0.25d$) or 5 mm cold-drawn rods ($d_{st}>0.2d$). If the reinforcement consists of a prefabricated block, 8 mm lateral ties must be used to satisfy welding requirements (see Table 4). Stirrup spacing:

$u=20d=20\times 2=40$ cm for prefabricated blocks;

$u=15d=15\times 2=30$ cm for tied blocks.

Illustrative problem 12. Given: longitudinal force $N=75$ t, grade 150-A concrete, bars to be of St-3 steel, $lo=5.2$ m. Find the column's sectional dimensions and bar area F_s.

Solution.

1) Gathering design data: $R_{pr}=70$ kg/cm², $R_s=2{,}100$ kg/cm², $m_s=1$.

2) Assume that $\varphi=1$, $m=1$, $\mu=0.01$ (1%).

3) From formula (117)

$$F_c=\frac{75{,}000}{70+2{,}100\times 0.01}=825 \text{ cm}^2;$$

$b=\sqrt{825}=28.6$ cm.

This is satisfied by a 30×30 cm column.

$$F_s=30\times 30=900 \text{ cm}^2.$$

4) Solving: $\frac{l_0}{b}=\frac{520}{30}=17.3$, and $\varphi=0.83$ according to Table 15.

5) When $m = 1$, formula (116) gives

$$F_s = \frac{\frac{75,000}{0.83} - 70 \times 30 \times 30}{1 \times 2,100} = 13.1 \text{ cm}^2, \text{ i.e.,}$$

$$\mu\% = \frac{13.1}{30 \times 30} \times 100 = 1.46\%$$

Therefore the formerly adopted sectional dimensions can be left unchanged and the bars will be 4ø20 ($F_s = 12.56$ cm²).

Illustrative problem 13. Given: an axially loaded 25 × 25 cm in-situ member (design length $l_0 = 5$ m) made of grade 200-A concrete and reinforced with St-5 intermittently deformed 4ø16 D ($F_s = 8.04$ cm²).

Find its carrying capacity.

Solution.

1) Gathering design data: R_{pr}=90 kg/cm², R_s=2,400 kg/cm², m_s=1, m=0.8 (the cross-section is less than 30 × 30 cm).

2) Solving: $\frac{l_0}{b} = \frac{500}{25} = 20$, from which φ=0.73 by Table 15.

3) From formula (114) the ultimate force borne by the member
N_{cs}=0.73 × 0.8 (90 × 25 × 25 + 1 × 2,400 × 8.04)=44,000 kg=44 t.

4) At a mean load coefficient n_{me} of 1.2, the allowed specified load

$$N^s = \frac{N}{n_{me}} = \frac{44}{1.2} = 36.7 \text{ t.}$$

Sec. 14. ECCENTRICALLY LOADED ELEMENTS

1. Their Design

Eccentric compression is created either when a longitudinal force N is applied outside the centre of gravity of a section, or when the latter is effected simultaneously by axial compression N and a bending moment M (Fig. 49, *a*). Here the eccentricity of the force, i.e., the distance from the centre of gravity to the force

$$e_0 = \frac{M}{N}. \tag{118}$$

Examples of eccentrically compressed construction are: members of bents, such as columns of frame buildings (Fig. 49, *b* and *c*), top chords of trusses loaded otherwise than via their joints (Fig. 49, *d*), and various other elements.

Eccentrically compressed members may be either rectangular, or I- or T-shaped. As a rule the height h (along the plane of the moment) is made greater than the width b and is usually a multiple of 5 cm. The reinforcement of such members consists of longitudinal effective bars and stirrups, the latter connecting the whole into a welded or a tied three-dimensional system.

The effective bars are placed along the short sides of the section (Fig. 50), F'_s and F_s denoting the bars nearer to, and further from, the force N, respectively. If the areas F'_s and F_s are not equal, the

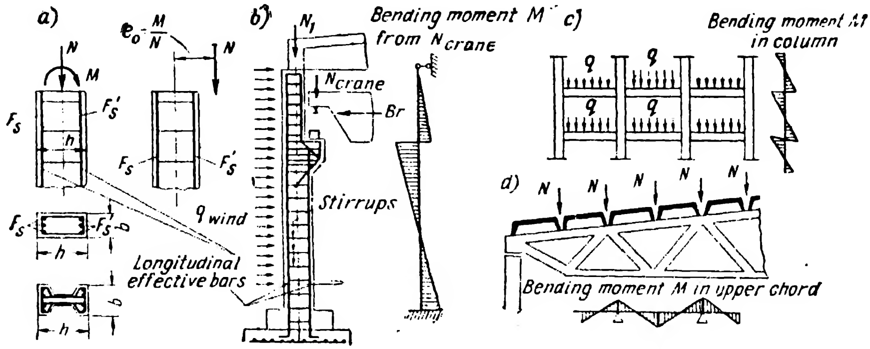


Fig. 49. Eccentrically compressed reinforced concrete members

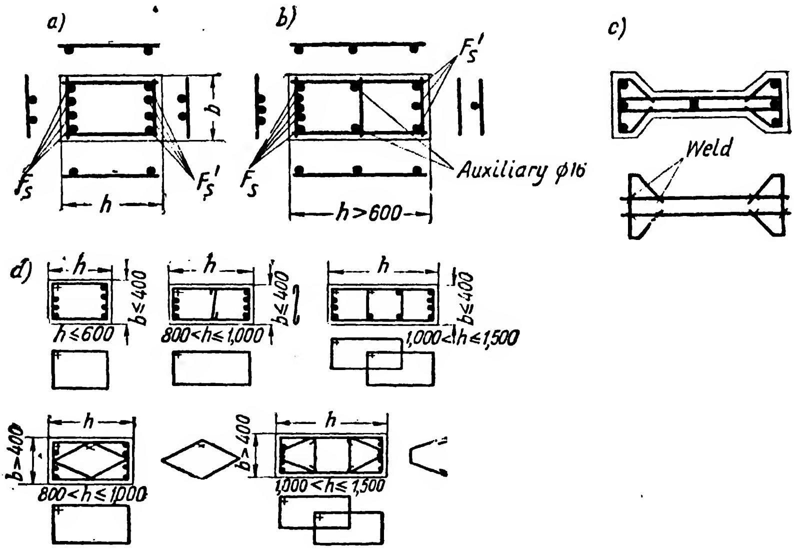


Fig. 50. Reinforcement of eccentrically compressed members

reinforcement is said to be unsymmetrical; and symmetrical if $F_s = F'_s$. The total bar area $(F'_s + F_s)$ is usually less in unsymmetrical reinforcement than in the symmetrical, but the former is very often more complicated than the latter. Consequently it is best to

symmetrise the reinforcement if it involves only a slight increase of bar area (up to 5%). Symmetrical reinforcement is also adopted if the bending moment might invert its sign.

When the height h exceeds 50-60 cm, auxiliary bars not less than 16 mm in diameter are arranged on the long side of the cross-section so that the distance between either effective or such supplementary bars does not exceed 50 cm (Fig. 50, *b*).

Reinforcement blocks are either completely fabricated by machine welding or made of individual frames welded together (Fig. 50, *a* and *b*). In irregular contours (Fig. 50, *c*) the stirrups are welded from separate rods. In tied blocks (Fig. 50, *d*), closed stirrups embrace the section's perimeter; if the effective bars are very numerous, rhombus or trapezoidal stirrups are added so that at least every other longitudinal bar is engaged in a stirrup corner. The stiffening rods on the long side of the member can be tied either with straight staples or grasped by the stirrups. Diameters and spacings of effective bars and stirrups (or upright bars of prefabricated blocks) are computed by the methods already given for axially compressed members. The least effective bar area on one side of the given section is established by code as 2% of the concrete area ($\mu=0.002$). The total bar area (F'_s+F_s) is usually not more than 3% of the section's concrete area.

2. Computation Formulae for Eccentrically Compressed Members

Experiments have proved that the behaviour of an eccentrically compressed element depends upon the magnitude of eccentricity of the longitudinal force $e_0=\frac{M}{N}$, in relation to the geometric axis. With a comparatively large eccentricity, the section will be partly under compression (near the applied force) and partly in tension (Fig. 51, *a*). Such behaviour is somewhat similar to a bending member, with failure occurring when the stresses in the tension steel F_s reach their ultimate value R_s. Here, just as in a beam, when the condition observed is $S_c\leqslant 0.8S_0$ ($x\leqslant 0.55h_0$ for a rectangular section), the stresses in the concrete's compression zone will attain R_{bc}. In the compression bars F'_s the stresses at failure will also equal R_s if the centre of gravity of the compression zone is not closer to the force N than the centre of gravity of the bars F'_s. For any kind of symmetrical shape, this results in the condition of $z\leqslant h_0-a'$ (and $x\geqslant 2a'$ for a rectangular shape). This type of failure in sections subject to eccentric compression is classified as *Case 1* (*behaviour under great eccentricity*).

When there is comparatively little eccentricity, then either the whole section will be under compression (Fig. 51, *c*), or its greater

part (Fig. 51, *b*). Failure will occur when ultimate stresses are attained in both the compression concrete and the compression bars F'_s. But the stresses will not reach their ultimate value in the bars F_s, which will be either in tension (Fig. 51, *b*) or compression (Fig. 51, *c*). This kind of failure is classified as *Case 2* (*behaviour under small eccentricity*) and takes place when $S_c > 0.8S_0$ ($x > 0.55h_0$ for a rectangular section).

Computation formulae for eccentrically compressed elements are derived through an analysis of the conditions of a balanced section subjected to design stresses in the concrete and steel.

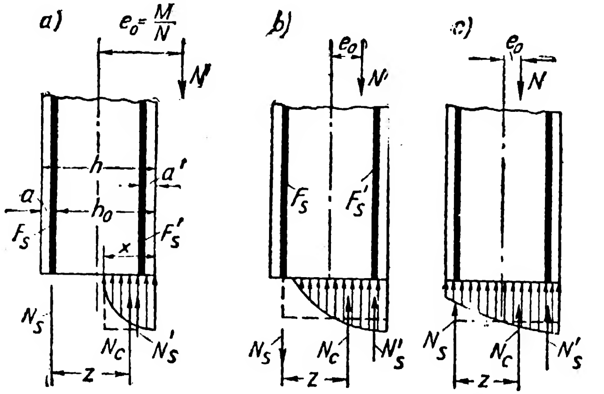


Fig. 51. Stresses in eccentrically compressed members

a) A Cross-Section of Any Symmetrical Shape. Notation:

S_0—static moment of the area of the entire concrete section in relation to the centre of gravity of the bars F_s;

S'_0—ditto in relation to the centre of gravity of the bars F'_s;

S_c—static moment of the compression-concrete area F_c in relation to the centre of gravity of the bars F_s;

$$S_c = F_c z; \tag{119}$$

e—distance from point of applied force N to the centre of gravity of the bars F_s;

e'—distance from point of applied force N to the centre of gravity of the bars F'_s.

Case 1 (The stresses are shown in Fig. 52, *a*).

Two conditions for resistance are evolved, i. e., 1) when the sum of all projected forces on the axis of the member is equal to zero, and 2) when the sum of the moments of all forces, in relation to the centre of gravity of the reinforcement F_s, is equal to zero:

$$N \leq m\,(R_{be}F_c + m_sR_sF'_s - m_sR_sF_s); \tag{120}$$

$$Ne \leq m\,[R_{be}S_c + m_sR_sF'_s\,(h_0 - a')]. \tag{121}$$

The position of the neutral axis is derived from condition (120).

As already stated, the condition to be observed to attain ultimate stresses in the bars F'_s is

$$z \leqslant h_0 - a'. \tag{122}$$

Case 2 (The stresses are given in Fig. 52, *b*).

Formulae for Case 2 are derived from empirical data: irrespective of the kind of stress diagram (either Fig. 51, *b* or Fig. 51, *c*), the

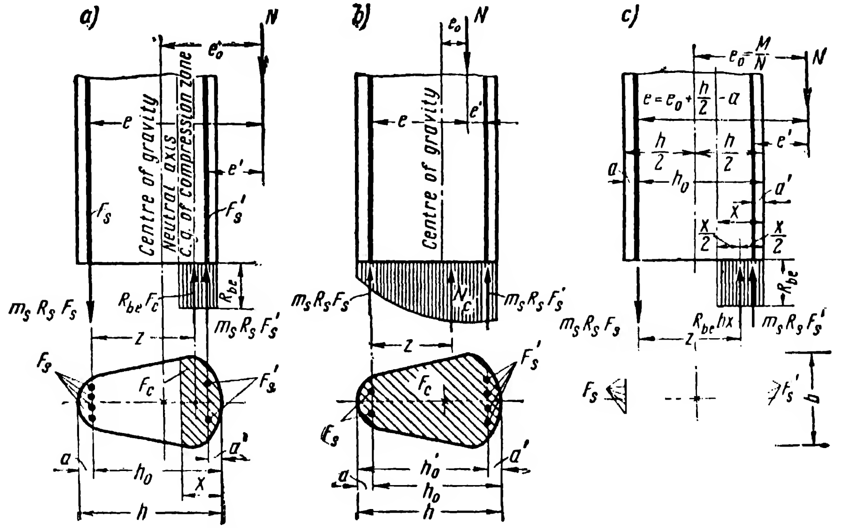


Fig. 52. Investigations of eccentrically compressed members

moment of the force N_c in relation to the centre of gravity of the bars F_s will be constant:

$$N_c z = \text{const} = R_{pr} S_0. \tag{123}$$

The condition of zero-equivalence of the sum of the moments of all forces in relation to the centre of gravity of the F_s and reinforcement F'_s, offers the formulae for resistance:

$$Ne \leqslant m\,[R_{pr} S_0 + m_s R_s F'_s\,(h_0 - a')]; \tag{124}$$

$$Ne' \leqslant m\,[R_{pr} S'_0 + m_s R_s F_s\,(h_0 - a')]. \tag{125}$$

When $S_c > 0.8 S_0$ and when the force is applied beyond the centres of gravity of the steel F_s and F'_s, i.e., when $e > h - a'$, the section's carrying capacity will be dictated only by condition (124).

b) A Rectangular Cross-Section. The practical way of finding the case for a rectangular eccentrically compressed cross-section

is by its eccentricity $e_0=\frac{M}{N}$: if $e_0>0.3h_0$, it will be Case 1; if $e_0\leqslant 0.3h_0$, it will be Case 2.

Case 1. $e_0>0.3h_0$; $x\leqslant 0.55h_0$; $x\geqslant 2a'$ (the stresses are shown in Fig. 52, *c*).

For a rectangular section

$$F_c=bx,$$

$$D_c=R_{bc}bx,$$

$$e=e_0+\frac{h}{2}-a,$$

$$e'=e-(h_0-a')$$

when the force N is applied beyond the space between the centres of gravity of the reinforcement F_s and F'_s,

$$e'=(h_0-a')-e;$$

when the force N is applied within the centres of gravity of the reinforcement F_s and F'_s,

$$z=h_0-\frac{x}{2},$$

$$S_0=bh_0\frac{h_0}{2}=0.5bh_0^2,$$

$$S'_0=0.5b(h'_0)^2,$$

$$S_c=bx\left(h_0-\frac{x}{2}\right).$$

Conditions (120) and (121) for resistance in rectangular sections give

$$N\leqslant m(R_{bc}bx+m_sR_sF'_s-m_sR_sF_s); \tag{126}$$

$$Ne\leqslant m\left[R_{bc}bx\left(h_0-\frac{x}{2}\right)+m_sR_sF'_s(h_0-a')\right]. \tag{127}$$

The position of the neutral axis is determined by condition (126).

Case 2. $e_0\leqslant 0.3h_0$; $x>0.55h_0$.

Condition (124) for resistance takes the form of

$$Ne\leqslant m[0.5R_{pr}bh_0^2+m_sR_sF'_s(h_0-a'), \tag{128}$$

and, correspondingly, condition (125), which must be observed if the force is applied between the centres of gravity of bars F_s and F'_s, takes the form of

$$Ne'\leqslant m[0.5R_{pr}b(h'_0)^2+m_sR_sF_s(h_0-a')]. \tag{129}$$

3. The Influence of Deflection in Eccentrically Compressed Members

An eccentrically applied load may cause considerable deflection in a long member and increase the initial eccentricity e_0 of the longitudinal force N (Fig. 53). This will increase the bending moment and failure will occur with less force N than in a similar, but shorter, section that would flex very little at failure. Development of plastic deformation in the concrete's compression zone also adds to this eccentricity. Long eccentrically compressed elements are calculated by already given formulae but with the introduction of a new, greater eccentricity* instead of the initial e_0:

$$e_0' = \eta e_0 \tag{130}$$

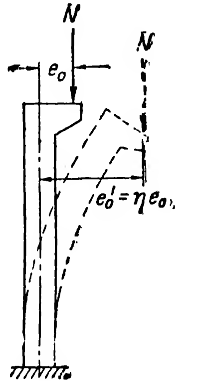


Fig. 53. How deflection of an eccentrically compressed element influences its behaviour under load

in which the coefficient η, introduced to increase the former initial value, is based on the amount of deflection of an eccentrically compressed homogeneous shaft, with allowances made for the plastic properties of concrete on the basis of empirical data.

For any symmetric shape

$$\eta = \frac{1}{1 - \frac{N}{m \times 4{,}800 \times R_{be} F}\left(\frac{l_0}{r}\right)^2}, \tag{131}$$

and for a rectangular shape

$$\eta = \frac{1}{1 - \frac{N}{m \times 400 R_{be} F}\left(\frac{l_0}{h}\right)^2},$$

where F—area of the section;
r—radius of gyration of the section;
h—dimension of the rectangular section in the plane of the moment.

Just as in axially loaded members, the design length of the element l_0 is determined in accordance with the type of the end connections.

The diagram in Fig. 54 is practical for determining values of the coefficient η.

Deflection is not taken into account when length is $l_0/r \leqslant 35$, which latter corresponds to a relationship of $l_0/h \leqslant 10$ in a rectangular section.

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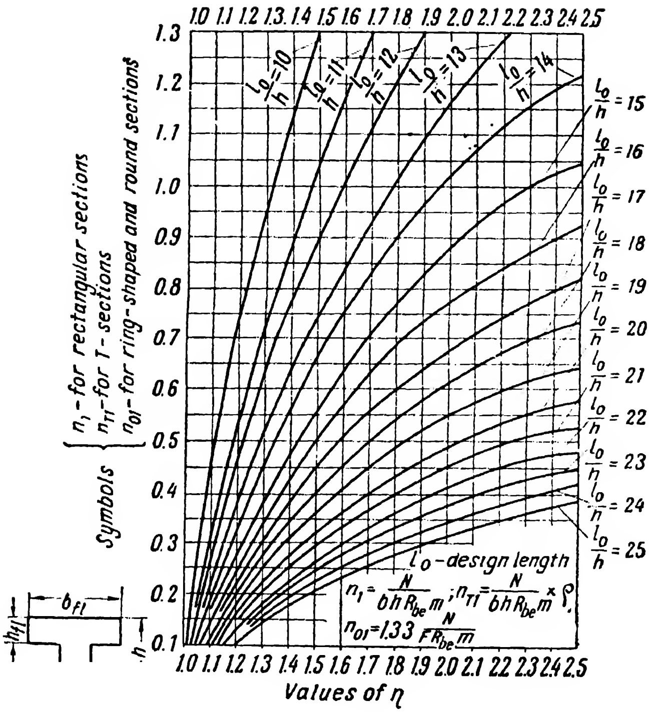
* With the exception of formulae (125) and (129).



b_{f1}/b \ h_{f1}/h	2	3	5	10	15
0.1	0.82	0.71	0.59	0.45	0.42
0.2	0.76	0.64	0.53	0.44	0.39
0.3	0.72	0.62	0.53	0.44	0.39
0.4	0.71	0.62	0.51	0.42	0.37
0.5	0.71	0.61	0.5	0.37	0.29

Fig. 54. Diagram for determining the coefficient η; the value ρ in the Table is for T-beams

The stability of eccentrically compressed members must be verified also along the plane perpendicular to the moment. Such an investigation concerns only the longitudinal force N (the bending moment is ignored) and is done through formula (114) just as for axially loaded elements.

4. Determining the Cross-Section of a Rectangular, Unsymmetrically Reinforced Eccentrically Compressed Element

The formulae given above can be used only for verifying the resistance of a known member. But usually the knowns are the forces M and N, the grades of materials, and the sectional dimensions, and it is required to find the reinforcement area F_s and F'_s.

Case 1.

$$e_0=\frac{M}{N}>0.3h_0.$$

The two equations (126) and (127) contain three unknowns: F_s, F'_s, and x and can therefore be solved through various values of F_s and F'_s. But in choosing the most economical solution, i.e., the one giving a minimum total area of reinforcement $(F_s+F'_s)$, a mathematical analysis indicates that such a minimum for compiling the formulae for F_s and F'_s will be when $x\approx0.55h_0$.

Thus, from formula (127)

$$F'_s=\frac{\frac{Ne}{m}-R_{be}bx\left(h_0-\frac{x}{2}\right)}{m_sR_s(h_0-a')}$$

which, after inserting $0.55h_0$ instead of x and performing a simple conversion, becomes

$$F'_s=\frac{\frac{Ne}{m}-0.4R_{be}bh_0^2}{m_sR_s(h_0-a')}\ . \tag{133}$$

From equation (126), when $x=0.55h_0$

$$F_s=\frac{-\frac{N}{m}+0.55R_{be}bh_0+m_sR_sF'_s}{m_sR_s}$$

and in its final form is

$$F_s=\frac{0.55R_{be}bh_0-\frac{N}{m}}{m_sR_s}+F'_s. \tag{134}$$

The results of formula (134) are justified only when formula (133) gives a sufficiently large value of F'_s for the cross-section. But formula (133) too often reveals a negative F'_s value, the physical meaning of which is that there is no need of compression bars (i.e., $x<0.55h_0$). In such a case, bars F'_s are installed as supplementary reinforcement on the basis of $\mu'_{min}=0.2\%$, i.e.,

$$F'_s=\frac{0.2\times bh_0}{100} \tag{135}$$

while the area F_s is computed as follows: the force N is transferred to the centre of gravity of the reinforcement F_s and a moment $M=Ne$ (Fig. 55) simultaneously applied. The stresses (disregarding the transferred force N) will then be the same as in a doubly reinforced beam. The resistance of the compression bars and their equal area in part of the tension bars will take the moment

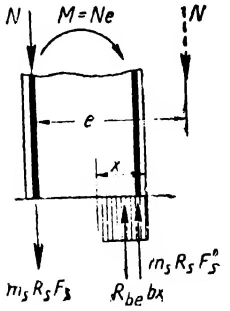


Fig. 55. Determining the area F_s of reinforcement when the bars F_s' are only used as supplementary reinforcement

$$M'=mm_s R_s F_s'(h_0-a'), \qquad (136)$$

while the remainder of the moment

$$M_1=Ne-M' \qquad (137)$$

will be resisted by the compression concrete and the rest of the tension bars: $F_{s1}=F_s-F_s'$. Under such circumstances, the condition $x \geqslant 2a'$ is obligatory.

The bar area F_{s1} is computed just as in a singly reinforced beam: $A_0=\frac{M_1}{mR_{be}bh_0^2}$ is solved, a corresponding γ_0 value is picked from Table 10, after which F_{s1} is evolved through formula (40). The full amount of tension steel will be

$$F_s=F_{s1}+F_s'-\frac{N}{mm_s R_s}. \qquad (138)$$

The last term of formula (138) represents the transference of the longitudinal compressive force N to the centre of gravity of the reinforcement F_s, with a corresponding decrease of tension in the latter.

The above calculations are also executed if the area of the compression bars F_s' installed as a supplementary element is more than that resulting from formula (133).

With a relationship of $\frac{l_0}{h}>10$, the eccentricity of the longitudinal force N in formulae (133) and (137) is calculated as

$$e=\frac{M}{N}\eta+\frac{h}{2}-a. \qquad (139)$$

Case 2. $e_0 \leqslant 0.3h_0$.

The formula for the area F_s' is given by equation (128),

$$F_s'=\frac{\frac{Ne}{m}-0.5R_{pr}bh_0^2}{m_s R_s (h_0-a')},$$

and takes its final form when R_{pr} is replaced by $0.8R_{be}$, i.e.,

$$F_s' = \frac{\frac{Ne}{m} - 0.4R_{be}bh_0^2}{m_s R_s (h_0 - a')},$$

which is the same as formula (133) in Case 1.

The area of the reinforcement F_s depends upon the stress diagram of the section.

Thus with a very small eccentricity, the entire section will be in compression and the bars F_s must satisfy compression requirements through a formula based on condition (129) after R_{pr} has been replaced by $0.8R_{be}$:

$$F_s = \frac{\frac{Ne'}{m} - 0.4R_{be}b(h_0')^2}{m_s R_s (h_0 - a')} \tag{140}$$

With an eccentricity of

$$0.3h_0 \geqslant e_0 > 0.15h_0,^*$$

the steel F_s will be either in compression or tension, but the stresses will be insignificant and the bars may be installed as stiffeners ($\mu_{min} = 0.2\%$).

With this eccentricity, if the grade of the concrete is low (less than 150) and if according to formula (133) the area of the compression bars F_s' is more than 2%, it will be well to check the reinforcement F_s by formula (140).

With a relationship of $\frac{l_0}{h} > 10$, the increased eccentricity of the longitudinal force, as given by formula (139), is valid only for computing the bar area F_s'. If the cross-section is entirely in compression and the steel F_s is determined through formula (140), then the influence of deflection on the rate of eccentricity will be negligible and can be ignored, since this means that $\eta = 1$.

1st type of problem. Given: sectional dimensions b and h, the length l_0, effective forces M and N and the grades of concrete and steel. To find: areas of reinforcement F_s and F_s'.

Method of procedure.

1) Design data is gathered: R_{be}, R_s, m_s, $e_0 = \frac{M}{N}$, $\frac{l_0}{h}$, $\frac{l_0}{b}$.

2) Solutions are found for e: if $\frac{l_0}{h} \leqslant 10$, then $e = e_0 + \frac{h}{2} - a$; if $\frac{l_0}{h} > 10$, then $e = e_0\eta + \frac{h}{2} - a$. The coefficient η is taken from the diagram in Fig. 54 or derived through formula (132).

* This condition will be $e_0 \leqslant 0.2h_0$ if the materials are grade 25Г2С steel and grade 200 (or lower) concrete.

3) F'_s is solved through formula (133); if it is a negative quantity or less than the structural minimum (0.2%), then it is computed by formula (135).

4) F_s is calculated; if $e_0 > 0.3h_0$ and formula (133) gives an area of the bars F'_s in excess of the structural minimum, then F_s is derived from formula (134); if $e_0 > 0.3h_0$ and F'_s is to be supplementary steel, then formula (138) is used to find F_s after F_{s1} has first been determined; if $0.3h_0 \geqslant e_0 > 0.15h_0$ with grade 150 (or higher) concrete and with $\mu' \leqslant 2\%$, then F_s is considered as supplementary steel ($\mu_{min} = 0.2\%$); if $e_0 \leqslant 0.15h_0$, or if $0.3h_0 \geqslant e_0 > 0.15h_0$ and with the concrete lower than 150 and with $\mu' > 2\%$, then F_s is found through formula (140).

5) The total percentage of reinforcement is found:

$$\mu_{tot} = \frac{F_s + F'_s}{bh_0} 100.$$

The given sectional dimensions $b \times h$ may remain unaltered if $0.4\% \leqslant \mu_{tot} \leqslant 3\%$. If $\mu_{tot} < 4\%$, the sectional dimensions may be lessened, whereas if $\mu_{tot} > 3\%$, they must be increased or a higher grade of concrete specified.

6) An additional verification is made, through formula (114), of the member's stability along the plane perpendicular to the plane of deflection if $\frac{l_0}{b} > 14$ after first having picked φ from Table 15.

2nd type of problem. This problem differs from the 1st type in that the area of the bars F'_s is given together with the other knowns. The method of procedure remains the same as above, except that the reinforcement F'_s is accepted as given. Hence, the area of the bars F_s, when $e_0 > 0.3h_0$, is quickly found by formula (138).

If a check is necessary of the strength of the given cross-section, the general equations are applied in the following order:

1) Working data is gathered: R_{be}, R_s, m_s, $e_0 = \frac{M}{N}$, $\frac{l_0}{h}$, $\frac{l_0}{b}$.

2) x is derived through formula (126).

3) If $x \leqslant 0.55h_0$, the problem is one of eccentric compression belonging to Case 1; hence, when $x \geqslant 2a'$, a check of strength must be made either by condition (127) if $\frac{l_0}{h} \leqslant 10$ after first solving $e = e_0 + \frac{h}{2} - a$, or through formula (139) if $\frac{l_0}{h} > 10$.

If $x < 2a'$, the strength of the section is verified, the reinforcement F'_s being disregarded.

If $x > 0.55h_0$, it will be a problem of eccentric compression belonging to Case 2; in this case resistance checking is done through condition (124), and additionally through condition (125) if $e < h_0 - a'$.

Illustrative problem 14. Given: longitudinal effective force $N=42$ t; effective bending moment $M=12.2$ t/m; cross-section $b=25$ cm, $h=40$ cm; effective length of member $l_0=6$ m; concrete, grade 200-A; reinforcement, St-5 intermittently deformed bars; $m=1$. To be determined: F'_s and F_s.

Solution.

1) Gathered design data: $R_{pr}=90$ kg/cm², $R_{be}=110$ kg/cm², $R_s=2,400$ kg/cm², $m_s=1$, $\frac{l_0}{h}=\frac{600}{40}=15>10$, $\frac{l_0}{b}=\frac{600}{25}=24>14$, $a=a'=3$ cm, $h_0=37$ cm,

$$e_0=\frac{M}{N}=\frac{12.2}{42}=0.29 \text{ m}=29 \text{ cm}.$$

2) A value is assumed for η by means of the diagram in Fig. 54 in order to determine e:

$$n_1=\frac{N}{mbhR_{be}}=\frac{42,000}{1\times 25\times 40\times 100}=0.382.$$

From the diagram: when $n_1=0.382$ and $\frac{l_0}{h}=15$, we find that $\eta=1.28$; and from formula (139) $e=29\times 1.28+\frac{40}{2}-3=54.1$ cm.

3) Solving through formula (133)

$$F'_s=\frac{42,000\times 54.1-0.4\times 110\times 25\times 37^2}{1\times 2,400\,(37-3)}=\frac{2,270,000-1,510,000}{2,400\times 34}=9.31 \text{ cm}^2,$$

which is satisfied by 2ø20D + 2ø14D ($F'_s=9.36$ cm²).

$$\mu'=\frac{F'_s}{bh_0}100=\frac{9.36\times 100}{25\times 37}\approx 1\%.$$

4) Inasmuch as $e_0\eta=29\times 1.28>0.3h_0=11$ cm and $\mu'>\mu_{min}=0.2\%$, the value of F_s is derived through formula (134):

$$F_s=\frac{0.55\times 110\times 25\times 37-42,000}{2,400}+9.34=5.83+9.34=15.17 \text{ cm}^2,$$

which is satisfied by 2ø20D + 2ø25D ($F_s=16.10$ cm²);

$$\mu=\frac{F_s}{bh_0}100=\frac{16.10}{25\times 37}100=1.75\%.$$

5) The total percentage of reinforcement $\mu_{tot}=\mu'+\mu=1+1.75=2.75\%<3\%$; therefore the cross-section is satisfactory.

6) Formula (114) serves for checking the member's stability: Table 15 shows that when $\frac{l_0}{b}=24$, $\varphi=0.62$; $m=0.8$ (inasmuch as $b<30$ cm).

The carrying capacity of the cross-section

$$N_{cs}=0.62\times 0.8\,(90\times 25\times 40+1\times 2,400\times 15.32)=64,240 \text{ kg}=64.24 \text{ t}>42 \text{ t}.$$

Hence stability of the member along the plane perpendicular to the action of the moment, is assured.

Illustrative problem 15. Given: the same as problem 14, except that $M=5.8$ t/m. Determine: F'_s and F_s.

Solution.

$$e_0=\frac{5.8}{42}=0.14\ \text{m}=14\ \text{cm};$$

$n_1=0.382$; and from the diagram (Fig. 54) $\eta=1.28$;

$$e=14\times 1.28+\frac{40}{2}-3=35\ \text{cm};$$

$$F'_s=\frac{42{,}000\times 35-0.4\times 110\times 25\times 37^2}{1\times 2{,}400\,(37-3)}=\frac{1{,}470{,}000-1{,}510{,}000}{2{,}400\times 34}<0;$$

therefore $F'_s=\mu_{min}\%\frac{bh^0}{100}=\frac{0.2\times 25\times 37}{100}=1.85$ cm², which is satisfied by 2ø12 D ($F'_s=2.26$ cm²).

Furthermore,

$e_0\eta=14\times 1.28>0.3h_0=11$ cm; hence F_s is solved through formula (138).

From formula (136)

$M'=1\times 1\times 2{,}400\times 2.26\,(37-3)=184{,}500$ kg/cm.

From formula (137)

$M_1=42{,}000\times 35-184{,}500=1{,}470{,}000-184{,}500=1{,}285{,}500$ kg/cm.

$$A_0=\frac{M_1}{mR_{be}bh_0^2}=\frac{1{,}285{,}500}{1\times 110\times 25\times 37^2}=0.34;$$

Table 10 gives $\gamma_0=0.782$; from formula (40)

$$F_{s1}=\frac{1{,}285{,}500}{1\times 1\times 2{,}400\times 0.782\times 37}=18.5\ \text{cm}^2,$$

and formula (138) finally gives

$$F_s=18.5+2.26-\frac{42{,}000}{1\times 1\times 2{,}400}=3.36\ \text{cm}^2,$$

which is satisfied by 3ø12 D ($F_s=3.39$ cm²).

Illustrative problem 16. Given: $N=120$ t; $M=14$ t/m; cross-section: $b=40$ cm, $h=60$ cm; concrete, grade 150-B; reinforcement, St-3 bars; $l_0=7.8$ m; $m=1$. To be determined: F_s and F'_s.

Solution.

1) Gathering design data: $R_{be}=80$ kg/cm², $R_{pr}=65$ kg/cm², $R_s=2{,}100$ kg/cm², $m_s=1$;

$$\frac{l_0}{h}=\frac{7.8}{0.6}=13>10;\quad \frac{l_0}{b}=\frac{7.8}{0.4}=19.5>14;$$

$$a=a'=4\ \text{cm};\quad h_0=60-4=56\ \text{cm};$$

$$e_0=\frac{M}{N}=\frac{14}{120}=0.116\ \text{m}=11.6\ \text{cm}.$$

2) Solving for e:

$n_1=\frac{120{,}000}{1\times 40\times 60\times 80}=0.625$; from the diagram (Fig. 54) $\eta=1.36$.

An identical value is given by formula (132):

$$\eta=\frac{1}{1-\frac{120{,}000\times 13^2}{400\times 80\times 40\times 60}}=1.36.$$

From formula (139)

$$e=11.6\times 1.36+\frac{60}{2}-4=41.8\ \text{cm}.$$

3) From formula (133)

$$F'_s=\frac{120{,}000\times 41.8-0.4\times 80\times 40\times 56^2}{1\times 2{,}100\,(56-4)}=9.15\ \text{cm}^2,$$

which is satisfied by 3ø20 (F'_s=9,41 cm²);

$$\mu'=\frac{9.41}{40\times 56}\,100=0.4\%>0.2\%.$$

4) Solving for F_s:

$e_0\eta=11.6\times 1.36=15.7$ cm, which is less than $0.3h_0=16.8$ cm and more than $0.15h_0=8.4$ cm.

Inasmuch as $0.3h_0>e_0\eta>0.15h_0$ and $\mu'<2\%$, therefore the area of the bars F_s will have only a supplementary importance,

$$F_s=\mu_{min}\frac{bh_0}{100}=0.2\,\frac{40\times 56}{100}=4.46\ \text{cm}^2,$$

and which is satisfied by 2ø18 (F_s=5.09 cm²).

5) Checking stability along the plane perpendicular to the moment action.

From Table 15, when $\frac{l_0}{b}=19.5$, $\varphi=0.75$; from formula (114), the carrying capacity of the cross-section

$N_{cs}=0.75\times 1\,[65\times 40\times 60+1\times 2{,}100\times(9.41+5.09)]=140{,}000$ kg $>$ 120 t, therefore stability is assured.

5. Determining the Cross-Section of a Rectangular, Symmetrically Reinforced Eccentrically Compressed Element

When the reinforcement is symmetrical (i.e., $F_s=F'_s$) the position of the neutral axis is found through formula (126), or

$$\frac{x}{h_0}=\frac{N}{mR_{be}bh_0}. \tag{141}$$

Case 1 must be dealt with if $\frac{x}{h_0}\leqslant 0.55$, and corresponding fundamental equations will yield

$$F_s=F'_s=\frac{e-h_0\left(1-0.5\frac{x}{h_0}\right)}{m_sR_s\,(h_0-a')}\times\frac{N}{m}. \tag{142}$$

Case 2 must be dealt with if $\frac{x}{h_0}>0.55$, and corresponding fundamental equations will give

$$F_s=F'_s=\frac{\frac{Ne}{m}-0.4R_{be}bh_0^2}{m_sR_s\,(h_0-a')}. \tag{143}$$

The influence of length is to be taken into account just as in the former examples.

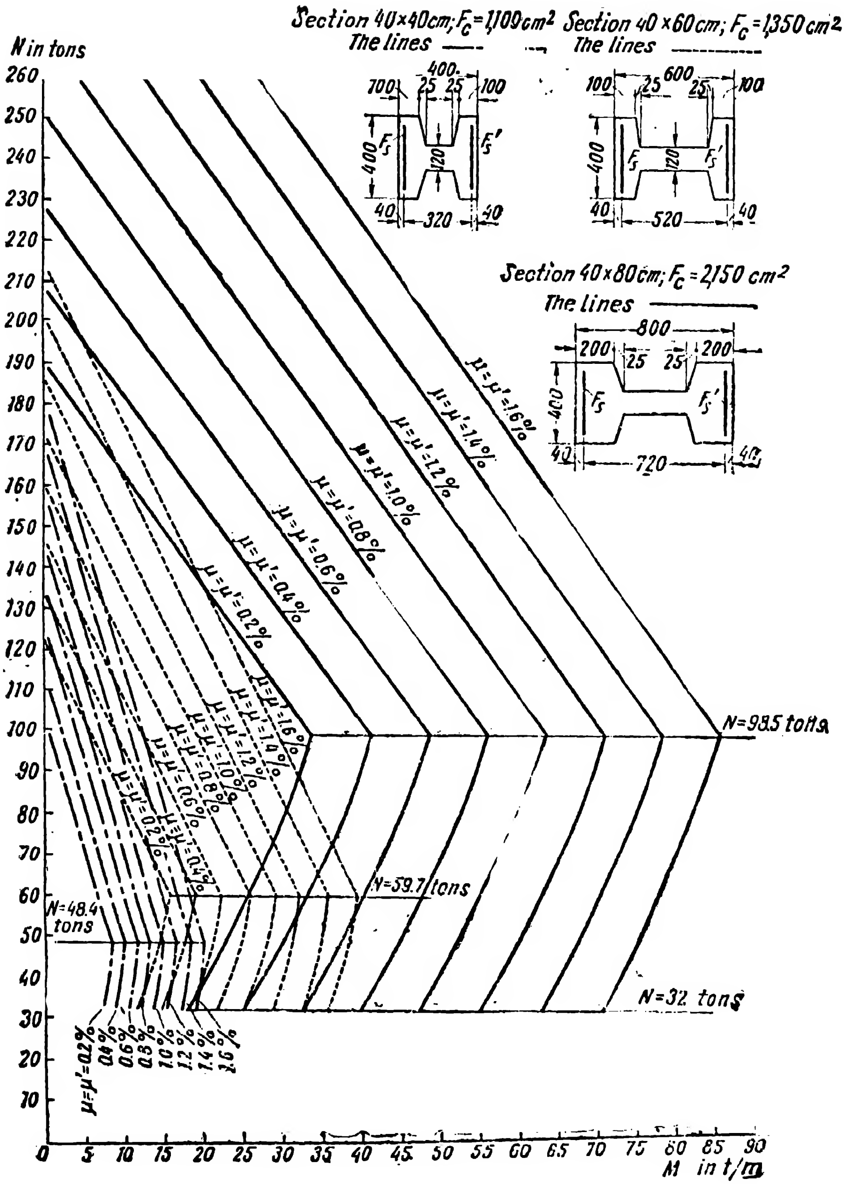


Fig. 56. Diagram for computing eccentrically compressed I-sections; grade 200 concrete and St-5 steel bars

6. T- and I-Shapes

Amongst the symmetrical shapes, other than rectangular, employed to bear eccentric compression, T- and I-sections are the most common. In computing them with the aid of fundamental equations, due regard must be given to the following factors.

When eccentricity is very great (when $S_c \leqslant 0.8S_0$), the flange disposed in the tension zone is disregarded in the calculations. Just as in T-shapes used for bending members, computations will establish the effective width of the compression flange. In determining S_0 when eccentricity is small (when $S_c > 0.8S_0$), the effective width of the flange situated at the bounds of less compression (or less tension) is limited by the condition

$$S_0 \leqslant 0.55bh_0^2.$$

If the neutral axis is contained within the flange that is situated on the side of greatest compression, the section is assumed as rectangular with a width of b_{fl}; but if the neutral axis bisects the web, the latter is considered as taking compression.

It must be noted that the use of fundamental equations in determining a T- or I-section is a complicated procedure usually demanding a whole series of approximations. Therefore various tables and diagrams are applied in practice to simplify the work. The diagram presented in Fig. 56 is for establishing bar areas for a number of symmetrically reinforced I-sections.

This diagram embraces standardised dimensions of symmetrically reinforced columns made of grade 200 concrete and intermittently deformed St-5 bars for one-storey industrial structures. In the application of this diagram when the knowns are M and N (shown on the coordinate axes), the corresponding percentages of reinforcement $\mu=\mu'$ are found at the intersections, after which the term to be solved is

$$F_s = F_s' = \mu F_c. \tag{145}$$

The values of F_c for each type of cross-section are also given in the diagram. The case of a long member is taken into account by increasing the effective moment M to a maximum of

$$M_{cor} = M \frac{e_0\eta + \frac{h}{2} - a}{e_0 + \frac{h}{2} - a}.$$

CHAPTER V

AXIALLY AND ECCENTRICALLY TENSIONED MEMBERS

Sec. 15. MEMBERS UNDERGOING AXIAL TENSION

1. Requirements in Their Design

Axial tension is created when a longitudinal pulling force N is applied to the centre of gravity of a cross-section (Fig. 57, *a*). Cases of this kind include ties and hangers of arches (Fig. 57, *b*), the lower chords and some of the struts in trusses, the walls of cylindrical tanks and pipes subjected to pressure of liquids from within (Fig. 57, *c*), and a number of other kinds of structural units.

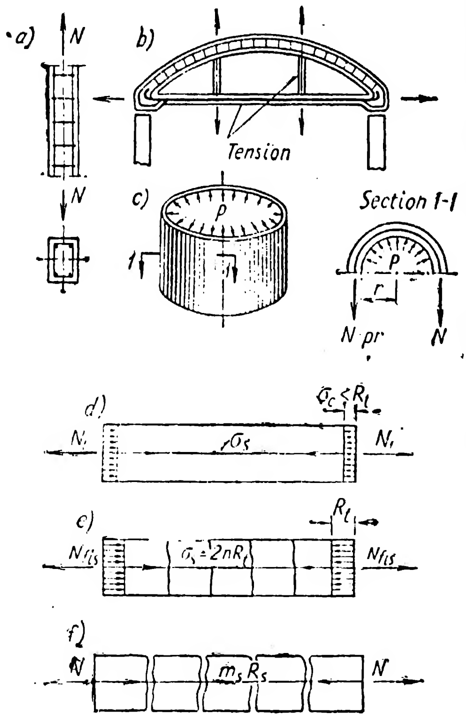


Fig. 57. Behaviour of reinforced concrete members undergoing axial tension

Members that bear axial tension are usually rectangular in cross-section and reinforced with longitudinal effective bars (the area of which is derived through computation) and lateral stirrups.

Square or rectangular sections possessing a dimensional relationship of $\frac{h}{b} \leqslant 2\text{-}2.5$ are reinforced with either welded or tied steel blocks; the reinforcement of sections whose width is considerably greater than their thickness (such as tank walls) consists of mats, either welded or tied. A great variation of diame-

ters, ranging from 3 to 30 mm and more, are used for the effective reinforcement. Stirrups and spacing bars are commonly 6 mm for tied mats, whereas in welded blocks and mats the minimum diameter of the lateral steel is dictated by welding requirements (see Table 4).

Splicing of longitudinal tensioned bars must be in broken step. Lapped joints in tied reinforcement are allowed only in slabs and walls, in which case the lap must be oversized: 40*d* for hot-rolled St-3 and St-5 steel bars, 45*d* for cold-notched steel, and 50*d* for St-25Г2С steel reinforcement.

2. Behaviour of a Member Under Axial Tension, Formulae

When the axial force N is first applied, both the concrete and the bars will bear the tensile stresses (Fig. 57, *d*). Subsequently, when the stresses become equal to the ultimate tensile resistance of the concrete, fissures will cut entirely through the cross-section (Fig. 57, *e*), and further resistance will be shown only by the longitudinal bars F_s. When stresses attain the ultimate tensile resistance of the bars, complete failure will occur (Fig. 57, *f*). From all this it follows that the condition for resistance of a member bearing a design load is

$$N \leqslant m m_s R_s F_s. \tag{146}$$

No fissure formation from specified exploitation loads N^s is allowed in structures undergoing axial tension from the pressure of liquids or gases (tanks, piping). Deformation in such units must be confined within the limits of the tensile resistance of the concrete.

The reader already knows (Sec. 8) that when the stresses in the concrete attain their ultimate R_t, the stresses in the bars will be $\sigma_s = 2nR_t$, where $n = \frac{E_s}{E_c}$. From the requirements of equilibrium (just prior to fissure formation)

$$N^s = m_{fis}(R_t F_c + \sigma_s F_s),$$

in which

F_c—area of concrete in cross-section;

m_{fis}—service coefficient. In calculating fissure anticipation of members subject to a maximum hydrostatic pressure of 1 atm, m_{fis} is assumed as 1.9; in all other cases its value is given in corresponding specifications.

There will be no fissure formation if

$$N^s \leqslant m_{fis}(R_t F_c + 2nR_t F_s), \tag{147}$$

which, after the brackets have been removed from $R_t F_c$, becomes

$$N^s \leqslant m_{fis} R_t F_c \left(1+2n \frac{F_s}{F_c}\right) \tag{148}$$

Just as in bending members, the width of fissures in members undergoing axial tension is determined by the mean deformation of the bars $\varepsilon_{s.me}$ in a length equal to the distance between fissures l_{fis} (see Chapter III, Sec. 11), i. e.,

$$a_{fis} = \varepsilon_{s.me} l_{fis} = \frac{\sigma_s}{E_{s.me}} l_{fis} = \psi \frac{\sigma_s}{E_s} l_{fis},$$

in which

ψ — a coefficient that takes into account the behaviour of the tension-stressed concrete between fissures and which is supplied by Table 16 in accordance with the value of the stresses in the bars

$$\sigma_s = \frac{N^s}{F_s} \tag{149}$$

it is also dictated by the following value:

$$\mu n = \frac{F_s}{F_c} \times \frac{E_s}{E_c} \tag{150}$$

Table 16

Values for the Coefficient ψ in Members Undergoing Axial Tension

μn	Values ψ for σ_s in kg/cm²					
	1,000	1,250	1,500	2,000	2,500	3,000
0.05	—	—	—	—	—	0.44
0.06	—	—	—	—	0.42	0.6
0.075	—	—	—	0.4	0.61	0.73
0.1	—	—	0.33	0.62	0.75	0.83
0.15	0.33	0.48	0.64	0.8	0.87	0.91
0.2	0.48	0.67	0.77	0.87	0.92	0.95
0.3	0.68	0.79	0.85	0.92	0.95	0.97
0.5	0.82	0.88	0.92	0.96	0.98	1

Note. When $\mu n > 0.5$, $\psi = 1$.

The distance between fissures is given by the formula

$$l_{fis} = \frac{u}{\mu}, \tag{151}$$

in which u is the relationship of the reinforcement F_s and the perimeter S of the cross-section:

$$u = \frac{F_s}{S}. \tag{152}$$

Just as in bending members, the magnitude of l_{fis}, as derived from formula (151), is multiplied by 0.5 when the bars are intermittently deformed, and by 1.25 if the reinforcement consists of cold-drawn prefabricated mats or blocks.

As already noted in Sec. 11 concerning bending members, the width of fissures need not be calculated in all cases.

3. Determining the Cross-Section of Elements Undergoing Axial Tension

Two types of problems are encountered in determining a cross-section when the knowns are the grades of concrete and steel and the pulling force N.

1st type of problem. It is required to find the bar area F_s and the sectional dimensions $n \times h = F_c$ when fissure formation is allowed.

Procedure.

1) Design data is gathered: R_s and m_s.
2) From condition (146) for strength requirements

$$F_s = \frac{N}{mm_s R_s}. \tag{153}$$

3) The cross-section $b \times h$ is chosen to satisfy structural requirements but with dimensions not less than needed to give the bars their protective covering.

2nd type of problem. It is required to find the bar and concrete areas F_s and F_c respectively, when fissure formation is prohibited.

Procedure.

1) Design data is gathered: R_t, R_s, m, E_s, E_c, $n = \frac{E_s}{E_c}$.

2) F_s, as required for strength, is computed by means of formula (153).

3) F_c, as required for fissure anticipation, is established through condition (147):

$$F_c = \frac{N^s}{m_{fis} R_t} - 2nF_s. \tag{154}$$

If the member subject to axial tension is known and it is required to *verify its resistance and fissure liability*, then general conditions (146) and (147) are observed.

Illustrative problem 17. Compute the wall of a tank that is to bear tension and determine F_s and F_c. Given: $N^s = 27$ tons, load coefficient $n = 1.1$, reinforcement of St-5 steel bars, $m = 1$, concrete of grade 200-B. Fissure formation is prohibited.

Solution.

1) Gathered design data: $R_t=6.4$ kg/cm², $R_s=2{,}400$ kg/cm², $m_s=1$.

$$E_s=2.1\times10^6\ \text{kg/cm}^2;\ E_c=2\times10^5\ \text{kg/cm}^2;$$

$$n=\frac{2.1\times10^6}{2\times10^5}=10.5.$$

2) Formula (153) solves for F_s:

$$F_s=\frac{27{,}000\times1.1}{1\times1\times2{,}400}=12.4\ \text{cm}^2,$$

which is satisfied by 8ø14 D ($F_s=12.31$ cm²).

3) Formula (154) solves for F_c:

$$F_c=\frac{27{,}000}{1.9\times6.4}-2\times10.5\times12.31=2{,}230-260=1{,}970\ \text{cm}^2.$$

With an effective sectional width $b=100$ cm, $h=19.7$ cm $\approx$ 20 cm.

Sec. 16. MEMBERS UNDERGOING ECCENTRIC TENSION

1. Requirements in Their Design

When a section is subject to a longitudinal pulling force N acting with an eccentricity of e_0, or when a pulling force acts simultaneously with a bending moment M (Fig. 58, *a*), the result will be eccentric tension and the eccentricity of the longitudinal force can be expressed as

$$e_0=\frac{M}{N}.$$

Instances of this kind include walls of rectangular tanks (Fig. 58, *b*), elements of certain types of portal frames, bunker walls, etc.

The manner of reinforcing such eccentrically tensioned units is contingent upon the rate of eccentricity of the longitudinal force: if the eccentricity is large, bar placement will be similar to that in eccentrically compressed members (Sec. 14); with little eccentricity, the analogy will embrace axially tensioned elements (Sec. 15).

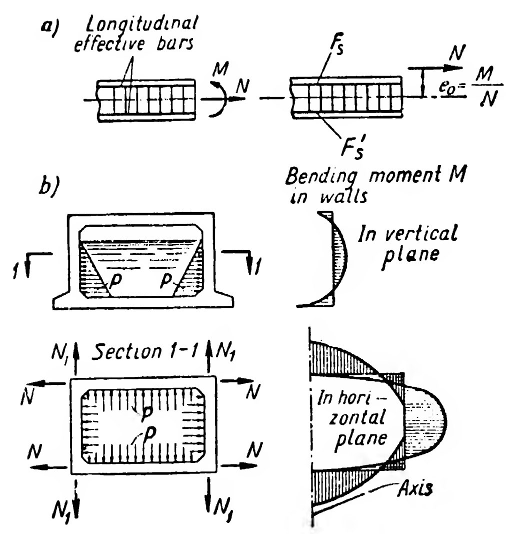


Fig. 58. Behaviour of reinforced concrete members subject to eccentric tension

Bars will be designated as F_s for those situated close to the force N, and F'_s for those further away from the force.

2. Behaviour of Members Undergoing Eccentric Tension

The behaviour of eccentrically tensioned members greatly depends upon the extent of eccentricity of the longitudinal force N. *With little eccentricity* and the pulling force applied between the respective centres of gravity of the F_s and F'_s reinforcement

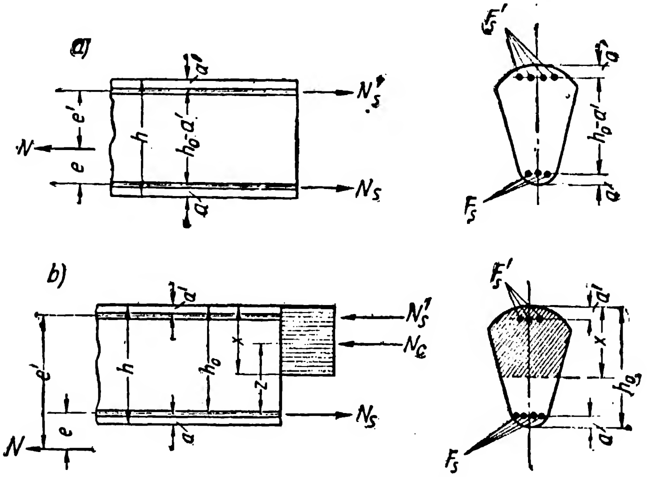


Fig. 59. Investigation of members undergoing eccentric tension
a—when there is little eccentricity; *b*—when there is much eccentricity

Fig. 59, *a*), the concrete will fissure throughout shortly after loading begins. When fissures are formed, only the effective bars will resist the force N, the latter being distributed between the steel F_s and F'_s according to the principles of a lever, i.e., in inverse proportion to the distances e and e'. Failure will set in when ultimate stresses are achieved in the bars F_s and F'_s.

With large eccentricity and the pulling force N acting beyond the bounds of the centres of gravity of the F_s and F'_s bars (Fig. 59, *b*), part of the section (away from the force N) will be compressed, and part will be under tension, a condition similar to heavily eccentric compression. The height of the compression zone, just as in an eccentrically compressed bending member, will be limited by the following conditions: $S_c \leqslant 0.8S_0$ if the shape has any symmetrical form, and $x \leqslant 0.55h_0$ for a rectangular section.

Failure will occur when ultimate stresses are attained either in the tension bars F_s, in the compression concrete (with a sectional area of F_c), or in the compression bars F'_s if the above-named con-

ditions are observed, or if $z \leqslant h_0 - a'$ when the section is of any symmetrical shape or $x \geqslant 2a'$ for a rectangular section.

3. Formulae for Cross-Sections of Any Symmetrical Shape

Computation formulae are based on design loads and evolved from the condition of equilibrium and the design stresses R_{bc} and $m_s R_s$ in the concrete and bars, respectively.

A case of little eccentricity (see Fig. 59, *a* for stresses). By reducing to zero the sum of the moments of all forces in relation to the centres of gravity of the bars F_s and F_s', two conditions are obtained for strength:

$$Ne \leqslant m m_s R_s F_s' (h_0 - a') \tag{155}$$

and

$$Ne' \leqslant m m_s R_s F_s (h_0 - a'). \tag{156}$$

A case of large eccentricity: $S_c \leqslant 0.8 S_0$; $z \leqslant h_0 - a'$ (see Fig. 59, *b* for the stresses). The conditions for resistance are identical to the case of large eccentricity in eccentric compression. But since the force N possesses an inverse sign (tension instead of compression), the signs are likewise changed in the conditions of strength, which latter are derived by reducing to zero the sum of all the forces projected on the axis of the member and the moment in relation to the centre of gravity of the reinforcement F_s:

$$N \leqslant m (m_s R_s F_s - m_s R_s F_s' - R_{be} F_c); \tag{157}$$

$$Ne \leqslant m [R_{be} S_c + m_s R_s F_s' (h_0 - a')]. \tag{158}$$

Condition (157) will serve to define the position of the neutral axis.

4. Formulae for Rectangular Cross-Sections

The case to be dealt with in eccentric tension is discovered immediately by its magnitude of eccentricity. Cases of little eccentricity are revealed when $e_0 \leqslant \frac{h}{2} - a$, and those of large eccentricity if $e_0 > \frac{h}{2} - a$.

A case of little eccentricity. The conditions for resistance are identical to those of cross-sections having any symmetrical shape, that is, conditions (155) and (156) are valid.

A case of large eccentricity: $x \leqslant 0.55 h_0$, $x \geqslant 2a'$. In the given instance

$$F_c = bx; \quad S_c = bx \left(h_0 - \frac{x}{2}\right).$$

Conditions of resistance (157) and (158) take the form of

$$N \leqslant m (m_s R_s F_s - m_s R_s F_s' - R_{be} bx) \tag{159}$$

and

$$Ne \leqslant m\left[R_{be} bx\left(h_0 - \frac{x}{2}\right) + m_s R_s F_s' (h_0 - a')\right]. \tag{160}$$

Condition (159) will be convenient for determining the position of the neutral axis.

5. Determining the Cross-Section of Eccentrically Tensioned Members

Computing the cross-section of eccentrically tensioned members usually consists in calculating the areas of the bars F_s and F_s' when the knowns are the effective forces M and N, the grades of materials, and the sectional dimensions of the concrete.

A case of little eccentricity. Method of procedure.

1) Design data is gathered: m_s, R_s, $e_0 = \frac{M}{N}$.
2) Solutions are found for

$$e = \frac{h}{2} - e_0 - a; \tag{161}$$

$$e' = e_0 + \frac{h}{2} - a'. \tag{162}$$

3) F_s' is calculated by observing condition (155)

$$F_s' = \frac{Ne}{m m_s R_s (h_0 - a')} \tag{163}$$

and F_s is computed by observing condition (156)

$$F_s = \frac{Ne'}{m m_s R_s (h_0 - a')}. \tag{164}$$

A case of large eccentricity. Method of procedure. Just as in eccentric compression, the minimum of $(F_s + F_s')$ is found when $x = 0.55 h_0$. Hence, formula (133) will give the area of the reinforcement F_s', whereas the area of the bars F_s is found through formula (134) after changing the sign of the member representing the force N, i.e., by the formula

$$F_s = \frac{\frac{N}{m} + 0.55 R_{be} b h_0}{m_s R_s} + F_s'. \tag{165}$$

If it should happen that the area of the bars F_s', as computed through formula (133), is either negative or less than the minimum for supplementary steel, then it is brought up to the supplementary minimum ($\mu_{min} = 0.2\%$), while the area of the bars F_s is determined in the same manner as in the case of heavily eccentric compression, i.e., through formula (138), except that an inverse

sign is placed before the member that represents the longitudinal force N, i. e.,

$$F_s = F_{sI} + F'_s + \frac{N}{mm_s R_s}. \tag{166}$$

Method of procedure.

1) R_{be}, R_s, m_s, and $e_0 = \frac{M}{N}$ are gathered as design data.

2) e' is solved through formula (162), and

$$e = e_0 - \frac{h}{2} + a. \tag{167}$$

3) F'_s is found through formula (133); if in this operation $F'_s < 0$ or $\mu < \mu_{min} = 0.2\%$, it is incorporated as supplementary reinforcement.

4) F_s is obtained; it is calculated through formula (165) if the area of the bars F'_s had been obtained through formula (133). But if F'_s had been computed as supplementary, then just as in an eccentrically compressed member, formula (166) will determine F_s after first having computed F_{sI}.

5) $\mu_{me} = \mu' + \mu$ is verified.

CHAPTER VI

PRESTRESSED MEMBERS

Sec. 17. PRINCIPLES OF CALCULATION AND DESIGN

1. Definition. The Object of Prestressing. Methods of Prestressing

Prestressed reinforced concrete, as a whole or in part, is a construction that is artificially stressed in the process of fabrication to create compression in the concrete and tension in the steel prior to the application of the service load.

Prestressing aims to save steel and also to make the concrete more fissure-proof. In ordinary reinforced concrete, hairline cracks will form in the tension zone (see Sec. 8) when the bar stresses $\sigma_{s.fis} = 200\text{-}300$ kg/cm². As the load is subsequently increased, the cracks (or fissures) will widen. Stresses in the reinforcement employed in ordinary reinforced concrete will attain a value of $\sigma_s = 1{,}700\text{-}2{,}500$ kg/cm², hence,

$$\frac{\sigma_s}{\sigma_{s.fis}} = \text{from 8 to 10.}$$

As a rule, fissures will widen insignificantly with such stresses and will neither cause bar corrosion nor hinder normal service of the member.

However, with the use of high-strength steels, the design resistance of which are 10,000-15,000 kg/cm² or more, the bar area may be reduced but the stresses instigated by the load will be $\sigma_s = 8{,}000\text{-}12{,}000$ kg/cm² and over. As before, fissures in the tension zone will form when the bar stresses $\sigma_{s.fis} = 200\text{-}300$ kg/cm², inasmuch as $\sigma_{s.fis}$ is related to the ultimate tensile resistance of the concrete and not to steel grades. In the given case $\frac{\sigma_s}{\sigma_{s.fis}} = 30\text{-}50$ and more, and with such a relationship the cracks will widen considerably and render the member unserviceable. For this reason high-strength bars cannot be incorporated into ordinary reinforced concrete.

The virtual cost of steel η is characterised by a relationship of its price Pr to its resistance R_s (in kg/cm²):

$$\eta = \frac{Pr}{R_s},$$

i. e., the price of steel rises very little with the incorporation of greater resistance and this actually means a decrease in virtual cost. Therefore the use of high-strength steels in reinforced concrete would signify an economy of that material.

In order to avoid fissure formation in the tension zone of the concrete and pave the way for the use of high-strength steel, the concrete must be artificially compressed in advance so as to lessen tension stresses when the loads are applied (Fig. 60). Furthermore, the elimination of fissures or reduction of their widths will increase the rigidity of the member. Another advantage is that prestressed construction can be erected from several units made separately beforehand and subsequently assembled by tying the reinfocement together.

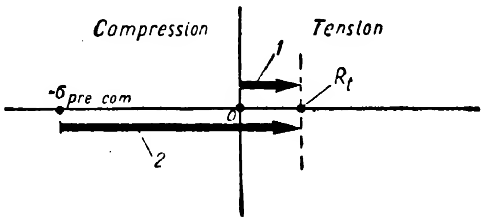


Fig. 60. Behaviour of concrete under tension when service loads are applied

1—in ordinary reinforced concrete; *2*—in prestressed reinforced concrete

The concrete becomes compressed simultaneously with the tensioning of the reinforcement, the desired stresses in the bars being first determined by computation.

Both precast and in-situ construction may be prestressed. When separate prestressed members are included in in-situ reinforced concrete, it is known as *composite* construction. Aside from tensioned bars (tendons), all prestressed members contain ordinary reinforcement, mostly in the form of prefabricated (welded) blocks and mats.

There are two methods of tensioning the bars in prestressing: 1) with the aid of abutments (pretensioning) and 2) against the hardened concrete (post tensioning).

In the first method, bar tensioning is done prior to concreting (Fig. 61, *a*). One end of the reinforcement (tendons) is fixed to the abutment while the stretching device (jack) grasps the other end which is subsequently also fixed to an abutment after the bar has been drawn to the desired stress (within the limits of elastic deformation). After the concrete has been poured and becomes cured to its predetermined strength (resistance) of R' (which is usually $R' \approx 0.7R$), the bar ends are released from the abutments, and the bar, in tending to return to its former length, compresses the surrounding concrete because of the bond between the two components. To make this bond dependable, the bars are either intermittently-deformed or otherwise

surface-worked. If the bars are plain, they are hooked at their ends.

The processes of installing and tensioning the bars in the forms are combined into one operation in *continuous reinforcement*. Here

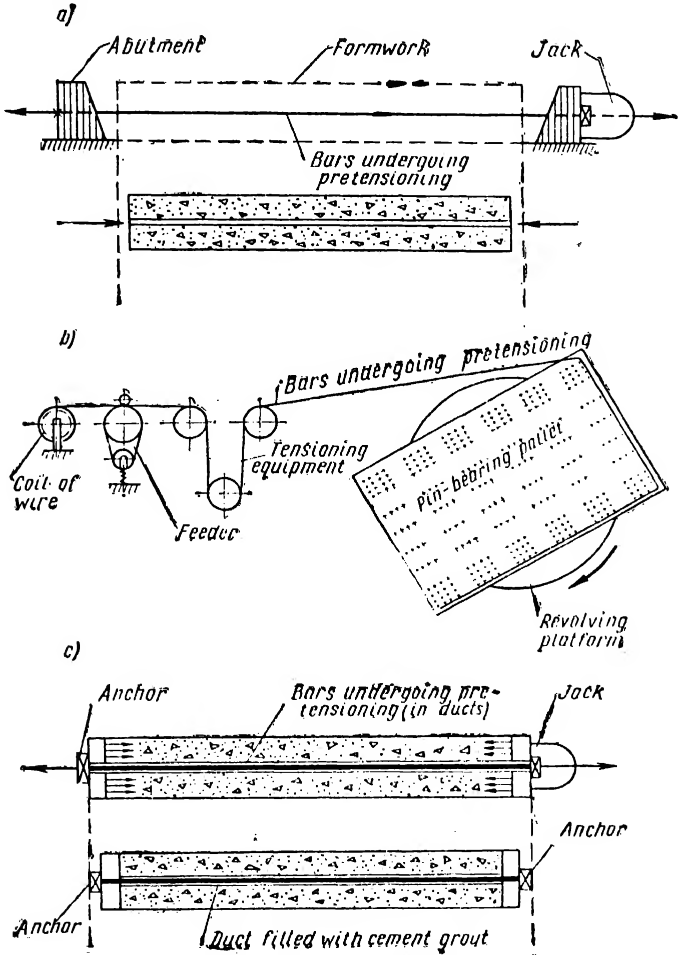


Fig. 61. Sketches showing methods of prestressing

a—tensioning with the aid of abutments (pretensioning), the process and the finished product; *b*—continuous (endless) reinforcement; *c*—tensioning against the hardened concrete (post tensioning), the process and the finished product

the tendons are made of thin wire and the bed (pallet) upon which the formwork is placed is furnished with pins whose positions correspond to the designed reinforcement layout. The bed is placed on a revolving platform equipped with special tensioning equipment.

As the revolving platform turns, the wire is wound around short steel pipes mounted upon the pins (Fig. 61, *b*), and since both ends of the tendons are anchored beforehand, they are tensioned to the needed value. After this operation the concrete is poured; when it has attained its desired strength the member is removed from the bed, the stress in the tendon being transferred to the concrete and compressing it.

Bar tensioning can also be done through electric heating: a current is passed through the prepared wire, heats it to 300-400°C and

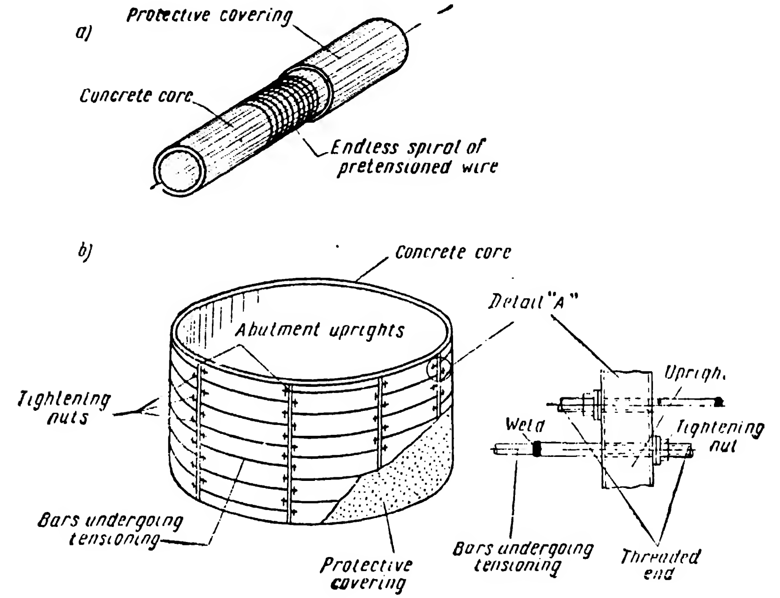


Fig. 62. Prestressing a member by tensioning the bars around the concrete

a—piping; *b*—a tank

lengthens it. The heated wire is installed in the formwork and anchored to the abutments. In cooling, it tends to return to its original length and is drawn taut between the abutments.

In the second method (tensioning against the hardened concrete) the tendons are pulled through ducts left in the body of the already hardened concrete. These ducts, 5-15 mm larger than the bar diameter, are made either by leaving corrugated steel tubing (casings) in the concrete, or by placing steel spirals, smooth sheet metal tubing, rubber hose, or other duct-forming units into the formwork and then removing them from the freshly hardened concrete. When the concrete has hardened, the wire is run through the duct, one of the wire's ends being furnished with an anchor which is fixed in the corresponding end of the member. The other end is grasped by a jack

braced against the opposite butt of the concrete. The jack pulls the steel and compresses the concrete simultaneously (Fig. 61, *c*). When the necessary tension has been attained in the tendon, the end held by the jack is also anchored to the concrete. Lateral mats are installed within the ends of the concrete member to insure proper distribution of local crushing compression. Bond between the tendons and surrounding concrete is subsequently achieved by pressure-fed cement grout or mortar forced through holes in the anchors and distributed via branching tee-fittings that had been placed at intervals of 10-12 metres along the shaft lengths before concrete pouring. This is known as the *injection method* of filling the shafts.

Tendons can also be installed concentrically (in hoops) along the outer contours of members (such as pipes and tanks), the operation of tensioning the steel and compressing the concrete being accomplished either by a winding machine (resulting in continuous reinforcement) or by means of tightening nuts (Fig. 62). In the latter method the ends of the tendons are threaded, and either abutment uprights or turnbuckles are spaced around the perimeter of the member. At the completion of the tensioning operation the concrete and bars are given a protective covering, applied under pressure (the gunite process), that achieves concrete-steel bonding and also guards the bars against corrosion.

2. Construction of Tendons and Methods of Their Anchorage

The tendons in prestressed construction can be made either of separate thick bars, tensioned between abutments or against the hardened concrete, or of various combinations of wire.* In the latter case, if the tendons are held by abutments, they may be made of in-

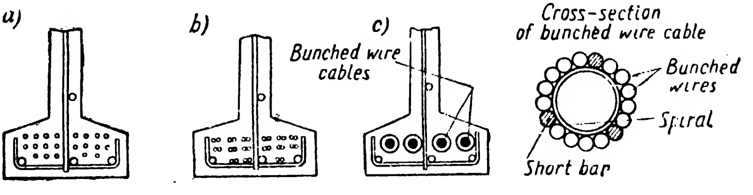


Fig. 63. Tendons in prestressed members

dividual wires (strings) or of groups of two wires (Fig. 63, *a* and *b*); but if tensioned against the concrete itself, they may be either stranded or bunched. Strands are woven of several wires, but bunching (Fig. 63, *c*) is done with a great number of straight wires grouped around a thin wire spiral with a number of gaps left to allow injec-

* See Sec. 2, Item 4.

tion of grout or mortar. Large bunched cables consist of several compact smaller bunches, likewise grouped around a spiral with spaces left to aid mortar or grout injection (Fig. 64).

When tendons are intermittently-deformed (bars or wire), they are tensioned against the abutments without additional bonding anchors; but if made of smooth bars they are furnished with additional anchorage in the form of short welded rods or washers (Fig. 65,*a* and *b*). If of smooth high-strength wire, link-anchors are used (Fig. 65, *c*). In continuous reinforcement, tubes play the role of anchors (Fig. 65, *d*). The beginning and end of windings can be either threaded and clamped, or bolted, etc. (Fig. 65, *e*).

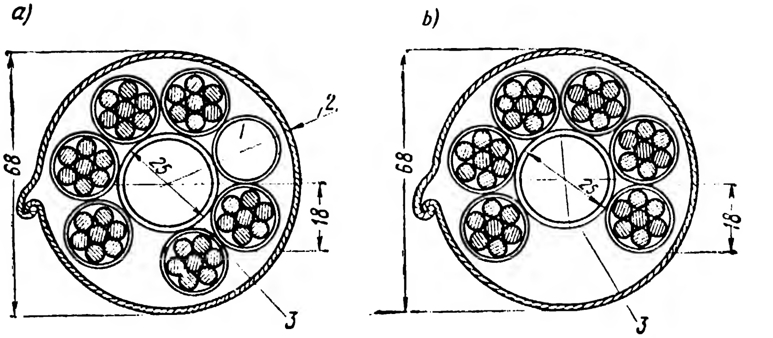


Fig. 64. Cross-section of large bunched cables

a—when injection is intended through anchor holes and branched tee-fittings; *b*—when only anchor holes are provided for injection; *1*—short pipes, Ø 18 mm, *l* = 10 cm, at intervals of 1 m; *2*—centre line of tee-fittings; *3*—spiral of 2.5 mm wire and with a pitch of 20 mm

When tensioning is accomplished directly against the hardened concrete, the construction of anchors will depend upon the kind of reinforcement and the type of tensioning equipment. If hydraulic jacks are used to pull the bunched tendons taut, the ends of the latter are fitted with a shoe (Fig. 65, *g*) or with a threaded socket (Fig. 65, *h*). The latter is made of a grooved end-rod with the bunched wires passing over the grooves, and a sleeve that becomes deformed when pulled through a ring of lesser diameter and thus clamps down on the wire ends. There are several ways of fastening a tendon: if the anchor is in the form of a shoe, washers are placed between it and the end of the member; if a threaded socket is used, a nut is screwed on and subsequently tightened.

When tensioning is accomplished by a double-acting jack (Fig. 66), the anchor (let into the butt of the member) is made of a steel plate with a tapered hole to accommodate the steel wedge, the latter being bored through so as to allow subsequent mortar injection. In tensioning bunched tendons, the jack is thrust against the steel plate and, upon completion of tensioning, runs out a wedging piston which

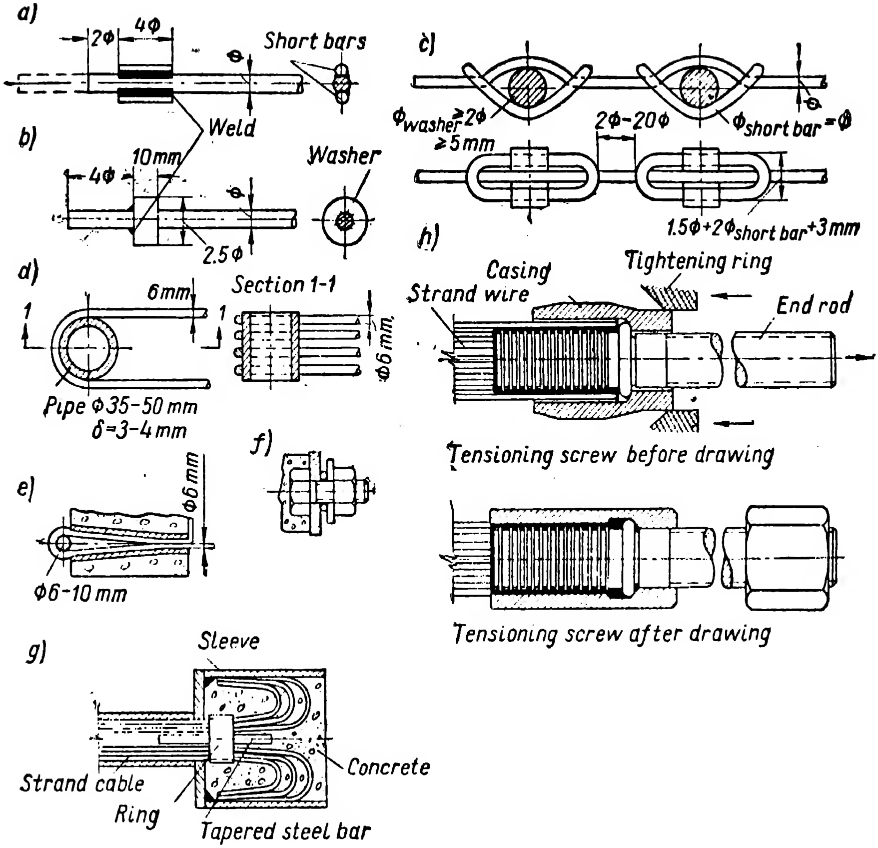


Fig. 65. Tendon anchors

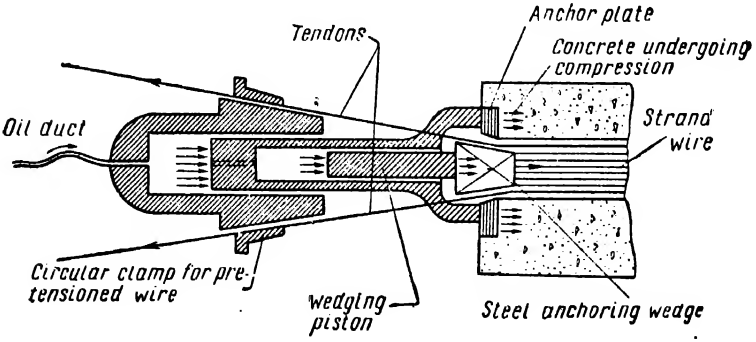


Fig. 66. Scheme of double-acting jack

plugs in a tapered wedge and permanently fixes the taut bunched wires (Fig. 67). Abroad, this type of anchor is made in the form of a reinforced concrete shoe, fitted into the butt end of the member, and a curved and tapered steel wedge.

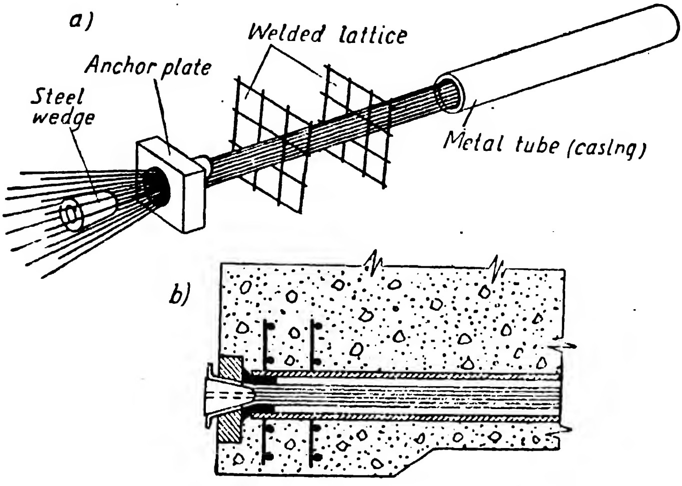


Fig. 67. Anchoring bunched tendons by means of a tapered steel wedge
a—before wedging; *b*—after wedging

If thick tendons are used and tensioned against the hardened concrete, thick threaded end-pieces are contact-welded to them for permanently securing the taut shank by means of a tightening nut. In some cases (such as tank construction) the tensioning process itself is achieved by tightening the nuts.

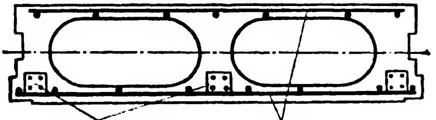


Fig. 68. Prestressed reinforced concrete inserts incorporated into reinforced concrete hollow-slab construction

In designing the length of a tendon, its anchoring devices must be kept in mind, nor must it be forgotten that it stretches during tensioning. Furthermore, its designed position in the concrete must be made convenient for tensioning operations.

In-situ operations of tensioning may be eliminated by the use of prefabricated prestressed reinforced concrete *insertions* reinforced with thin wire that has been tensioned between abutments (Fig. 68).

These insertions, when concreted within the form (together with any necessary untensioned bars), become well-bonded with the

fresher concrete and assure subsequent mutual action under load. Since fissure formation is hampered in the insertions it is also reduced in the composite bonded mass.

3. General Principles in Calculation

Prestressed reinforced concrete design must include computation of carrying capacity and fissure anticipation if fissures are forbidden, and fissure width calculations if they are allowed. In bending members, deformation is also calculated to determine the degree of deflection under load.

Two schemes of behaviour must be considered in the calculations:

1) when the forces acting upon the member are a combination of the external load and the acquired prestress;

2) when the forces acting upon the member are due only to the acquired prestress (this occurs when the member is being made).

As far as the possibilities of fissure formation are concerned, prestressed construction is grouped into three categories.

1st category. This is construction required to be impermeable (pressure piping and industrial tanks).

2nd category. This is either construction that need not be impermeable but which is reinforced with high-strength bars possessing a specified resistance of 10,000 kg/cm² or more, or construction intended for an aggressive environment.

3rd category. This includes all other types of construction.

Fissures are prohibited in the 1st and 2nd categories, but are allowed within certain limits in the 3rd category.

In carrying-capacity computations, the loads chosen are design loads (acting together with the prestress), the same also entering into calculations of 1st-category fissure-proof construction. But for 2nd-category non-fissure units specified loads are employed, as also in computations of deformation and fissure widths.

The carrying capacity of prestressed elements is determined, just as for ordinary reinforced concrete, by the failure-stage method. In dealing with deformation, fissure avoidance, and fissure widths, identical general prerequisites are adopted as for non-prestressed members: the Hypothesis of Plane Sections (Navier's Hypothesis) is considered valid, allowances are made for the development of plastic deformation, etc. The maximum stress of tensioned high-strength bars σ_0 must not exceed $0.65R^s$, nor $0.9R^s$ * in the case of mild steels.

Service coefficients. Qualified design resistance of concrete and reinforcement. In most calculations concerning carrying capacity of

* In certain cases $\sigma_0=0.75\ R^s$ is allowed for hard steels, and $\sigma_0=R^s$ for mild steels.

prestressed members the service coefficient $m=1$, but for pressure piping it is taken as 0.9. For tensioned reinforcement (tendons) the former ordinary designations of service coefficients m_s and m_{di} become m_{pres} and $m_{pres.di}$ respectively. Their values are indentical to untensioned bars,* except that for cold-drawn wire in prefabricated mats and blocks and cold-notched bars, m_s (or m_{pres}) equals 0.7 instead of 0.65. For prestressed high-strength wire (both plain and intermittently deformed) m_{pres} is likewise 0.7.

In computing the strength of the pre-compressed concrete, a higher value is taken for its design resistance by multiplying it by the service coefficient of the concrete $m_c=1.2$. In handling service loads, $m_c=1$.

If minus-deviations should occur in the predetermined stresses of tensioned bars, it will lead to earlier cracking. Hence in fissure-anticipation calculations a coefficient of $m_{dev}=0.9$ is attached to the magnitude of the stresses in the tendons.

The designation for design resistance of ordinary (untensioned) reinforcement remains R_s, but in prestressed bars it becomes R_{pres}. Design resistance is prescribed to enter calculation formulae together with the service coefficients. But in order to simplify the formulae for prestressed construction, a *qualified design resistance* is adopted, which is the product of design resistance of concrete or steel and a respective service coefficient.

For ordinary effective bars $R_{s.qu}=R_s m_s$; in tensioned bars $R_{pres.qu}=$ $=R_{pres}m_{pres}$. When analysing the resistance of a diagonal plane in determining untensioned upright bars, $R_{s.qu}=R_s m_s m_{pres}$; for prestressed upright bars in similar computations $R_{pres.qu}=R_{pres}m_{pres}m_{pres.di}$. Identical values of qualified design resistance are adopted for the compression reinforcement, except that its magnitude is further limited by the condition attending ultimate concrete compression.**

The qualified design resistance of concrete $R_{pr.qu}=R_{pr}m_c$ for axial compression calculations, and $R_{be.qu}=R_{be}m_c$ for compression due to bending. The qualified design resistance of concrete in tension is taken from Table 17, independent of production methods (A or B).

Table 17

Qualified Design Resistance of Concrete in Tension $R_{t.qu}$ in kg/cm²

Grade of concrete	150	200	300	400	500	600
$R_{t.qu}$	8	10	15	18	20	21

Note. For concretes containing aluminiferous cements $R_{t.qu}$ is combined with a coefficient of 0.7.

* See Sec. 4, Item 4.

** See Sec. 13, Item 2, but instead of 4,200 kg/cm² 3,600 kg/cm² is adopted.

Qualified design resistance for concrete $R_{pr.qu}$ and $R_{be.qu}$ and reinforcement $R_{pres.qu}$ and $R_{s.qu}$ can be evaluated by multiplying corresponding design strength (Tables 6 and 8) by the respective service coefficient.

Modulus of elasticity of tendons. In prestressed construction the stresses are very much conditioned by the modulus of elasticity of the tensioned reinforcement (tendons). Consequently, in accordance with empirical findings, the magnitude of the modulus of elasticity $E_s^s(E_s)$ has been differentiated for various kinds of bars:

2,100,000 kg/cm² for St-0, St-3 and St-5 hot-rolled steels;
2,000,000 kg/cm² for 25Г2С and 30ХГ2С hot-rolled bars;
1,800,000 kg/cm² for cold-drawn wire (either plain or intermittently deformed), bunched cold-drawn wire and cold-notched bars;
1,700,000 kg/cm² for strands and cable.

The transformed cross-section. For convenience in calculating precompression, fissure anticipation, and deformation in beams, the cross-section of prestressed beams is assumed to be reduced to a homogeneous section known as the transformed cross-section.

This substitution is possible because, deformation of the concrete and bars being identical prior to fissuring, the stresses in both components are proportional to their moduli of elasticity, and the centre of gravity of the transformed and actual bar areas coincide. All geometrical data concerning the cross-section and that is needed for calculations (areas, centres of gravity, moment of inertia, and resisting moment) are determined as for a transformed cross-section, e. g.,

$$F_{tr}=F_c+F_s\frac{E_s}{E_c^s}. \tag{168}$$

Loss of prestress in tendons. Due to various factors, tension losses take place in the tendons when they are drawn taut to their controlled prestress $\sigma_{init.contr}$. Part of this loss $\sigma_{loss\,1}$ occurs before the concrete is compressed, and partly after, $\sigma_{loss\,2}$.

A summary of such losses, as given in special specifications, is presented below.

1) *Loss due to concrete shrinkage* — σ_1. Concrete will eventually shorten somewhat because of shrinkage, and the tendons, also decreasing in length, will lose some of their prestress. The magnitude of such losses

$\sigma_1=400$ kg/cm² when the bars are tensioned between abutments;
$\sigma_1=300$ kg/cm² when they are tensioned against the hardened concrete.

2) *Loss caused by concrete creep* —σ_2. Prestressed concrete will eventually be effected by creep deformation which will shorten the member somewhat and cause a loss of prestress. The amount of this loss will depend upon the degree of prestress in the concrete σ_c, specimen

strength (grade of concrete) at the time the member undergoes compression —R', the relationship between the moduli of elasticity of the steel and concrete, and upon the kind of bars.

In tensioning against abutments and when $\sigma_c \leqslant 0.5R'$,

$$\sigma_2 = \frac{kE_sR}{E_c^sR'}\sigma_c; \tag{169}$$

and when $\sigma_c > 0.5R'$,

$$\sigma_2 = \frac{kE_sR}{E_c^sR'}\left[\sigma_c + 3R'\left(\frac{\sigma_c}{R'} - 0.5\right)\right]; \tag{169a}$$

whereas when tensioning is done against the hardened concrete and when $\sigma_c \leqslant 0.5R'$,

$$\sigma_2 = 0.75\frac{kE_sR}{E_c^sR'}\sigma_c; \tag{170}$$

and when $\sigma_c > 0.5R'$,

$$\sigma_2 = 0.75\frac{kE_sR}{E_c^sR'}\left[\sigma_c + 3R'\left(\frac{\sigma_c}{R'} - 0.5\right)\right]. \tag{170a}$$

The coefficient k in formulae (169) and (170) will equal 0.8 for hot-rolled bars, and 1.0 for cold-drawn wire.

Pre-compression in the concrete σ_c is determined by ordinary formulae of Strength of Materials on the assumption that the resultant of the forces throughout the tendons act upon the section, with the stresses in the reinforcement being equal to $\sigma_{\text{init.contr}} - \sigma_{\text{loss 1}}$. This resultant is assumed as an outer force compressing the transformed cross-section of the member.

3) *Losses stemming from relaxation of stresses in the metal* — σ_3. Bars made of cold-drawn wire and subjected to considerable stresses will experience changes of internal structure and, under constant deformation, will lead to a lowering of the prestress. This is known as stress relaxaton; the amount of such loss, when $\sigma_0 \leqslant 0.65R^s$, is proportional to the prestress σ_0 in the bars:

$$\sigma_3 = 0.05\sigma_0; \tag{171}$$

and when $\sigma_0 > 0.65R^s_{\text{pres}}$,

$$\sigma_3 = 0.05\sigma_0 + 0.2(\sigma_0 - 0{,}65R^s_{\text{pres}}). \tag{171a}$$

In case of hot-rolled reinforcement, $\sigma_3 = 0$.

4) *Loss provoked by anchor deformation* —σ_4. Anchor deformation and compression in the washers placed between the anchors and concrete incur a shortening of the tendons and a loss of prestress. According to Hooke's Law, the value of this loss is in proportion to the amount of the shortening:

$$\sigma_4 = (\lambda_1 + \lambda_2)\frac{E_s}{l}, \tag{172}$$

where λ_1—compression of washers located between the anchors and concrete and assumed as 1 mm to each anchor; if the anchors consist of tightened nuts or of vee-washers, $\lambda_1=0$;

λ_2—deformation of anchors used with bunched-wire tendons and of anchor washers and bar grasps, and assumed as 1 mm to each anchor or grasp;

l—length of tendon in mm.

5) *Loss resulting from friction between tendons and shaft walls* —σ_5. Friction between tendons and shaft walls causes a prestress loss that can be determined by specific computations.

6) *Loss accounted to giving way of the concrete under the turns of spiral or circular windings* —σ_6. This type of loss is encountered in piping and industrial tanks. $\sigma_6=300$ kg/cm² when the diameter of the unit $d \leqslant 3$ m, but equals zero if $d > 3$ m.

7) *Loss incurred by temperature differences between tendons and abutments* —σ_7. This loss takes place only if the newly made member is subject to heat from steam curing or any other thermal treatment. The amount of loss will depend upon temperature differences between the reinforcement and the abutments Δt and the coefficient of thermal expansion of the steel, and is expressed as

$$\sigma_7 = 20\Delta t. \tag{173}$$

8) *Loss caused by repeated loading* —σ_8. This kind of loss is also determined by specific computations; when the tendons are held between abutments the loss prior to compression of the concrete will be

$$\sigma_{loss\,1} = \sigma_3 + \sigma_4 + \sigma_7; \tag{174}$$

after compression of the concrete,

$$\sigma_{loss\,2} = \sigma_1 + \sigma_2 + \sigma_8. \tag{175}$$

When tensioning is performed against the hardened concrete,

$$\sigma_{loss\,1} = \sigma_4 + \sigma_5; \tag{176}$$

$$\sigma_{loss\,2} = \sigma_1 + \sigma_2 + \sigma_3 + \sigma_6 + \sigma_8. \tag{177}$$

The total amount of loss is computed in accordance with the above data, i.e.,

$$\sigma_{loss} = \sigma_{loss\,1} + \sigma_{loss\,2},$$

but must not be assumed as less than 1,000 kg/cm².

Sec. 18. COMPUTING PRESTRESSED MEMBERS

1. Members Subject to Axial Tension

Phases of stresses provoked within tendons tensioned between abutments are illustrated in Fig. 69, *a*.

Phase I. The tendons are laid in the formwork. Tendon stresses are null.

Phase II. The tendon is pulled until stressed to its predetermined (controlled) value $\sigma_{init.contr}$.

Phase III. The concrete is poured but the tendon is held taut by the abutments; a prestress loss $\sigma_{loss\,1}$ occurs, hence the tendon stresses will be $\sigma_{init.contr}-\sigma_{loss\,1}$.

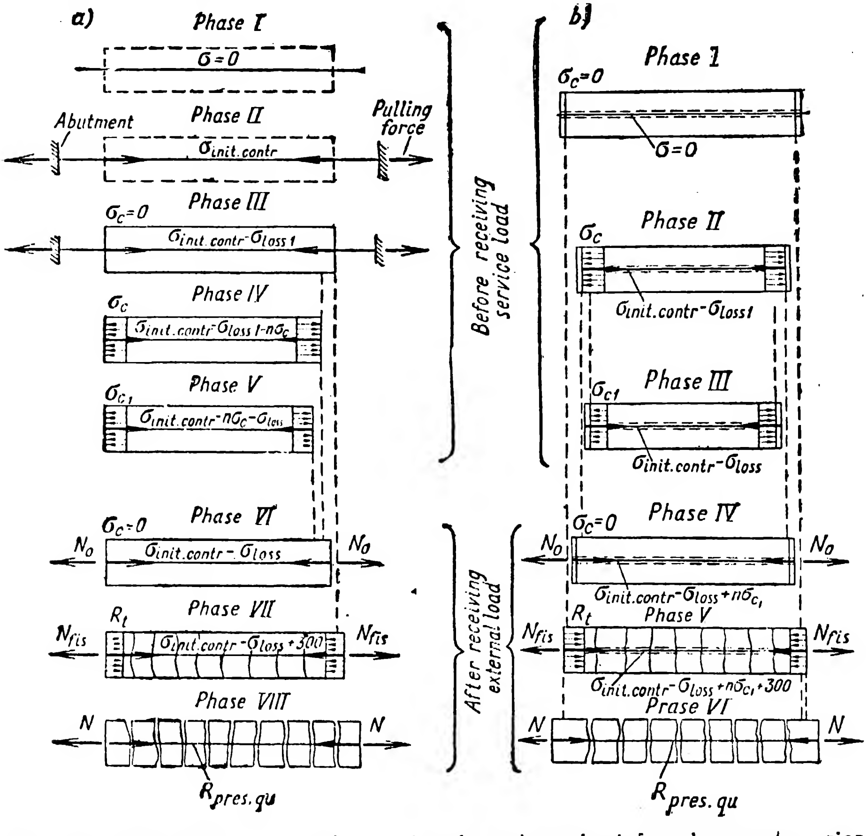


Fig. 69. Stress phases in axially tensioned prestressed reinforced concrete, prior to and after, application of loads

a—when prestressing against abutments; *b*—when prestressing against the hardened concrete

Phase IV. The tendons are released from the abutments and the concrete becomes compressed to a stress of σ_c which somewhat shortens it (Δ or elastic compression). The tendons, because of their bond with the concrete, undergo a similar shortening and their prestress is slackened by

$$\Delta\sigma_s=\varepsilon_s E_s, \text{ in which } \varepsilon_s=\varepsilon_c=\frac{\sigma_c}{E_c};$$

hence,

$$\Delta\sigma_s = \frac{\sigma_c}{E_c} E_s = n\sigma_c.$$

Thus in Phase IV the tendon stresses are

$$\sigma_{init.contr} - \sigma_{loss\ 1} - n\sigma_c.$$

Phase V. A prestress loss $\sigma_{loss\ 2}$ occurs in the tendons, which leaves a prestress remainder of

$$\sigma_{init.\ contr} - \sigma_{loss\ 1} - n\sigma_c - \sigma_{loss\ 2} = \sigma_{init.\ contr} - n\sigma_c - \sigma_{loss}.$$

Phases I to V concern members before they are regularly loaded.

When the pulling force N is applied, the member will lengthen and incur a lessening of pre-compression in the concrete and a corresponding increase of tension in the tendon.

Phase VI. When the external pulling force N has increased so as to attain a certain value, the stresses in the concrete will have reached zero. At this point, tendon stresses will be $\sigma_{init.\ contr} - \sigma_{loss}$, which means that those prestresses are restored which were lost because of elastic compression of the concrete.

Phase VII. When the external force is further increased to N_{fis}, the concrete will undergo tensile stresses which will cause cracking when ultimate tensile strength of the concrete R_t is attained. In the interval during which the concrete stresses grow from zero to R_t, there will be a mean increase of stresses in the tendons of 300 kg/cm²*. Hence, in Phase VII tendon prestresses will be

$$\sigma_{init.\ contr} - \sigma_{loss} + 300.$$

Phase VII serves as the basis for computing fissure anticipation.

Phase VIII. Just as in non-prestressed construction, after fissure formation in the concrete the tendons alone will resist the external pulling force, and when the stresses in the steel reach the latter's ultimate resistance, failure will occur.

Carrying capacity computations are based on Phase VIII.

From the above it is seen that prestressing does not influence the carrying capacity of construction as computed by the failure-stage method, but only postpones fissuring.

The behaviour of a member that is tensioned against the hardened concrete is shown in Fig. 69, *b*.

Phase I. The slack tendon is manoeuvred into the prepared duct within the concrete. The stresses of both components are null.

Phase II. The tendons are pulled, attain their controlled prestress $\sigma_{init.contr}$ and actuate compression σ_c in the concrete. The shaft is filled with cement mortar or grout. The tendons experience their first prestress loss $\sigma_{loss\ 1}$, the stresses being lowered to $\sigma_{init.contr} - \sigma_{loss\ 1}$.

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* See Sec. 8.

Phase III. Prestress loss $\sigma_{loss\,2}$ takes place in the tendons and their remaining prestresses are $\sigma_{init.contr}-\sigma_{loss\,1}-\sigma_{loss\,2}=\sigma_{init.contr}-\sigma_{loss}$, while the prestresses in the concrete are $\sigma_{c.1}<\sigma_c$.

Phases I to III are valid up to the time the member receives its designated service load.

Phase IV. The external pulling force N is applied which, when it reaches a given value, neutralises compression in the concrete, whereas the stresses in the tendons correspondingly increase by a value equal to $n\sigma_{c.1}$ and attain $\sigma_{init.contr}-\sigma_{loss}+n\sigma_{c.1}$.

Phase V. As the external force is further increased, tensile stresses are created in the concrete and reach a value of R_t when the external force attains $N_{f.s}$. Fissures form in the concrete as tendon stresses achieve $\sigma_{init.contr}-\sigma_{loss}+n\sigma_{c.1}+300$. This phase serves as the basis for calculation of fissure anticipation.

Phase VI. Tendon stresses attain R_s and failure occurs. This phase serves as the basis for computing carrying capacity (strength).

The formula for calculating carrying capacity of axially tensioned construction is evolved from the condition of equilibrium between external and internal forces, i.e., from Phase VIII if the reinforcement is tensioned between abutments, or from the corresponding Phase VI if the tendon is tensioned against the hardened concrete. Moreover, the stresses in the tendons are assumed as for qualified design resistance:

$$N \leqslant mR_{pres.\,qu}F_{pres}, \tag{178}$$

where N—longitudinal effective force;

m—service coefficient of the member;

$R_{pres.qu}$—qualified design strength of tendons;

F_{pres}—cross-sectional area of tendons.

If the member also embraces untensioned reinforcement with an area F_s and a qualified design resistance $R_{s.qu}$, then formula (178) takes the form of

$$N \leqslant m\,(R_{pres.\,qu}F_{pres}+R_{s.\,qu}F_s). \tag{179}$$

The formula for calculating fissure anticipation in axially tensioned members is likewise evolved from the condition of equilibrium between external forces and internal stresses, that is, from Phase VII when tensioning is done between abutments, and from Phase V if it is done against the hardened concrete. The stresses in the concrete just prior to cracking will be equal to qualified design resistance $R_{t.qu}$. The outer forces responsible for fissure creation in the concrete shall be designated as N_{fis},* and a deviation coefficient $m_{dev}=0.9$ shall be attached to the tension value in the tendons in order to unify the formulae. The stresses in the tendons just prior to the time when the

* As noted in Sec. 17, Item 3, $N_{fis}=N$ for 1st category fissure-proof construction, and $N_{fis}=N^s$ for 2nd category members.

concrete stresses are neutralised shall be designated as σ_0, regardless of production methods.

With all the above taken into account, the formula becomes

$$N_{fis} \leqslant F_c R_{t.\,qu} + F_{pres}\,(m_{dev}\sigma_0 + 300). \tag{180}$$

When tensioning is performed between abutments,

$$\sigma_0 = \sigma_{init.\,contr} - \sigma_{loss}, \tag{181}$$

and when it is done against the hardened concrete,

$$_{init.\,contr} - \sigma_{loss} + n\sigma_{c.1}, \tag{182}$$

where

$$\sigma_{c.1} = \frac{F_{pres}\,(\sigma_{init.\,contr} - \sigma_{loss})}{F_c} \tag{183}$$

If the member should also contain ordinary (untensioned) bars F_s, then when the concrete stresses reach zero, these bars will be found compressed to a value of σ_s because of shortening of the member from shrinkage and creep; the compression of such ordinary bars will be equal in value to the prestress loss in the tendons, likewise caused by shrinkage and creep, i. e., $\sigma_s = \sigma_1 + \sigma_2$. Upon further increase of the external load, 300 kg/cm² of tensile stresses will be added to the reinforcement F_s just prior to fissuring of the concrete. Thus, at the point of concrete fissuring, the stresses in the steel F_s will be

$$-\sigma_s + 300 \text{ kg/cm}^2.$$

Hence, because of the stresses in the ordinary bars, formula (180) becomes

$$N_{fis} \leqslant F_c R_{t.\,qu} + F_s\,(300 - \sigma_s) + F_{pres}\,(m_{dev}\sigma_0 + 300). \tag{184}$$

2. Bending Members

The tendons of prestressed bending members are placed at a considerable distance from the section's centre of gravity (usually beyond the kern). Therefore the force applied to compress the member during its manufacture might lead to cracking in the zone intended to withstand compression (Fig. 70, *a*). To avoid this, bending members are equipped not only with tendons F_{pres} but also with tendons F'_{pres} situated in the zone of future compression (Fig. 70, *b*). As a rule, $F'_{pres} = (0.2\text{-}0.25)\,F_{pres}$.

In prestressed beams where some of the tendons are diverted to the upper zone at the supports (Fig. 70, *c*), a decrease of tensile stresses during fabrication (in the intended compression zone) is achieved through the horizontal component of the diagonal forces created when the aforesaid diverted tendons are tensioned. The vertical component of the same diagonal forces produces lateral compression which

renders the diagonal planes less liable to fissuring. It must be noted that carrying capacities are somewhat reduced by this tensioning, inasmuch as the tendons may remain taut at the moment of failure and thus lower the ultimate stresses in the compression zone.

Let us examine the behaviour of a beam whose tendons have been tensioned between abutments (Fig. 71).

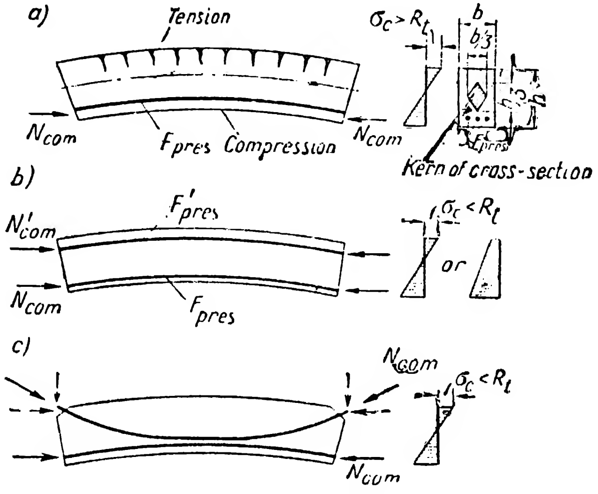


Fig. 70. Stress conditions in a prestressed beam during its fabrication

Phase I. The tendons F_{pres} and F'_{pres} are placed in the formwork; their stresses are equal to zero.

Phase II. The tendons are drawn taut to their predetermined (controlled) stresses $\sigma_{init.contr}$.

Phase III. The concrete is poured; the first prestress loss $\sigma_{loss\,1}$ takes place as the tendons are still held by the abutments. Stresses in the tendons are equal to $\sigma_{init.contr} - \sigma_{loss\,1}$.

Phase IV. The ends of the tendons are released from the abutments, resulting in compression of the concrete. The stress diagram for the concrete may have either one or two signs, depending upon the position of the centre of gravity and the area relationship of the tendons F_{pres} and F'_{pres}; with two signs, the stresses in the tension zone will not exceed $R_{t.qu}$. The beam will bend in the direction opposite to that incurred by the future load. Elastic compression in the concrete will lower the stresses in the tendons F_{pres} and F'_{pres}.

Phase V. Shrinkage and creep develop in the concrete and incur an additional prestress loss $\sigma_{loss\,2}$ in the reinforcement.

Phases I to V predate service loading.

After loading, a bending moment is created which causes tensile stresses in the zone harbouring upon the bars F_{pres}, as well as compressive stresses in the zone where the tendons F'_{pres} are situated, each of the stress groups being superimposed upon their respective prestresses.

Phase VI. When the growing load has imparted a definite value to the bending moment, pre-compression in the concrete will be decreased by the tensile stresses and will align with the stresses in the tendons F_{pres}. The stresses in these latter bars when concrete compression has become zero, shall be signified as σ_0 (just as in an axially tensioned member).

Phase VII. As the load (bending moment) continues to grow, tensile stresses are produced in the concrete zone of the bars F_{pres}; when these stresses attain $R_{t.qu}$, the concrete will fissure. The stress diagram is assumed as triangular for the concrete in the compression zone and rectangular for the concrete in the tension zone; this infers that the development of plastic deformation is recognised just as in non-prestressed beams (see Fig. 28). When the stresses in the concrete will have grown from zero to $R_{t.qu}$, those in the reinforcement will have increased by 300 kg/cm² and will be equal to σ_0+300.

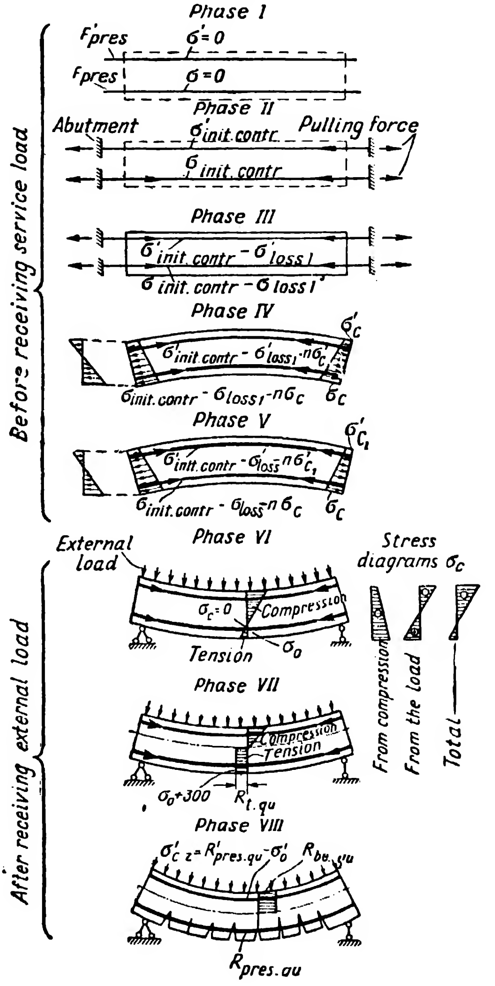


Fig. 71. Stress conditions in a prestressed reinforced concrete beam, tensioned between abutments, before and after application of the load

Phase VII is assumed as the basis for fissure-anticipation computations.

Phase VIII. With further growth of the load (bending moment), tensile stresses in the tendons F_{pres} will increase, with a corresponding

stress addition in the compression-zone concrete, where the stress line, just as in ordinary beams, will become distorted. Tensile stresses will relax in the reinforcement F'_{pres}. When the stretched tendons F_{pres} and the concrete in the compression zone attain their ultimate stress values $R_{pres.qu}$ and $R_{be.qu}$ respectively, failure will occur. At the moment of failure the stresses in the tendons F'_{pres} will be

$$\sigma'_{c.\,z} = R'_{pres.\,qu} - \sigma'_{0}, \qquad (185)$$

in which $R'_{pres.\,qu}$—the reinforcement's qualified design resistance under compression;

σ'_0—prestress in the reinforcement F'_{pres}, with allowances for loss.

Phase VIII serves for computing carrying capacity.

Thus, just as in the preceding case, prestressing of beams only postpones fissure formation but does not increase carrying capacity as determined through failure-stage computations.

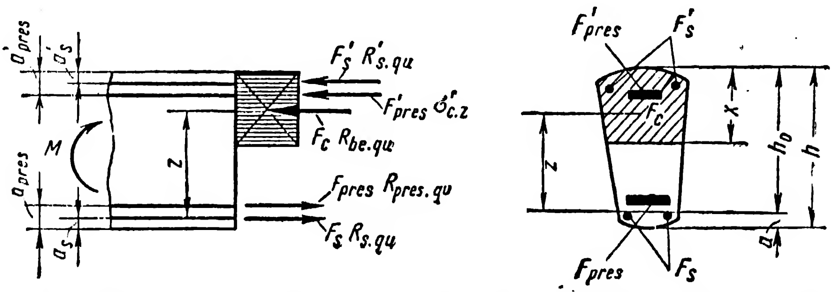


Fig. 72. Investigation of resistance in prestressed reinforced concrete beams

The same thing is true when tensioning of reinforcement is performed against the hardened concrete of the beam.

Computing carrying capacity at a normal cross-section. The stresses corresponding to Phase VIII (failure stage) in a cross-section of any symmetrical shape are illustrated in Fig. 72.

If the section contains non-prestressed bars F_s and F'_s, their stresses will attain $R_{s.qu}$ at the moment of failure. The formula (condition for resistance) stems, as usual, from the conditions attending moment equilibrium of outer and internal forces in relation to the centre of gravity of the bars in the tension zone:

$$M \leqslant m\,[R_{be.\,qu}F_c z + \sigma'_{c.\,z}F'_{pres}(h_0 - a'_{pres}) + R_{s.\,qu}F'_s(h_0 - a')]. \qquad (186)$$

If the results of multiplying areas and moment-point distances are replaced by corresponding static moments, then the condition for resistance will be expressed as

$$M \leqslant m\,(R_{be.\,qu}S_c + \sigma'_{c.\,z}S'_{pres} + R_{s.\,qu}S'_s). \qquad (187)$$

The position of the neutral axis is derived through the condition of equilibrium when all forces are projected on the axis of the member

$$R_{be.qu}F_{pres}+R_{s.qu}F_s-\sigma'_{c.z}F'_{pres}-R_{s.qu}F'_s=R_{be.qu}F_c. \qquad (188)$$

Computing carrying capacity at diagonal planes. The character of failure in a diagonal plane of a prestressed member is identical to that in a non-prestressed beam. Therefore all formulae as well as methods of computing stirrups and bent-up bars, as presented in Sec. 10, are valid for prestressed construction, with the stresses in the tendons being assumed as equal to qualified design resistance $R_{pres.qu}$ attended by a service coefficient $m_{pres.di}$. Some of the tendons, turned upward at the supports, are assumed as functioning as bent-up bars.

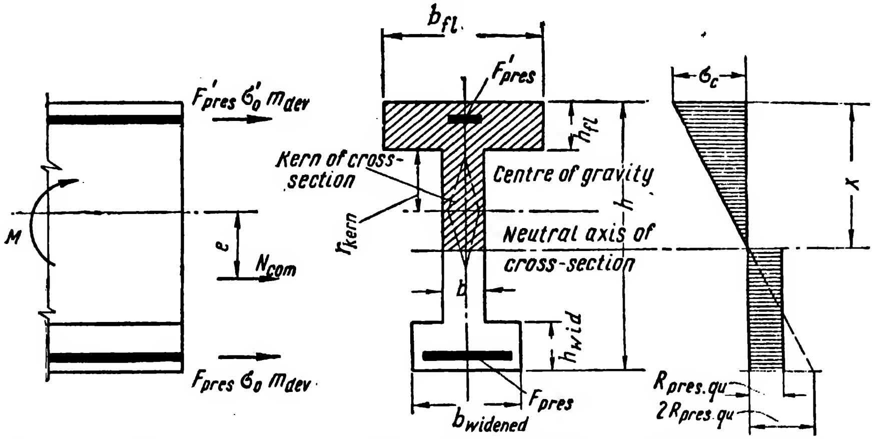


Fig. 73. Computing fissure anticipation in a prestressed reinforced concrete beam

Fissure-anticipation calculations. In Phase VII the stress diagram is considered as triangular for the concrete in the compression zone of the transformed cross-section, and rectangular for the tension concrete, with a stress of $R_{t.qu}$ (Fig. 73).

The aim in fissure-anticipation calculations is to see that the external loads and internal compression do not cause higher stresses than $R_{t.qu}$ in the tension-zone concrete.

The bending moment due to the load shall be designated as M.

The resultant of forces in the tendons F_{pres} and F'_{pres} that compress the section,

$$N_{com}=F_{pres}\sigma_0 m_{dev}+F'_{pres}\sigma'_0 m_{dev}, \qquad (189)$$

applied with an eccentricity e in respect to the centre of gravity of the section, will produce an inverse-sign moment. The total moment in the section

$$M_{tot}=M-N_{com}e. \qquad (190)$$

The section will undergo eccentric compression from the united action of the moment M_{tot} and the longitudinal force N_{com}, with the

result that the stresses in the concrete's tension zone (according to the research of the Soviet scientists A. A. Gvozdyov and S. A. Dmitreyev) will be

$$\sigma_{c.t}=\frac{M_{tot}-N_{com}r_{kern}}{W_c}, \qquad (191)$$

where $r_{kern}=\frac{W_0}{F_{pr}}$ expresses the distance from the centre of gravity of the transformed section to a point of the kern at its border;

W_0—resisting moment of the transformed cross-section in respect to the extreme tensile fibre as determined for elastic materials through formulae in Strength of Materials;

W_c—resisting moment of the transformed cross-section with allowances for plastic properties of the concrete in the tension zone, as determined by the formula

$$W_c=W_0\gamma; \qquad (192)$$

γ —a coefficient evaluated in accordance with the shape of the cross-section: $\gamma=1.75$ for rectangular shapes and T-beams whose flanges are in the compression zone.*

Let $N_{com}(e+r_2)=M^{2}_{com}$ be designated as the moment of the forces compressing the section in respect to a point of the kern.

Then the condition for fissure anticipation will take the following form:

$$\sigma_{c.t.}=\frac{M_{tot}-M^{kern}_{com}}{W_c}\leqslant R_{t.qu}, \qquad (193)$$

the final expression of which is

$$M_{tot}-M^{kern}_{com}\leqslant R_{t.qu}W_c. \qquad (194)$$

Checking maximum diagonal stresses. A check of beam resistance must include maximum diagonal stresses, both tensile $\sigma_{di.t}$ and compressive $\sigma_{di.com}$. Such a verification of transformed sections is conducted through the usual formulae given in Strength of Materials and with the fulfilment of the following conditions:

$$\begin{aligned}&\sigma_{di.t}\leqslant 1.5R_{t.qu};\\&\sigma_{di.com}\leqslant R_{pr.qu}\ \text{for 1st category construction};\\&\sigma_{di.com}\leqslant 0.8R_{pr.qu}\ \text{for 2nd category construction.}\end{aligned} \qquad (195)$$

* For I-sections, when $3<\frac{h_{fl}}{b}<8$ and $\frac{b_w}{b}<4$, $\gamma=1.5$;

when $\frac{b_w}{b}>4$ and $\frac{h_w}{h}>0.2$, $\gamma=1.5$;

and

when $\frac{b_w}{b}>4$ and $\frac{h_w}{h}<0.2$, $\gamma=1.25$.

Deformation computations. Deflection in prestressed beams devoid of fissures in the tension zone is determined through ordinary formulae given in Structural Mechanics and at a rigidity $B_0=0.85E_c^sJ_{pr}$; this means that rigidity is computed as for a homogeneous, transformed section but with a decreased coefficient of 0.85. When B_0 is computed in connection with a constant load, a coefficient of not less than 1.0 is introduced as given in Chapt. III, Sec. 11 [formula (102)] when the value of $\theta=0.5$.

Deflection in prestressed beams that fissure in the tension zone (3rd category) is assumed as the sum of two deflections: 1) deflection f_1 at a rigidity of $E_c^sJ_{pr}$ from part of the load applied prior to neutralisation of the prestress compression in the concrete, and 2) deflection f_2, calculated as with the existence of fissures in the tension zone and at a rigidity of B.*

3. Determining the Effect of Forces Created During Production of Prestressed Members

The process of prestressing reinforcement involves either axial or eccentric compression in the concrete, depending upon the position of the resultant of compressive forces.

With an axially applied compressive force, a check of the member's resistance will mean fulfilment of the condition

$$F_{pres}(\sigma_0-\sigma_{loss})\leqslant\varphi(F_cR_{pr.\,qu}+F_sR_{s.\,qu}), \qquad (196)$$

where φ—a coefficient of longitudinal deflection.

If tensioning is conducted between abutments, $\varphi=1$, whereas if it is accomplished against the hardened concrete, φ is taken from Table 15.

With an eccentrically applied resultant of compressive forces, condition (194) must be observed in checking fissure avoidance, assuming that $M_{tot}=0$.

4. Illustrative Problems

Illustrative problem 18. Given: a cylindrical tank whose walls are subject to axial tension; $N^s=20$ t; $N=22$ t; wall thickness $h=10$ cm; $b=100$ cm; diameter of tank $D=15$ m; grade 200-B concrete; tendons of high-strength round carbon wire, 5 mm dia. (State Standard 7348-55); tensioning to be performed on the concrete's outer surface (Fig.74, *a*), untensioned reinforcement $F_s=6ø6$ of St-3 steel.

Compute the prestressed walls.

Solution.

1) *Gathering design data*. From Table 17, $R_{t.qu}=10$ kg/cm²; from Table 6 $R_{pr}=80$ kg/cm²: for checking pre-compression $m_c=1.2$; $R_{pr.qu}=80\times$

* See Sec. 11.

$\times 1.2 = 96$ kg/cm²; from Table 3 $R^s_{pres} = 17.000$ kg/cm²; from Table 8 $R_{pres} = 13{,}500$ kg/cm²; $m_{pres} = 0.7$; $R_{pres.qu} = 13{,}500 \times 0.7 = 9{,}450$ kg/cm²; from Table 8 $R_s = 2{,}100$ kg/cm²; $m_s = 1$; $R_{s.qu} = 2{,}100$ kg/cm²; $F_s = 6 \times 0.283 = 1.7$ cm².

For untensioned bars $E_s = 2{,}100{,}000$ kg/cm². For tensioned tendons $E_s = 1{,}800{,}000$ kg/cm². $E^s_c = 290{,}000$ kg/cm². In computing stress loss σ_2 through formula (169), $k = 1$; $R' = R$.

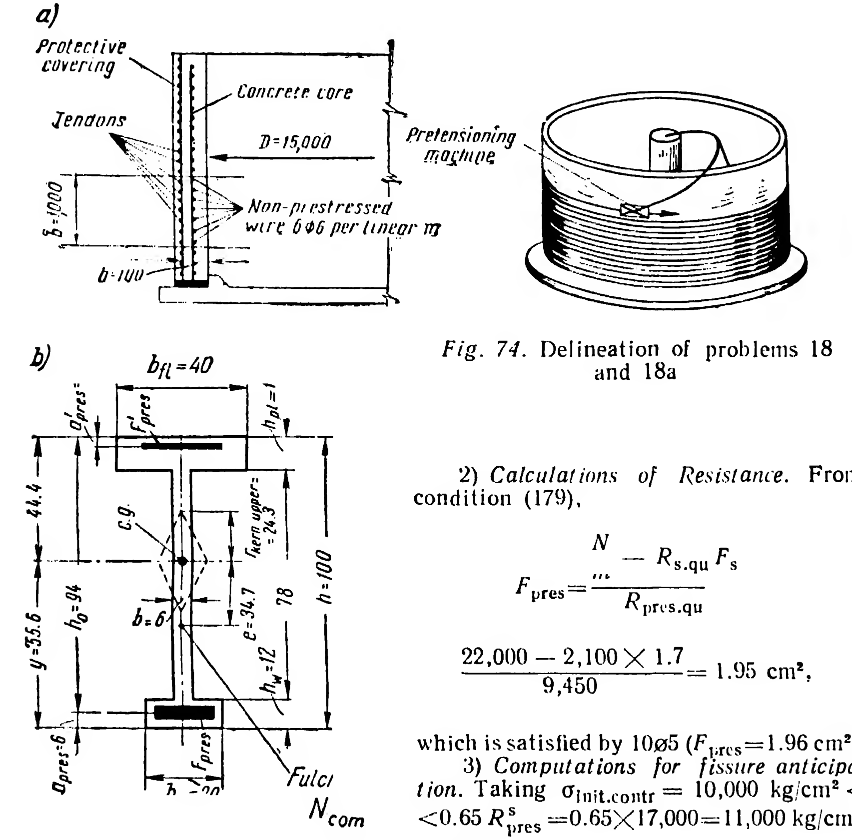


Fig. 74. Delineation of problems 18 and 18a

2) *Calculations of Resistance.* From condition (179),

$$F_{pres} = \frac{N_{m} - R_{s.qu} F_s}{R_{pres.qu}}$$

$$\frac{22{,}000 - 2{,}100 \times 1.7}{9{,}450} = 1.95 \text{ cm}^2,$$

which is satisfied by 10ø5 ($F_{pres} = 1.96$ cm²).

3) *Computations for fissure anticipation.* Taking $\sigma_{init.contr} = 10{,}000$ kg/cm² $< 0.65 R^s_{pres} = 0.65 \times 17{,}000 = 11{,}000$ kg/cm².

The area of the transformed cross-section

$$F_{tr} = F_c + F_s n + F_{pres} n_1 = 10 \times 100 + 1.7 \frac{2.1 \times 10^6}{2.9 \times 10^5} + 1.96 \frac{1.8 \times 10^6}{2.9 \times 10^5} = 1{,}024 \text{ cm}^2.$$

Determining loss of prestress.

Loss prior to compressing the concrete: $\sigma_{loss\,1} = \sigma_4 + \sigma_5 = 0$, inasmuch as the reinforcement is wound continuously and located on the outer surface of the wall instead of in ducts.

Loss after compression of the concrete:

$$\sigma_{loss\,2} = \sigma_1 + \sigma_2 + \sigma_3 + \sigma_6;$$

$$\sigma_1 = 300 \text{ kg/cm}^2;$$

$$\sigma_c = \frac{(\sigma_{init.contr} - \sigma_{loss\,1}) F_{pres}}{F_{c.tr}} = \frac{10{,}000 \times 1.96}{1{,}024} = 19.4 \text{ kg/cm}^2;$$

from formula (170)

$$\sigma_2 = \frac{0.75 \times 1 \times 1.8 \times 10^6 \times 200}{2.9 \times 10^5 \times 200} \times 19.4 = 90 \text{ kg/cm}^2;$$

from formula (171),

$$\sigma_3 = 0.05 \times 10{,}000 = 500 \text{ kg/cm}^2;$$

when $d > 3\text{m}$, $\sigma_6 = 0$;

$$\sigma_{\text{loss } 2} = 300 + 90 + 500 = 890 \text{ kg/cm}^2.$$

Total stress loss

$$\sigma_{\text{loss}} = \sigma_{\text{loss } 1} + \sigma_{\text{loss } 2} = 890 \text{ kg/cm}^2 < 1{,}000 \text{ kg/cm}^2;$$

σ_{loss} is therefore taken as 1,000 kg/cm².

From formula (183) the stresses in the concrete

$$\sigma_{c.1} = \frac{1.96\,(10{,}000 - 1{,}000)}{1{,}024} = 17.5 \text{ kg/cm}^2.$$

From formula (182), when the stresses in the concrete drop to zero, the stresses σ_0 in the tendons $F_{\text{t.res}}$ will be:

$$\sigma_0 = 10{,}000 - 1{,}000 + \frac{1.8 \times 10^6}{2.9 \times 10^5}\, 17.5 = 9{,}110 \text{ kg/cm}^2.$$

The stresses in the bars F_s at the moment of fissure formation in the concrete $\sigma_s = \sigma_1 + \sigma_2 = 300 + 90 = 390$ kg/cm².

In 1st category construction, which includes the given tank, $N_{\text{fis}} = N = 22{,}000$ kg; $m_{\text{dev}} = 0.9$. From condition (184)

$$N_{\text{fis}} = 10 \times 100 \times 10 + 1.7\,(300 - 390) + 1.96\,(0.9 \times 9{,}110 + 300) = 26{,}500 > 22{,}000 \text{ kg},$$

which indicates that the wall will be fissure-proof.

It is not necessary to verify, through formula (196), the resistance of the concrete *vs.* the force causing its compression, inasmuch as the magnitude of pre-compression is very small ($\sigma_c = 19.4 \text{ kg/cm}^2 < 0.5R'_{\text{pr. qu}}$).

(Illustrative problem 18a)

Compute a prestressed I-beam (Fig. 74, *b*). Given: grade 400 concrete, $R_{\text{be. qu}} = 210$ kg/cm², $R_{\text{t. qu}} = 18$ kg/cm², $E_c^s = 3.8 \times 10^5$ kg/cm²; reinforcement of 5 mm high-strength cold-drawn intermittently deformed wire (State Standard 8480-57), $R^s_{\text{pres}} = 15{,}000$ kg/cm², $R_{\text{t res. qu}} = 8{,}400$ kg/cm²; $E_s = 1.8 \times 10^6$ kg/cm². Prestressing to be performed between abutments. The work to be done on a bed with heat-cured concrete. The difference in temperature between the reinforcement and its abutments $\Delta t = 30°$. The reinforcement (tendon) is to be released when the concrete's resistance $R' = 0.7R = 0.7 \times 400 = 280$ kg/cm². Beam span $l = 11.7$ m. Specified uniformly distributed load $q^s = 2{,}400$ kg/m, design load $q = 3{,}300$ kg/m. Service coefficient $m = 1.0$.

a) *Computing normal-plane resistance:*

$$M = \frac{ql^2}{8} = \frac{3{,}300 \times 11.7^2}{8} = 56{,}450 \text{ kg/m} = 56.45 \text{ t/m}.$$

For a tentative calculation, assume that $F_{\text{pres}} = 7.84$ cm² (40ø5), $F'_{\text{pres}} = 1.57$ cm² (8ø5). Supplementary untensioned reinforcement F_s and F'_s is to be ignored in the calculations because of its small influence.

Assuming $\sigma_{\text{init. contr}} = \sigma'_{\text{init. contr}} = 0.65R^s_{\text{pres}} = 0.65 \times 15{,}000 = 9{,}750$ kg/cm².

Tensioning losses: from relaxation, according to formula (171), $\sigma_3 = \sigma_3' = 0.05 \times 9{,}750 = 490$ kg/cm²; from a drop of temperature, according to formula (173), $\sigma_7' = 20 \times 30 = 600$ kg cm²: loss $\sigma_{loss\,1} = \sigma_3 + \sigma_7 = 490 + 600 = 1{,}090$ kg/cm²; from shrinkage $\sigma_1 = \sigma_1' = 400$ kg/cm².

Determining σ_c in order to compute loss due to creep:

$$n = \frac{E_s}{E_c^s} = \frac{1.8 \times 10^6}{3.8 \times 10^5} = 4.75.$$

Through formula (168) the transformed area of the section

$F_{c.\,tr} = 78 \times 6 + 40 \times 10 + 20 \times 12 + (7.84 + 1.57) \times 4.75 = 1{,}153$ cm².

Static moment of the transformed cross-section in relation to the lower fibre:

$$S_{tr} = 20 \times 12 \times 6 + 78 \times 6 \times 51 + 40 \times 10 \times 95 + 7.84 \times 4.75 \times 6 + 1.57 \times 4.75 \times 96 = 64{,}180 \text{ cm}^3.$$

Distance from lower fibre to centre of gravity,

$$y = \frac{S_{tr}}{F_{c.\,tr}} = \frac{64{,}180}{1{,}153} = 55.6 \text{ cm}.$$

Moment of inertia of transformed section

$$J_{c.\,tr} = \frac{20 \times 12^3}{12} + 20 \times 12 \times 49.6^2 + \frac{6.78^3}{12} + 6.78 \times 4.6^2 + \frac{40 \times 10^3}{12} + 40 \times 10 \times 39.4^2 + 7.84 \times 4.75 \times 49.6^2 + 1.57 \times 4.75 \times 40.4^2 = 1{,}567{,}200 \text{ cm}^4.$$

Compressive force

$N_{com} = (F_{pres} + F'_{pres})\, \sigma_0 = (7.84 + 1.57) \times (9{,}750 - 1{,}090) = 81{,}500$ kg.

Force of compression applied to the centre of gravity of all the longitudinal reinforcement at a distance e from the centre of gravity of the transformed cross-section

$$e = 55.6 - \frac{7.84 \times 6 + 1.57 \times 96}{7.84 + 1.57} = 34.7 \text{ cm}.$$

The moment of the compressive force

$$M_{com} = N_{com} \times e.$$

The stresses of pre-compression in the concrete at the level of the applied force N_{com}

$$\sigma_c = \frac{N_{com}}{F_{c.\,tr}} + \frac{M_{com} e}{J_{c.\,tr}} = \frac{N_{com}}{F_{c.\,tr}} + \frac{N_{com} e^2}{J_{c.\,tr}} = \frac{81{,}500}{1{,}153} + \frac{81{,}500 \times 34.7^2}{1{,}567{,}200} = 133 \text{ kg/cm}^2 < 0.5R' = 140 \text{ kg/cm}^2.$$

Loss from creep according to formula (169):

$$\sigma_2 = \frac{1 \times 1.8 \times 10^6 \times 400}{3.8 \times 10^5 \times 280} \times 133 = 910 \text{ kg/cm}^2.$$

Loss $\sigma_{loss\,2} = \sigma_1 + \sigma_2 = 400 + 910 = 1{,}310$ kg/cm².

Total loss $\sigma_{loss} = \sigma_{loss\,1} + \sigma_{loss\,2} = 1{,}090 + 1{,}310 = 2{,}400$ kg/cm².

The stresses $\sigma'_{c.z}$ in the bars F'_{pres}, introduced in computing resistance, are determined through formula (185):

$$\sigma'_{c.z} = 3,600 - (9,750 - 2,400) = -3,750 \text{ kg/cm}^2.$$

When the width of the section $b_{fl} = 40$ cm, the height of the compression zone is found through condition (188) where $F_s = F'_s = 0$. and $F_c = b_{fl}x$.

$$x = \frac{8,400 \times 7.85 + 3,750 \times 1.57}{210 \times 40} = 8.6 \text{ cm}.$$

$x < h_{fl} = 10$ cm, hence the section is considered rectangular and with a width of b_{fl}.

When $F_c = b_{fl}x$, $z = h_0 - 0.5x$, and $F'_s = 0$, formula (186) will determine the moment borne by the cross-section:

$$M = 1 \times [210 \times 40 \times 8.6 (94 - 4.3) - 3,750 \times 1.57 (94 - 4)] =$$

$= 5,930,000$ kg/cm $= 59.3$ t/m $\quad 56.45 < 59.3$, hence there is sufficient resistance.

b) *Computing diagonal-plane resistance.*

$$Q = \frac{ql}{2} = \frac{3,300 \times 11.7}{2} = 19,300 \text{ kg}.$$

From formula (75)

$$q_{st} = \frac{19,300^2}{0.6 \times 210 \times 6 \times 94^2} = 55.7 \text{ kg/cm}.$$

which is satisfied by reinforcing the beam with one prefabricated frame whose upright bars are 10 mm intermittently deformed St-5 bars: $R_{s.qu} = 1,900$ kg/cm², $f_{st} = 0.78$ cm².

According to formula (76) required spacing

$$u = \frac{1,900 \times 0.78}{55.7} = 27 \text{ cm}.$$

Checking condition (79):

$$u = \frac{0.1 \times 210 \times 6 \times 94^2}{19,300} = 55.7 > 27,$$

hence the spacing of upright bars $u = 27$ cm (25 cm).

c) *Computing fissure resistance under load.*

$$M^s = \frac{q^s l^2}{8} = \frac{2,400 \times 11.7^2}{8} = 41,200 \text{ kg/m} = 41.2 \text{ t/m}.$$

The resisting moment of the transformed section (its lower part)

$$W_0 = \frac{J_{c.tr}}{y} = \frac{1,567,200}{55.6} = 28,000 \text{ cm}^3.$$

The distance from the centre of gravity of the transformed cross-section to the upper point of the kern

$$r_{\text{kern upper}} = \frac{W_0}{F_{c.tr}} = \frac{28,000}{1,153} = 24.3 \text{ cm}.$$

The resisting moment of the transformed cross-section, with due regard to concrete plasticity in the tension zone, is determined by formula (192):

$$\text{when } \frac{b_{fl}}{b} = \frac{40}{6} = 6.7 \text{ and } \frac{b_w}{b} = \frac{20}{6} = 3.7, \ \gamma = 1.5;$$

$$W_c = 28,000 \times 1.5 = 42,000 \text{ cm}^3.$$

The stresses in the upper and lower bars when $m_{dev} = 0.9$:

$$m_{dev}\sigma_0 = m_{dev}\sigma_0' = 0.9\ (9{,}750 - 2{,}400) = 6{,}600\ \text{kg/cm}^2.$$

In order to calculate M_{com}^{kern}, N_{com} must be determined, including all losses:

$$N_{com} = (7.84 + 1.57)\ 6{,}600 = 62{,}000\ \text{kg}.$$

$$e = 34.7\ \text{cm};\ M_{com}^{kern} = N_{com}^{kern}(e + r_{kern\ upper}) = 62{,}000\ (34.7 + 24.3) =$$
$$= 3{,}660{,}000\ \text{kg/cm} = 36.6\ \text{t/m}.$$

$$R_{t.\ qu}W_c = 18 \times 42{,}000 = 755{,}000\ \text{kg/cm} = 7.55\ \text{t/m}.$$

Checking fissure resistance, according to condition (194):

$$41.2 - 36.6 \leqslant 7.55;\quad 4.6 < 7.55,$$

hence, fissure resistance is assured.

d) *Determining beam deflection.*

$$\text{Rigidity}\ B_{zero\ mo} = 0.85\ E_c^s J_{tr} = 0.85 \times 380{,}000 \times 1{,}567 \times 200 =$$
$$= 5.05 \times 10^{11}\ \text{kg/cm}^2.$$

At a constant load, $0.5\vartheta = 0.5 \times 2 = 1$, hence, $B_{zero} = B_{zero\ mo} = 5.05 \times 10^{11}$ kg/cm².

Deflection in midspan from the load

$$f_1 = \frac{5q^s l^4}{384 \times B_{zero}} = \frac{5 \times 24 \times 1{,}170^4}{384 \times 5.05 \times 10^{11}} = 1.17\ \text{cm}.$$

Determining retroflexion of the beam when the tendon is released from the abutments:

when resistance of the concrete $R' = 0.7R$, $E_c^s = 3.3 \times 10^5$;

hence,

$$B'_{zero} = B_{zero}\frac{3.3 \times 10^5}{3.8 \times 10^5} = 4.37 \times 10^{11}\ \text{kg/cm}^2.$$

$$f_{retro} = \frac{N_{com}el^2}{8B_{zero}} = \frac{81{,}500 \times 34.7 \times 1{,}170^2}{8 \times 4.37 \times 10^{11}} = 1.1\ \text{cm}.$$

Total deflection

$$f = f_1 - f_{retro} = 1.17 - 1.01 = 0.07\ \text{cm}.$$

CHAPTER VII

DESIGN PRINCIPLES IN REDISTRIBUTION OF BENDING MOMENTS

Sec. 19. DEFINITIONS OF A PLASTIC PIVOT AND OF BENDING MOMENT REDISTRIBUTION

When the applied load of a reinforced concrete bending member has imparted a certain amount of stresses to the cross-section, the tension bars will begin to yield if they are of mild steel. When yielding commences, plastic deformation along a given length of the bars will deform the member (mutually twisting the parts on both sides of the given length) without further addition of the load. This phenomenon is known as the *creation of a plastic pivot.*

The stresses in the compression-zone concrete of the said cross-section will increase together with plastic deformation in the tension steel, and if the member represents a statically determinate system (such as the single-span freely supported beam shown in Fig. 75,*a*), the stresses in the concrete will reach ultimate compression in bending and cause failure.

If the beam is statically-indeterminate, e. g., if one end is restrained and the other is freely supported (Fig. 75, *b*), the concrete stresses in the compression zone of the plastic pivot might not attain the value of ultimate resistance, as heightening of plastic deformation and crushing of the concrete in the compression zone will be hindered by the restrained support, functioning as an additional tie.

From this it is seen that the creation of a plastic pivot in a statically indeterminate system merely reduces its degree of static indeterminateness. The above beam, with one end restrained and the other supported freely, will become statically-determinate after the formation of the first plastic pivot. Subsequently, as the load grows, there will be a loss of geometric stability and a second plastic pivot will appear at the support B.

A plastic pivot experiences an unchanged bending moment of

$$M = mm_s R_s F_s z, \tag{197}$$

when the load increases, because the stresses will have gained the yield point in the bars (Fig. 75,*c*).

The first plastic pivot will originate in the plane where the bars yield because of insufficient bar area, unable to sustain the bending moment in its elastic stage. The location of such a plane will depend upon the methods of either mid- or end-section reinforcement.

In statically indeterminate systems the creation of a plastic pivot will engender redistribution of bending moments amidst the various

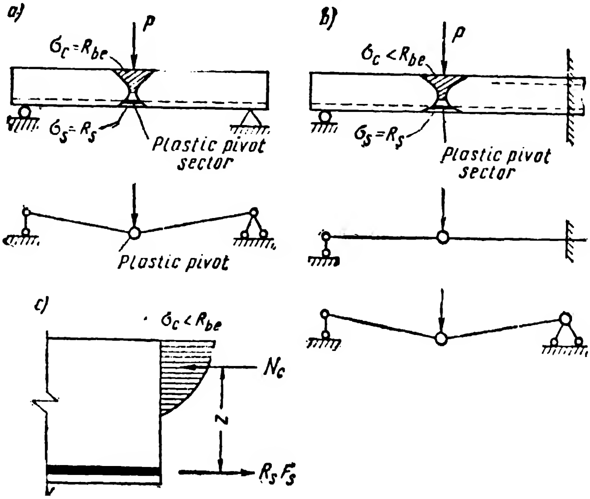


Fig. 76. Creation of a plastic pivot in reinforced concrete beams

a—a single-span statically determinate beam; *b*—a statically indeterminate beam with one restrained end and the other end freely supported; *c*—stress diagram in a plastic pivot

cross-sections. To explain this, a statically uni-indeterminate beam will be taken as an example (Fig. 76,*a*).

The creation of a plastic pivot, due to the load *P*, at the support *B* will render the member statically-determinate and with any further growth of the load it will continue to function under this new scheme: the bending moment will increase only in the interspan and remain constant at the support M_B. At a given addition to the load, a plastic pivot will also form in the interspan, the beam will be transformed into a geometrically unstable system and it will fail.

Directly before failure, the condition of equilibrium will cause the bending moment in the span (Fig. 76, *c*) to be

$$M_{sp}=M_0-M_B\frac{a}{l},$$

which gives

$$M_{sp}+M_B\frac{a}{l}=M_0, \tag{198}$$

where M_0—the moment of a simple, single-span beam.

Similarly, when both supports are restrained (Fig. 76,*d*), the condition of equilibrium will cause the bending moment in the interspan, just prior to failure, to be

$$M_{sp}=M_0-M_A\frac{b}{l}-M_B\frac{a}{l}, \tag{199}$$

which gives

$$M_{sp}+M_A\frac{b}{l}+M_B\frac{a}{l}=M_0. \tag{200}$$

Thus, the creation of a plastic pivot and an increase of load in a statically indeterminate system will change the moment-ratio between the interspan and support, that is, there will be a redistribution (or

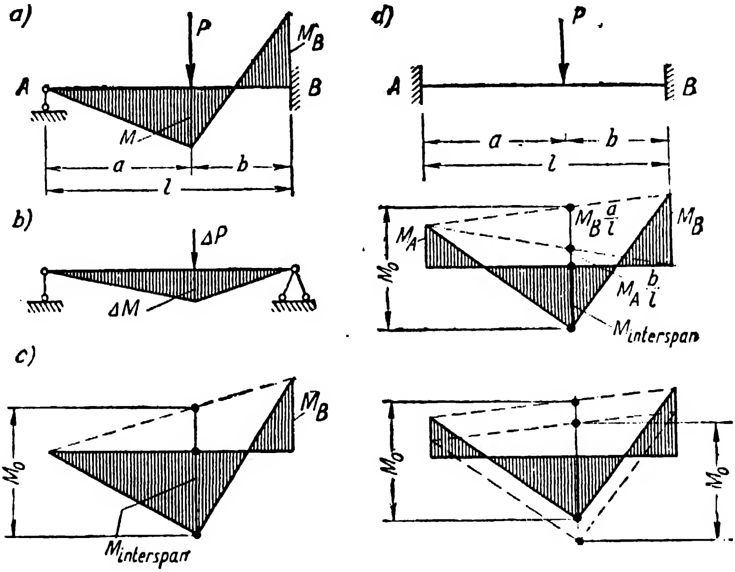


Fig. 76. Redistribution of bending moments in statically indeterminate beams

equalisation) of bending moments amidst the various planes. Under such circumstances, condition (200)—equilibrium during moment redistribution—is always fulfilled: the sum of the moment in the interspan section M_{sp} plus the fractions of moments at both end sections $M_A\frac{b}{l}$ and $M_B\frac{a}{l}$, corresponding to the interspan moment, will be equal to the moment in a simple beam M_0.

The sequence of formation of plastic pivots at the inter- and end-span cross-sections may be arbitrary only if condition (200) be observed. Nevertheless, the extent of bending moment redistribution will influence the width of fissures in the first plastic pivot. Fig. 76,*d*

indicates possible variants of bending moment redistribution in a beam when condition (200) is fulfilled.

The relationship of bending moments must be ascertained when designing statically indeterminate reinforced concrete, the sequence and location of plastic pivot formation being noted with the aim of attaining a rational share of reinforcement between the interspan and end-sections.

Calculating reinforced concrete with proper regard to redistribution of bending moments—or, as it is called, *moment redistribution computations*—will permit, firstly, a decrease of bars in some cross-sections while increasing them in others (this being especially important for simplified joints at the supports in precast construction), and secondly, unification of reinforcement in many instances at the mid- and end-sections by the introduction of prefabricated (welded) mats and blocks (which is impossible when computing by the elastic diagram) and thus standardising the reinforcement of continuous slabs and beams.

In order to restrict fissure widths in the tension-stressed concrete owing to bar lengthening in the plastic pivot, a 30% maximum moment redistribution is allowed. This means that an equalised moment must be at least 70% of the moment calculated by means of the elastic diagram. But when cold-notch bars are used, 15% will be the limit of moment redistribution, since the properties of this type of bars are very close to that of hard steels which have no horizontal sector of yield in their stress-deformation diagrams. When fissures are prohibited by the intended use of a structure, computations are done through moments determined for the elastic stage.

The planes where plastic pivots will form must not fail, because of crumbling of the compression-zone concrete, before the application of ultimate loads. Tests have shown that in order to achieve this, the reinforcement percentage must be limited to $\alpha \leqslant 0.3$. This determines the cross-section's critical percentage of reinforcement where plastic deformation would begin developing in the bars and cause a plastic pivot:

$$\mu\,\% = \alpha \frac{R_{\mathrm{be}}}{m_{\mathrm{s}} R_{\mathrm{s}}} 100 \leqslant 30 \frac{R_{\mathrm{be}}}{m_{\mathrm{s}} R_{\mathrm{s}}} \,\%. \tag{201}$$

Sec. 20. EQUALISATION OF BENDING MOMENTS IN CONTINUOUS BEAMS

There are several ways of equalising bending moments in statically indeterminate continuous beams. Let us examine two of the principal methods.

In continuous equal-span cross beams reinforced with prefabricated blocks and mats, and in continuous slabs reinforced with prefabricated roll-type mats, it is best to arrange an equal-moment system with similar moments in the interspan and at the supports: $M_{\mathrm{sp}}=$

$=M_A=M_B$. Thus in a uniformly loaded 5-span beam q (Fig. 77,*a*), the moments at the supports at each end of the centre span are equalised to satisfy symmetry. The cross-section in the middle of the centre span, where $a=b=\frac{l}{2}$, is derived by adapting the moment-equalising condition (200):

$$M_{sp}+\frac{1}{2}M_A+\frac{1}{2}M_B=M_0. \tag{202}$$

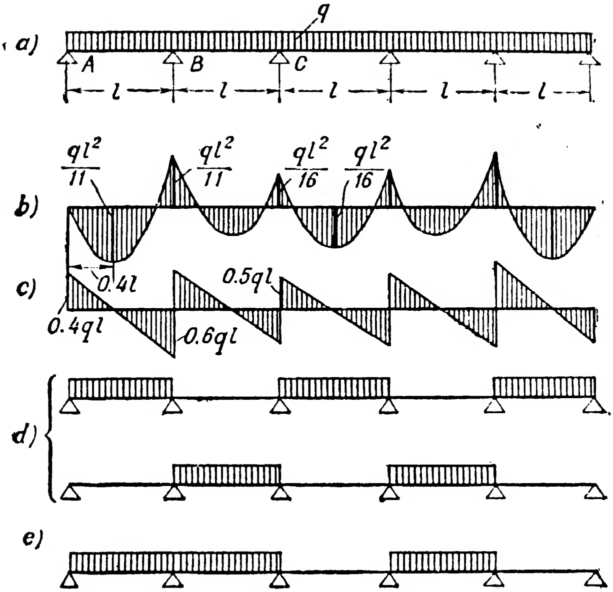


Fig. 77. Equalisation of moments in a continuous beam

For a simple uniformly loaded beam

$$M_0=\frac{ql^2}{8}.$$

Since the problem requires that

$$M_{sp}=M_A=M_B=M,$$

then equation (202) gives

$$2M=\frac{ql^2}{8} \quad \text{or} \quad M=\frac{ql^2}{16}. \tag{203}$$

Fig. 77, *b* illustrates the moment curve of a multi-span beam: at the extreme unrestrained support, $M_A=0$. The greatest moment in the first span is in the cross-section located at a distance of $a=x\approx0.4l$

from the support. By adapting the moment-equalising condition (200) for this cross-section, we obtain

$$M_{sp}+0.4M_B=M_0. \qquad (204)$$

When $x=0.4l$, the moment for a simple beam

$$M_0=\frac{qx(l-x)}{2}=0.12ql^2.$$

Since the problem demands that $M_{sp}=M_B=M$, then from equation (204) we obtain

$$1.4M=0.12ql^2$$

or

$$M=\frac{ql^2}{11.65},$$

which, after trimming the denominator (which gives a slight positive error to the moment), we obtain

$$M=\frac{ql^2}{11}, \qquad (205)$$

and for which the reactions at the supports (Fig. 77, *c*) will be

$$Q_A=\frac{ql}{2}-\frac{M}{l}=0.4ql \qquad (206)$$

and

$$Q_B^L=\frac{ql}{2}+\frac{M}{l}=0.6ql. \qquad (207)$$

For heavily loaded continuous beams and girders (main beams) other methods are employed where it is desirable, because of structural requirements, to lighten the reinforcement at the supports by means of a corresponding strengthening of interspan bars. In such cases, equalisation of moments is achieved by adopting a moment diagram obtained from live load distribution that gives a maximum moment in the interspans.

The dead loads are disposed in each span of continuous beams, but the live loads can be arranged in varied sequence. Thus, if the live load is placed in every other span (Fig. 77, *d*), a maximum interspan moment is obtained. If the live load is assigned to the first two spans and then to every other span (Fig. 77, *e*), the maximum moment will be at the support between two adjacently loaded spans. However, if condition (200) is observed when equalising the moments, the same moment at the support can be obtained for loading schemes as represented by Fig. 77, *e*, as for those shown in Fig. 77, *d*.

Hence, when computing moment equalisation in continuous beams, it will be sufficient to allot the live loads to every other span and calculate the moment curve as for an elastic design. Such computations are commonly done with the aid of tables (see Supplement V) and by fulfilling all stipulated limitations.

Equalised bending moments in continuous equal-span beams when computed as above are usually not less than 70% of the moments gotten by the elastic diagram, including cases when the live load is not more than $p \leqslant 1\text{-}1.3\ g$. For greater live loads borne by heavy beams and girders (main beams), moment equalisation is carried out through methods of Structural Mechanics (superimposing equalised curves, etc.) which procure the desired limits of moment redistribution.

For greater live loads in slabs and cross beams in in-situ floors, moments are evolved through formulae (203) and (205) because unfavourable distribution of the live loads is neutralised by the monolithic rigidity of all the members as a whole.

CHAPTER VIII

BASIC PRINCIPLES IN DESIGN OF REINFORCED CONCRETE TO SATISFY INDUSTRIAL METHODS AND ECONOMICS

Sec. 21. STANDARDISATION AND UNIFICATION

1. General Outline

A building is made up of separate members, each of which has its special function and is liable to various load stimulations. The roof carries its cladding and snow load and is subject to the action of bending; the floors support the building's occupants, equipment, goods, etc., and are also subject to bending; the columns hold up the floors and roof and are subject either to axial or eccentric compression and transmit their burdens to the foundation.

The main vertical and horizontal stays of a building are the columns and girders; they sustain the members that bear the roof and floors and create the skeleton of the structure, known as the frame (Fig. 78).

A building must be designed so that all its members embody the necessary carrying capacity (strength and stability), rigidity, and the required degree of crack resistance, while the structure as a whole must possess skeletal rigidity. Skeletal rigidity is understood to mean the ability to horizontally resist the action of various loads and physical forces.

A number of structural methods are employed to impart skeletal rigidity to a building: systems of geometric stability (trusswork) (Fig. 79, *a*), framework consisting of bents (Fig. 79, *b*) or of tied vertical and horizontal diaphragms (Fig. 79, *c*), etc.

Reinforced concrete members of a building must not only possess strength and stability, they must also be economical and lend themselves easily to methods of industrial fabrication and machine erec-

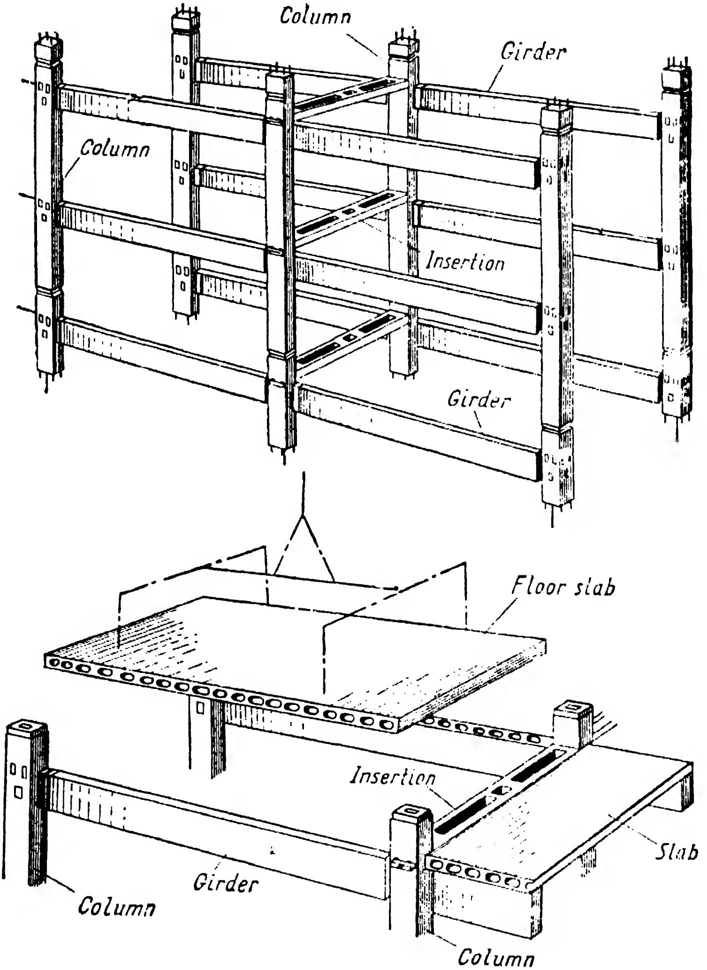


Fig. 78. Reinforced concrete frame of a multi-storey building

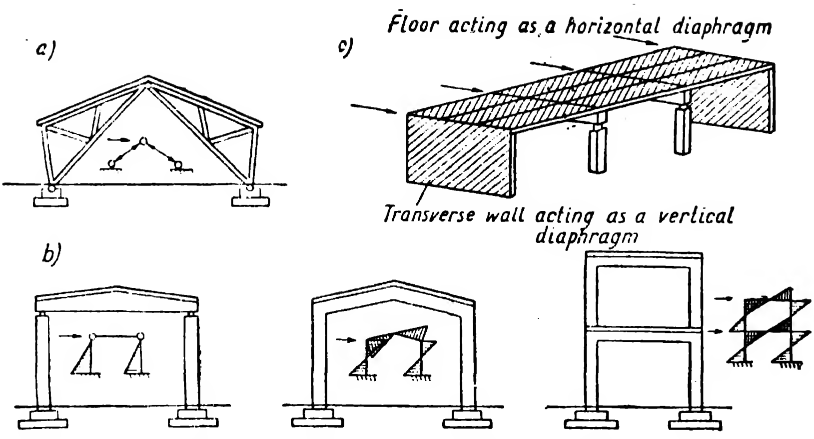


Fig. 79. Structural systems that impart skeletal rigidity to a building

tion with a minimum expenditure of material and manpower. These demands are best met with by precasting methods, which have found wide application both in this country and abroad.

2. Standardisation of Precast Members

The most efficient way of making precast members by factory methods is to produce them in single-type series. This permits perfection of technological processes, reduces manual labour, and lowers the cost and improves the workmanship of the finished product. Moreover, a very important conclusion that follows from all this is that the number of detail types for building construction must be reduced to a minimum but must be used in great quantities on all kinds of structures.

All this may be achieved by standardising structural details. This means that the best types must be selected as dictated by experience and as compared with design-evaluation factors of similar types (expenditure of component materials, gross weight, comparative difficulty of manufacture, and cost of finished product). The details thus chosen are then used in mass production.

The results of this standardisation of precast reinforced concrete elements are then presented, for use of the building designer, in the form of catalogues. Such catalogues will necessarily undergo alterations from time to time as the details are either improved or new or better ones created on the basis of technical progress and accumulated building experience.

3. Unification of Spacing Multiples and Structural Layouts

To make possible wide incorporation of standard members into all kinds of buildings, the latter's spacing parameters (column centering and building heights) must be unified, i. e., they must be reduced to a limited number of definite standard dimensions. Such a gauge is provided by the Standard System of Modules (SSM) which has established 100 mm module variations and enlarged special modules.

At present this unification of spacing lengths for one-storey factory buildings has resulted in a 6 or 12 m longitudinal centering (column spacings), and a 3 m enlarged module for lateral centering of 6, 9, 12, 15, 18, 24, 30, and 36 m (Fig. 80, *a*).

Unified grids for multi-storey industrial structures are 6 × 6 or 6 × 12 m [exceptions such as (7 + 3 + 7) × 6 m are employed in

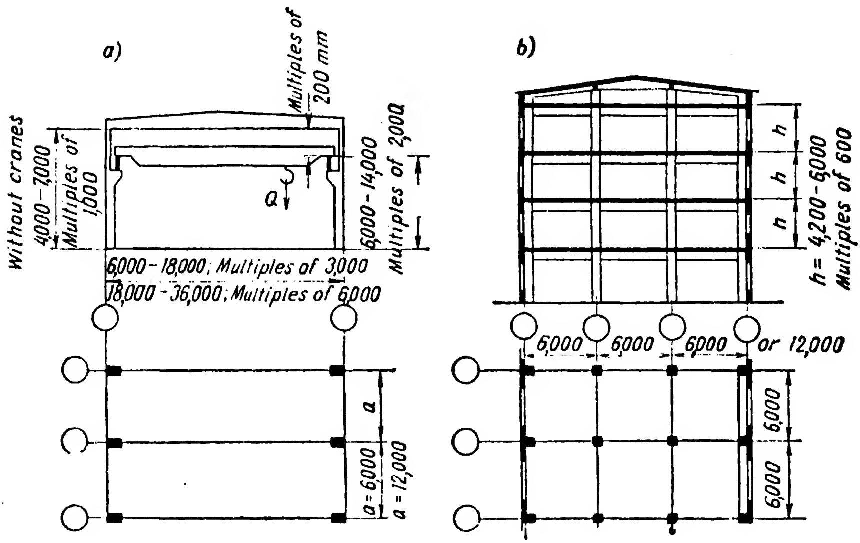


Fig. 80. Unified spacing multiples for industrial buildings
a—for a one-storey structure; *b*—for a multi-storey building

individual cases because of equipment clearances], and enlarged height multiples of 600 mm, as for instance 4.2, 4.8, and 6 m (Fig. 80, *b*).

An enlarged module of 400 mm embraces apartment houses and civic buildings with grid layouts ranging from 2 to 6.8 m for longitudinal (spacing) and lateral (span) dimensions. An enlarged

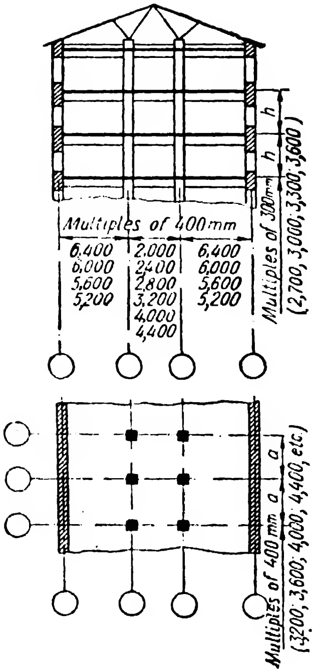


Fig. 81. Unified dimensions for multi-storey apartment houses and civic buildings

height module of 300 mm includes such heights as 2.7, 3, 3.3, and 3.6 m (Fig. 81).

This standardisation of spacing multiples has made it possible to adopt a limited nomenclature of unified structural layouts for the typification of frames and their connections, embracing the structural blocks of a great number of building designs.

All this has permitted the design of standard building projects for mass construction. Dimensions of all standard members are coordinated by the SSM, which is based on three dimension categories: nominal, structural, and actual.

The *nominal* dimensions are for grid spacing of buildings.

The *structural* dimensions differ from the nominal by a difference of clearances required for joints. For example, a ribbed floor panel with a nominal dimension of 6,000 mm will have a structural dimension of 5,970 mm, since 30 mm is discounted for the joint (Fig. 82). Joint clearances depend upon conditions and methods of erection, and must provide for convenience in assembly of each member and, if necessary, its grouting. In the latter instance the joint is made 30 mm.

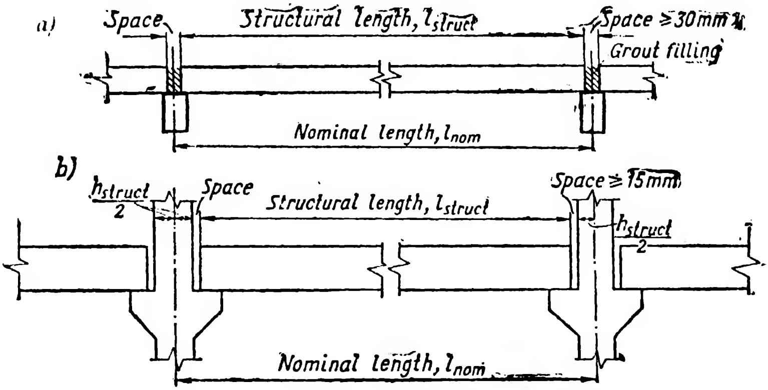


Fig. 82. Nominal and structural dimensions of precast members
a—floor panel; *b*—girder

The *actual* dimensions are those of the finished member which, depending upon the exactness of factory workmanship, will differ somewhat from structural dimensions by an amount known as the *allowance* (from 3 to 10 mm). Structural dimensions must be chosen for the design with full consideration of joint clearances and specified allowances.

4. Amalgamated Units

In the design of precast members the trend should be towards amalgamated (large) units. There are a number of advantages in erecting a building of such large parts: 1) lifting and placing operations are reduced to a minimum, 2) the number of erection joints are decreased, and 3) the degree of prefabrication is increased with a subsequent reduction of finishing at the site.

Thus, in appartment house construction the rational floor slab would be one of room-size, while wall blocks should be of one-storey height and the width of a room with completely finished surfaces. In industrial structures the best roof slab would be one that fits from truss to truss without the aid of purlins. Limiting factors for the size of such large members are the maximum weights and overall dimensions that could be handled by transporting and hoisting machinery, and also methods of delivery. In order that full advantage be taken of cranes, the enlarged units should be amalgamated so as to be all of approximately the same weight, close to the maximum capacity of the crane.

Since in most cases enlarging of units is limited by allowable ultimate weights, they should be designed with light cross-sections, thin walls, hollow slabs, and other lightening features and with the incorporation of prestressed high-strength steels and high-density concretes. In some cases it will be found advisable to use lightweight aggregate for the concrete.

Sec. 22. MANUFACTURE AND ERECTION

1. Producibility of Precast Members

When the design of a precast member lends itself readily to a mass technological process in a given factory or casting yard where modern efficient machinery is used without additional hand operations, it is classified as having the attribute of *producibility*.

Precast reinforced concrete elements are made in accordance with one of the following three schemes:

1. *Assembly-line technology*. The members are made in metal forms on individual wheeled trays that move along rails from one machine to another with the necessary operations being performed consecutively.

2. *Consecutive machine technology*. The manufacturing processes are executed in separate shops of the factory with the forms moved by crane or on rolling platforms from one machine to the next.

3. *Stand technology*. The specific feature of this method is that production and thermal treatment of a product are conducted on fixed stands or beds while the machines that perform the various operations are consecutively moved to the stand.

The exact technology chosen depends upon the design and size of the individual precast members.

a) Giant prestressed members for industrial structures (beams, roof panels, overhead-crane girders, etc.) are made on *straight-line stands*. The latest type of this stand produces prestressed members reinforced with various types of tendons: bunched-wire, single-bar, and braided. The members are made in a horizontal position and the concrete is compacted by vibration. After setting they are subjected to heat curing.

b) Roof panels and other parts with continuous reinforcement are very efficiently made by Professor Mikhailov's *stamping and vibrating method*; the ДН-7 machine winds high-strength wire upon pegs fixed in the form, after which the ДВ-57 machine pours, vibrates, and stamps the concrete. The stamper of the mechanism is made to correspond to the outline of the member, and the stamp's intensive pressure and simultaneous vibration exerted upon the stiff mix gives the member its required density and shape. Directly after completion of this operation the stamp is detached from the concrete and the sides of the form open up without disturbing the pressed concrete shape.

c) Wide application is being given in this country and abroad to the *chamber method of moulding reinforced concrete floor and wall panels* for apartment house construction. By this method a package of from 8 to 10 and more panels are produced on a stand in vertical metal moulds. Setting up and subsequent stripping of the moulds is accomplished mechanically, the preassembled reinforcement of the panels being placed during the setting-up operation. A stiff concrete mixture is pneumatically piped to the chambers and tightly packed by vibrators, this operation being followed by steam curing (injection of steam into the thermal bays of the moulds). The concrete attains its required strength in several hours and is stripped from the moulds in a vertical position.

The mould method of producing both flat and ribbed slabs results in a considerable saving of factory (production) space, eliminates the need of edging the panels, and produces even, smooth surfaces.

d) An invention of the Soviet engineer N.Y. Kozlov for the further improvement of industrially produced reinforced concrete is the *method of machine rolling and vibro-packing of thin-walled panels* for floors and partitions of apartment houses. The essence of this progressive manufacturing process is that the reinforced concrete members are made on an endless conveyor belt whose surface serves as the formwork for multi-ribbed members.

After placement of the reinforcement blocks, the concrete mixture

is delivered, rolled, and vibro-packed by means of rollers situated above the belt. The successively rolled pieces, covered at the top and heated from below, attain required strength in the course of their two-three hour trip on the conveyor and, after cooling on racks, are transferred to the finished products warehouse.

The complete cycle of production, beginning with mixing of the concrete and ending with issuance of the finished product, is conducted on one rolling-mill installation entirely subordinated to the speed of the casting belt. This method of uninterrupted vibro-rolling achieves a high degree of mechanisation, productivity, and geometric precision that permits the fabrication of members having walls as thin as 10 mm.

e) An alternative of the above endless vibro-roller belt is the *vibro-roller stand* for the manufacture of floor and wall panels entering into the construction of industrial buildings. This method is suited to the casting of giant prestressed panels with tendons tensioned in two directions by a winding machine.

However, the fabrication of the entire list of reinforced concrete elements cannot be accomplished through any one individual method, and plants engaged in such production usually combine the various methods.

Three of the most efficient production lines for the fabrication of precast reinforced concrete for industrial buildings have been developed in the U.S.S.R.

The first line is a combination of three production schemes: consecutive machines for making members up to 6 m in length, a straight-line stand for very long prestressed parts, and a stand for long members with ordinary reinforcement (columns, ground beams, etc.). The second line is the same as the first, but with the addition of a vibro-roller production line for giant prestressed slabs. The third line consists of consecutive machine production, a stand equipped for continuous reinforcement and vibration-stamping, and one for ordinary reinforced concrete.

All of the above open the way for the manufacture of a wide assortment of reinforced concrete units and also allow for any subsequent alterations required by future improvements in design.

Methods of production must absolutely be taken into account in reinforced concrete design, because a member suitable for one technological scheme may prove badly adapted to another. For example, when dividing the frame of a multi-storey structure into separate elements (Fig. 83, *a*), it is possible to cut the girders at the planes having the least moments (Fig. 83, *b*), but the columns will have to be made with brackets reinforced by cages placed in two intraperpendicular directions. The breadth of such a column at its brackets is several times wider than the column proper and under conditions of assembly-line consecutive machine production lacks producibility for the following reasons: 1) the volume, in m^3, of goods handled by the convey-

or is lowered because the width of the conveyor truck allows placing of only one column; 2) the necessity of arranging reinforcement in two directions complicates the placing of the bars, and 3) manual labour is added to the operation of pouring the concrete into the brackets.

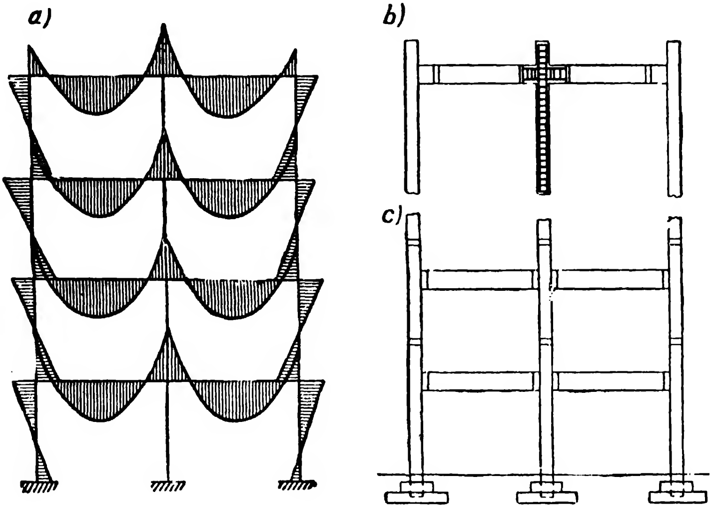


Fig. 83. Division of a multi-storey frame into precast members
a—moment diagram induced by vertical loads; *b*—sketch showing division of girders at zero-moment planes; *c*—division of the frame into straight.

In assembly-line and consecutive machine fabrication the division of the frame into straight units (Fig. 83, *c*) attains greater producibility, and although in this case bending moments and lateral forces are greatly increased at the planes of the cuts and consequently demand better workmanship during erection, nevertheless this division considerably raises production of factory-made precast elements of the frame and is therefore accepted as a standard. However, if such columns are made in a casting yard, the bracket-type of division is a comparatively simple matter and attains producibility.

2. Producibility in Erection

The producibility of reinforced concrete must not only embrace fabrication in the factory or casting yard but also erection, i.e., the design must anticipate convenience of erection in the piece's actual position.

In certain cases the principal factor determining the method of dividing the frame will be producibility in erection. For example, it is very convenient during erection to place the dividing joint of the

column at a level of 600-700 mm above the floor (Fig. 83, *b*, *c*), in spite of the fact that the static behaviour of the said column makes a mid-height plane, where the bending moment has a minimum value (Fig. 83,*a*), more rational. From this it follows that the position of joints of precast units is controlled not only by the question of static resistance, but also by practical erection requirements. As a rule the joint must require neither heavy erection welding nor difficult subsequent concreting.

The design of precast members must not omit arrangements for their transportation and hoisting, such as loops of mild steel properly attached to the bars, holes that pierce the member, etc.

The positions of loops and holes must be designed with allowances for static behaviour during transportation and hoisting of the given member.

3. Static Behaviour of Reinforced Concrete Elements During Transportation and Erection

Precast reinforced concrete units are subject to various force stimulations prior to their final placing: when they are taken from their forms, lifted, moved, stored, etc., all this being reflected in design diagrams that differ from the regular design diagram of each member.

In correctly designed structures, erection stresses should not be the cause of additional bars or greater cross-sections.

When calculating erection stresses, the dead weight of the member is commonly considered its principal load and possessing a dynamic character, because it is applied in full and instantaneously during hoisting operation.* Because of this, and also because the member is liable to additional dynamic forces (jarring and impact), a dynamic coefficient of 1.5 is attached to the dead weight of the member when computing erection stresses.

The most typical member whose stresses in its normal position differ entirely from those caused during handling, is the column. In its normal working position it undergoes either axial or eccentric compression, but in moving or hoisting it is subject to bending by the uniformly distributed load of its own weight (Fig. 84,*a*). Hence the stresses imparted during handling may considerably alter the design cross-section of the member as compared to that calculated for its normal position.

The design cross-section of a column taken as an example is given in Fig. 84, *b*, but inasmuch as it is fabricated horizontally (lying flat), during hoisting operations its effective cross-sectional height will fall short of these dimensions, and bar areas F_s and F_s' will also change (Fig. 84,*c*). Therefore the reinforcement evolved for the col-

* Static loads are assumed as being applied gradually, beginning with a zero value and ending with their final magnitude.

umn's normal position may prove insufficient for erection operations. The designer must, in such a case, include such measures as will alter the member's design diagram during the course of lifting operations so that it will not be necessary to add more bars that will be useless when the member is in place.

There are a number of such possible measures: shifting the position of grappling hooks, specifying the positions of supporting chairs used in transportation and stacking, use of patent tackle (cross pieces, grappling hooks), etc.

For example, if the hoisting loops in the column were shifted from the ends towards the centre, when lifted the element will behave as a single-span beam with overhangs (Fig. 84,*a*) and the maximum bending moment will be less than if it were without overhangs.

In lifting members designed to act normally as bending elements, it will likewise be found convenient to place the grappling loops or holes somewhat away from the ends. This will create an allowed tension in the member's upper zone if the installation bars in that part of the concrete can bear the stress. But this must be verified by computations in each individual case.

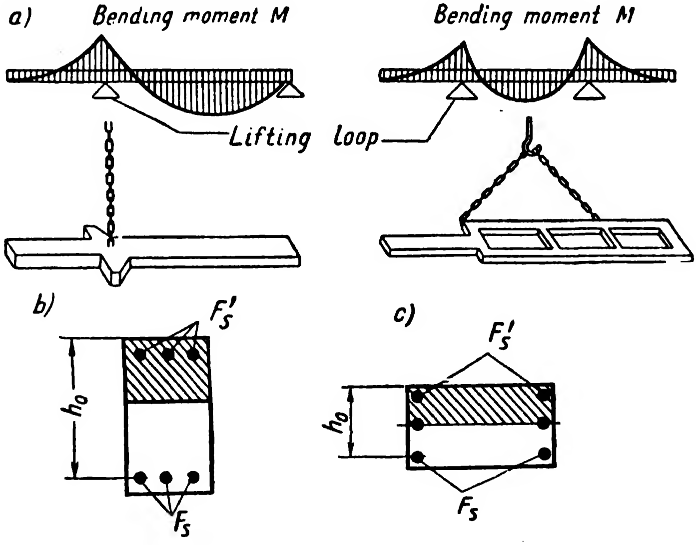


Fig. 84. Structural diagram of columns during erection operations

The above suggestions concerning grappling positions during hoisting operations are also valid for supporting chairs (blocks) used in moving or stacking of members.

Units that are very high and narrow, such as high beams, trusses, wall panels, etc., are usually transported or lifted "edge up". When

such units are made in a horizontal position at the plant, they are squared off before being stripped from the forms.

In the design of a building made of precast members, its stability must be guaranteed during erection, using temporary bracing if required. If the structure is statically indeterminate, the designer must also dictate erection sequence.

Forces imposed upon a member during erection are assumed as additional load combinations and, with the exception of the element's dead weight, are therefore computed with a coefficient of 0.9.

Sec. 23. PRINCIPLES IN DESIGNING JOINTS OF PRECAST ELEMENTS

A structure assembled of correctly jointed separate elements will, under load, conduct itself as a monolithic whole. The joints must therefore be strong enough to resist the forces stimulated within them and also embody producibility, both in fabrication and erection.

Three kinds of forces may act within the joints: a bending moment M, a lateral force Q and a longitudinal force N. The decisive forces determining the design of a given joint will depend upon the frame construction. Thus, the decisive forces will be M and Q in a column-girder connection (Fig. 85,*a*), N and M in the joint of an eccentrically compressed column (Fig. 85, *b*), and the longitudinal force N in a hinge joint (Fig. 85,*c*).

The joints of some members will bear no design load and are merely supplementary, examples of such being a single-span beam bearing directly upon the butt-end of a column, joints of adjacent floor panels, etc.

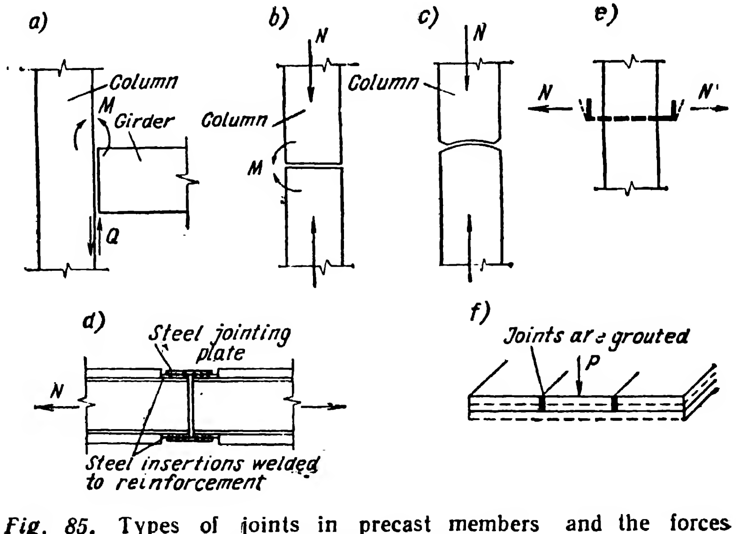


Fig. 85. Types of joints in precast members and the forces stimulated within them

The transference of forces in a joint is implemented either via welded steel insertions, or concrete placed into the joint during erection, depending upon the kind of forces being dealt with. Welded steel insertions (Fig. 85,*d*) are the common connection when the greatest stimulations are bending moments, lateral forces, or tensions. This kind of connection transfers the tensile forces from the reinforcement of one precast element to the bars of another. A welded joint of this sort must be so constructed as to avoid distortion of the insertions or bearing plate when the forces are conveyed through the connection (Fig. 85,*e*).

In joints effected only by compression, the stresses may be transferred either directly through butted concrete surfaces, or by means of additional concrete grout, or even through inserted and welded steel parts.

Concrete poured into the joint during erection may either bear design stresses, or serve as a supplementary connection. In the first instance the grouted concrete must not be less than grade 200, while in the latter case (supplementary grout) grade 100 is used. To pour a reliable joint, the clearance must not be less than 30 mm. Experience has proved that an excellent bond is obtained between the newly poured grout and precast concrete if the work is well done (when surfaces are cleaned and moistened, etc.). Thus, well grouted joints of floor panels will assure unified action even when one of the panels receives a concentrated load (Fig. 85,*f*).

Local high temperatures are developed during welding of steel insertions in the joints, thus subjecting the surrounding concrete to the action of heat. Tests have shown that this somewhat impairs the mechanical strength of the concrete; nevertheless the effect is only local and will not harm the carrying capacity of the jointed connections.

To render steel insertions corrosion- and fire-proof, they are treated with a 2-3 cm protective cement covering applied on metal lath to avert subsequent peeling. Furthermore, the steel insertions must be embedded within the precast members so that a minimum of projections will result after application of the protective covering. Insertions must be made as small as the computed length of the weld allows.

Butt connections are implemented by tensioning the tendons within the ducts of the hardened concrete parts intended to be joined (see Sec. 17).

Sec. 24. SPECIFIC PROBLEMS IN THE DESIGN OF IN-SITU REINFORCED CONCRETE

Industrial methods should also be applied to in-situ reinforced concrete: a considerable part of the work should be performed in specialised plants, while erection on the site proper should be accomplished through coordinated mechanical means. With this in

mind, in-situ reinforced concrete should consist of simple cross-sections and its fulfilment should involve re-useable formwork, prefabricated reinforcement blocks and mats, self-supporting reinforcement, and amalgamated blocks of bars—both bare and together with the formwork—all ready-made in a factory.

Amongst the kinds of in-situ reinforced concrete units that are adaptable to the above demands is the thin-slab type of roof construction for elongated industrial buildings, fulfilled with the aid of a truck on rails which supports the arched form and raises, lowers, and moves it horizontally (Fig. 86).

Another way to increase industrialisation of in-situ construction is by the adoption of composite reinforced concrete, which consists of precast elements incorporated into in-situ sectors which embrace the entire job.

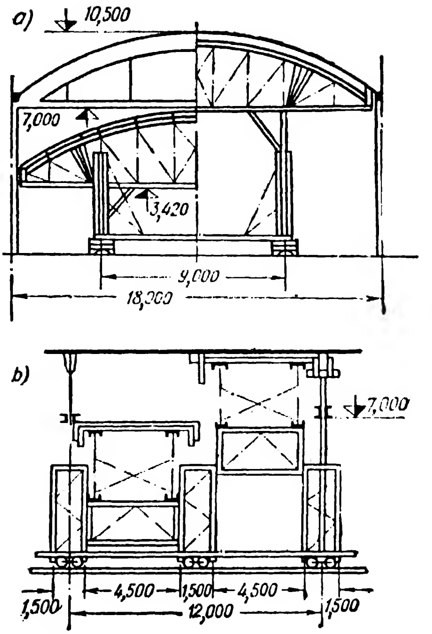


Fig. 86. Travelling formwork for the construction of in-situ reinforced concrete buildings

Sec. 25. COMPENSATION JOINTS

In order to minimise the forces in a structure arising from temperature changes and concrete contraction (shrinkage), the building must be divided into sections by means of compensation joints (shrinkage-

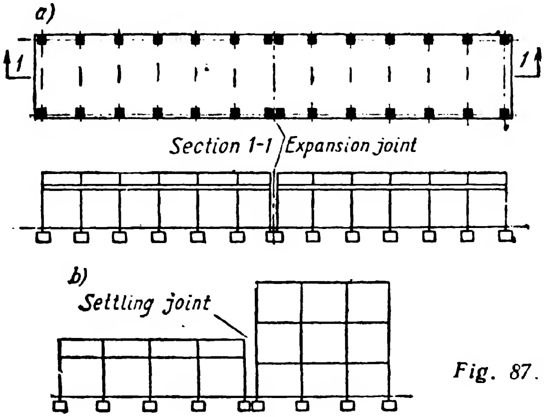


Fig. 87. Compensation joints

temperature joints) (Fig. 87,*a*). The width of such joints is usually 20 mm, and their spacing established so that the forces provoked by shrinkage and temperature changes (computed as additional stress combinations) do not initiate a need for extra reinforcement in the members.

A list of such spacing is presented in Table 18.

Enclosed structures or those built in the ground are less liable to temperature changes than if open-built, for which reason their spacing of joints is made greater than in the latter case. Joints in frames assembled of precast elements are arranged at greater intervals than

Table 18

Spacing of Shrinkage-Temperature Joints

Type of construction	Spacing in metres	
	In enclosed structures and those built in the ground	In open structures
In-situ reinforced concrete frames of heavy concretes	50	30
Ditto, precast construction	60	40
Structures entirely of in-situ heavy concretes	40	25
Ditto, of lightweight concretes	30	20
Framed reinforced concrete buildings combined with either wooden or steel roof cladding construction	60	40

in cast-in-place frames, because some of the temperature and contraction deformation of the former is absorbed before the final grouting of the joints. More frequent joint spacing is practised for buildings erected of light concretes because they contract more intensively than those of heavy concretes.

Structures in which the authorised spacing of compensation joints is exceeded, must be checked by computation.

A building will settle unevenly if it rests on different bearing soils or has different numbers of storeys. To forestall dangerous stresses from uneven settlement, the reinforced concrete structure is divided into sectors (compensation blocks) by means of settling joints which start at the very foundation (Fig. 87, *b*). Such settling joints also function as shrinkage-temperature joints.

Details of compensation joints are presented in Chapters XIII and XIV.

CHAPTER IX

HORIZONTAL SLAB CONSTRUCTION

Reinforced concrete floor construction (horizontal slabs) is the most prevalent of all floor structures used in dwelling, and civic and industrial buildings. As far as structural layout is concerned, present types of reinforced concrete floors can be divided into two main groups: ribbed and flat-slab construction.

In ribbed construction, the girders—or girders and beams—act together with the slabs, or panels, that they support, whereas flat slabs are supported neither by beams nor girders but rest directly on the columns, around which the slab is thickened to form a drop panel (or capital).

Either in-situ or precast methods are applicable to both groups, but the structural layout will be different.

Sec. 26. HORIZONTAL RIBBED CONSTRUCTION

A. PRECAST ONE-WAY RIBBED FLOORS

1. Choosing the Structural Layout

Precast one-way ribbed floors consist of panels and their supporting members, known as girders or main beams (Fig. 88,*a*). The girders bear upon either columns or walls and may run longitudinally (i.e., lengthwise with the building), or laterally (Fig. 88,*b*). The girders and the columns collectively make up the frame of the building. The floor structure may be divided into two or three lateral spans in apartment houses and civic buildings, and up to five or six lateral spans in industrial buildings. The division of longitudinal spans will depend upon column spacing and the given length of the building. As already stated in Chapter VIII, dimensions of column spacing (the grid) are established on the basis of the Standard System of Modules.

Choosing (or arranging) the structural layout of any floor construction consists in determining the direction of girders and the dimensions of the grid, all of which fixes the spans of girders and panels. The

following factors must be borne in mind when choosing the structural layout of the floor:

1. The building's intended use (whether industrial, civic, or dwelling), its plan and architectural composition, and its technical requirements (machinery arrangement, width of building, live loads, etc.).

2. The general structural layout. If skeletal rigidity is to be attained laterally through a rigid joint frame, the girders are laid laterally and the panels longitudinally. But if a longitudinal frame system is planned (primarily to satisfy the layout of partitions in apartment

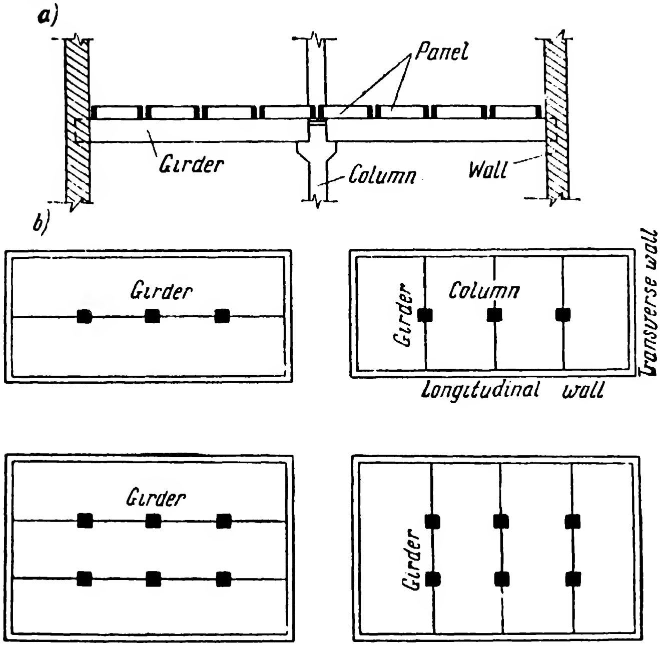


Fig. 88. Precast rubbed-floor construction

houses and civic buildings), then the girders are placed longitudinally and the panels laterally, a corresponding grid being determined in each case.

3. Technico-economic evaluation factors. There must be a minimum of both size ranges of members and volume of reinforced concrete expended on the floor as a whole, while at the same time members must be made as large as can be handled by lifting equipment and transport facilities.

The total amount of concrete and steel in a reinforced concrete floor is the sum of materials entering the panels, girders and columns. But the lion's share—about 65% of the whole—goes into the panels, which means that the design evaluation factors of the latter acquire the greatest importance.

2. Design of Floor Panels

Economy in panel design is achieved through structural lightness of hollow or ribbed panels. Let us consider the panels illustrated in Fig. 89, *a*, which are subject to bending as a beam and whose

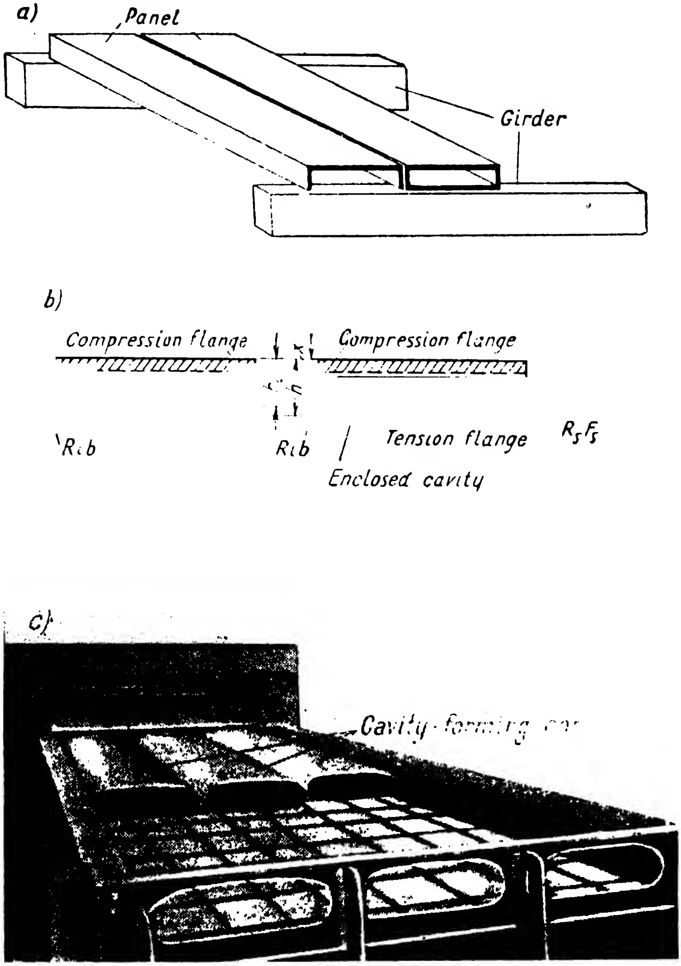


Fig. 89. Structural floor panels

ends rest upon girders. Their cross-section indicates that compression forces are borne by the concrete-compression zone, and tensile stresses by the reinforcement F_s (Fig. 89,*b*). Idle concrete, except for that in the vertical ribs, is absent in the tension zone. The width of the

ribs is computed to satisfy diagonal plane resistance and to accommodate the welded blocks of bars. The flange in the compression zone of the cross-section also takes local (perpendicular) bending stresses occurring in the span between the ribs. If a flat ceiling is required, the lower flange is retained in the tension zone, thus forming an enclosed hollow space. Such hollows (cavities) may be either oval (Fig. 89,*c*) or round in section, depending on the type of cavity-forming cores on hand.

The general principles underlying the design of panels, no matter what their type of cross-section, are: exclusion of a maximum amount of concrete from the tension zone, with the exception of vertical ribs; assurance of diagonal-plane resistance; keeping within the technological possibilities of the panel manufacturer.

Panel types are classified according to their cross-sections into oval- and round-cavity panels, upper- and lower-rib panels, and solid panels. Cross-sections as taken from panel standards are illustrated in Fig. 90, and technico-economic evaluation factors are given in Table 19.

The least flange thickness of oval-cavity panels is 25-30 mm and the minimum rib width is 30-35 mm.

Solid panels are cast in two-ply concrete, the lower ply (or layer) enclosing the effective bars and consisting of heavy concrete, while the upper ply is made of lightweight, grade 150 concrete.

Floor panels are made in multiples of 200 mm with nominal widths and lengths up to 3.6 and 6.4 m, respectively. Structural dimensions are 10 mm less than nominal widths and 20-30 mm less than nominal lengths. For apartment houses and civic buildings, structural lengths are made 140 mm less than nominal dimensions of 4.8, 6.0, and 6.4 m, to accommodate the passage of ventilation ducts through floors.

Economy of a panel may be judged by the reduced denomination of its thickness, a figure obtained as the result of dividing panel volume by projected area.

Given a specified load of 600-700 kg/m^2, the panel that contains the least concrete is the oval-cavity type, its reduced denomination of thickness being 8.4 cm as compared with 12 cm for round-cavity sections.

Of all types of panels, the one requiring the least amount of concrete is the upper-rib panel, its reduced denomination of thickness being 8 cm. However, the absence of a compression-zone flange weakens the arm of its internal resisting couple and renders it less economical in the matter of reinforcement. Another thing to keep in mind is that since the cost of floor construction includes the price of the flooring, its price is somewhat heightened because the upper ribs make it necessary to lay an underfloor.

In industrial buildings, where the floors carry heavy live loads (1,000-1,500 kg/m^2), downward-rib panels are commonly employed

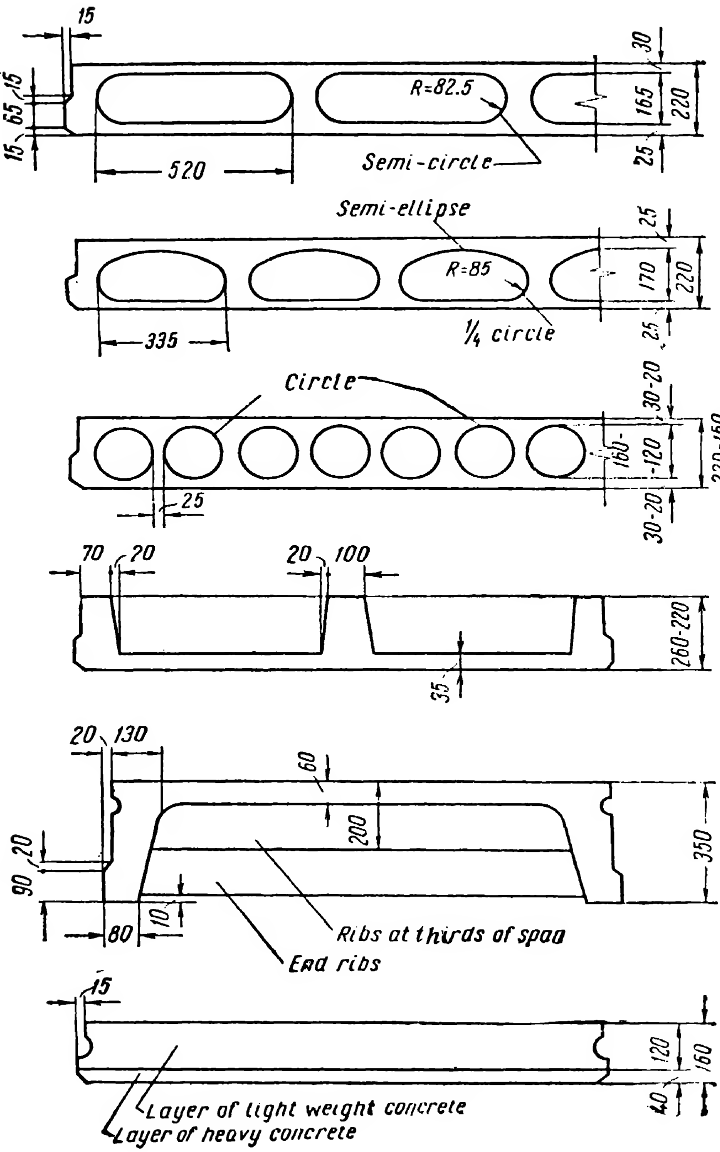


Fig. 90. Cross-sections of floor panels

Table 19

Technico-Economic Evaluation Factors for Floor Panels with a Nominal Length of 6,000 mm

Type of panel	Weight in kg per m²	Height of section in mm	Grade of concrete	Transformed thickness of concrete in cm	Kind and grade of reinforcement	Steel expenditure in kg per m² at a specified load of 600-700 kg/m²	
						Ordinary welded mats and blocks	Prestressed reinforcement
With enclosed oval cavities, 520 mm wide	195-210	220	200	7.7-8.4	Hot-rolled intermittently-deformed, grade St-5 steel	7.55	—
	195-210	220	300	7.7-8.4	High-strength wire	—	3.4
Ditto, 335 mm wide	250	220	200	10	Hot-rolled intermittently-deformed, grade 25Г2С steel	7	—
	250	220	200	10	Ditto, grade 30ХГ2С steel	—	4.95
With enclosed round cavities, 160 mm dia.	300	220	200	12	Hot-rolled intermittently-deformed, grade 25Г2С steel	—	5.15
	300	220	300	12	High-strength tensioned wire enclosed in ready-made prestressed reinforced concrete inserts	—	3.1
With ribs upward	200	260	200	8	Hot-rolled intermittently-deformed, grade St-5 steel	9.1	—
	200	260	300	8	Ditto, grade 25Г2С steel, subsequently cold-drawn	—	6.4
With ribs downward	305	350	200	12.1	Hot-rolled intermittently-deformed, grade St-5 steel	10.9; 14.9; 20.3*	—
Solid, two-ply concrete	315	160	Grade 300 for lower layer, and 150 for upper layer	16	High-strength wire	—	5.1

* Steel expenditure is given according to specified loads 900, 1,400 and 1,900 kg/m², respectively.

even though their content of concrete is not economical (their reduced denomination of thickness is 12.1 cm).

Grade 200 concrete is used in non-prestressed reinforced panels, and grade 300 when the steel is tensioned.

The effective span l_0 of a panel is considered as the distance between the centres of its supports (Fig. 91,*a*). When supported on the

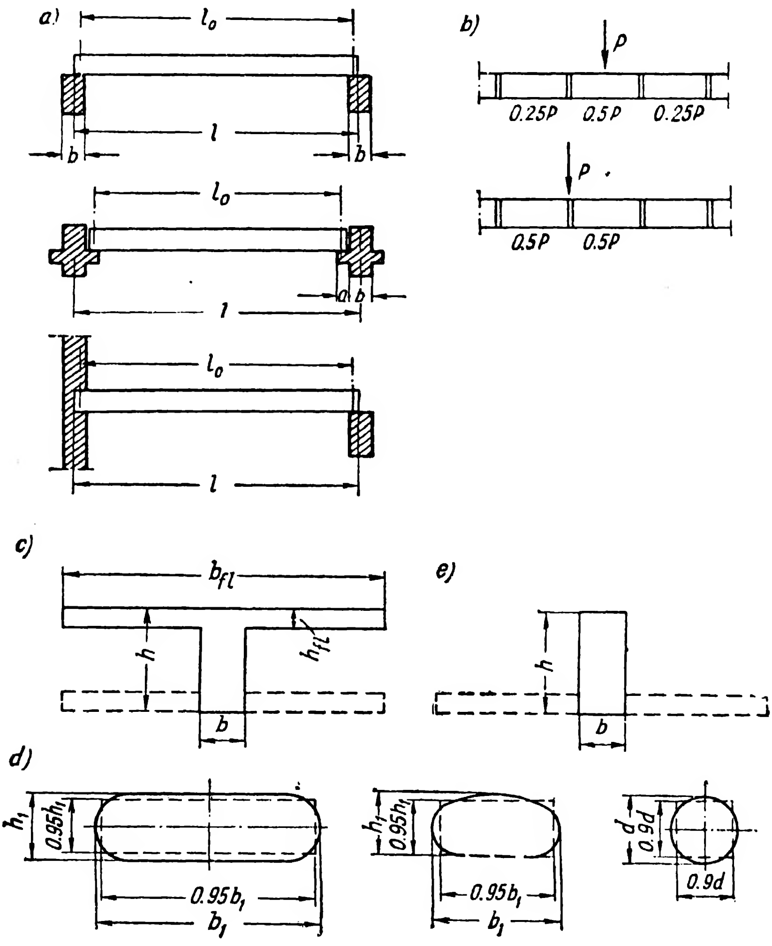


Fig. 91. Effective spans and cross-sections of panels

top of a girder $l_0=l-\frac{b}{2}$; when the supports are the flanges of the girders $l_0=l-a-b$; when the panel rests with one end on a girder and the other in a brick wall the effective span is taken as from the centre of the wall bearing to the centre of the girder bearing.

The dead load will be the sum of the dead weights of the panel (220-230 kg/m²), the floor construction, and the partitions. Live loads are dictated by building codes or are computed from data given in the technological programme if the proposed structure is to be a factory. A concentrated load P, such as a partition, machine, etc., bearing directly on one panel, will also be supported by adjacent panels because of the monolithic action of the entire floor construction (Fig. 91,*b*). Tests have shown that for the purpose of computation $0.5P$ may be assumed for the loaded panel and $0.25P$ for each neighbouring panel, except when the concentrated load occurs on the joint between two panels, in which case the load is considered as equally divided between the two.

The height h of the panel must not only satisfy the condition of resistance, but also that of rigidity (deflection). Panel deflection must not exceed from $\frac{1}{200}$ to $\frac{1}{300}\,l$ *; if the panel's span is about 5 or 6 m, its height will be dictated mostly by deflection limits. A preliminary height h that will satisfy both resistance and rigidity requirements can be established by means of the formula

$$h = cl_0 \frac{m m_s R_s}{E_s} \times \frac{g^{s} + p^{s}}{q^{s}}, \qquad (208)$$

where the coefficient c will be 18-22 for cavity panels, and 30-36 in the case of a ribbed panel whose flange is in the compression zone. A greater c-value is used when the reinforcement is of St-5 steel, while a lesser c-value is taken if the grade of steel is 25Г2С.

In the case of prestressed panels, a preliminary figure for h may be assumed as

$$h = \frac{1}{20} - \frac{1}{30}\,l.$$

Cavity and lower-rib panels are calculated just as for T-beams, for which purpose the panel's actual cross-section is replaced by the effective cross-section of a T-beam with a similar height of h (Fig. 91,*c*). The flange width b_{fl} of the T-beam's effective cross-section is assumed as equal to the width of the panel's flange, while the width of the web b is taken as the sum of all the panel ribs.

For the purpose of computing resistance and rigidity, the width of intermediate ribs and thicknesses of flanges may be determined by reducing the said cavities to their rectangular equivalents having similar areas and moments of inertia. Moreover, the value of S_c must be alike both for the equivalent and actual sections. Such rectangular equivalents may be practically computed according to Fig. 91,*d* when the panel height h is 15-25 cm and the maximum width of the cavity is 50 cm.

* See Sec. 11.

When the effective T-section has been established, b_{fl} (effective width of the flange) must be corrected; if $\frac{h_{fl}}{h} \leq 0.1$, then the maximum flange width

$$b_{fl} = 12(n-1)h_{fl} + b, \tag{209}$$

where n—number of ribs in the panel's cross-section,
b—width of the flange in the effective T-section.

When $\frac{h_{fl}}{h} > 0.1$ or if there are intermediate ribs, the absolute width of the ribs is entered into the computations.

An upper-rib panel will behave just as a rectangular beam because its underflange lies in the tension zone and does not aid carrying capacity. The effective width b of the rectangular section (Fig. 91,*e*) will be the sum of the widths of the ribs.

Diagonal-plane resistance is computed by assuming that b is equal to the absolute widths of all the ribs in the section. In computing local bending athwart the panel, the flange is assumed as a partly restrained slab with a span of l_0 equal to the rib spacing. In lower-rib panels the slab is finally restrained by the grouting of concrete into the joints and thus prevents the ribs from twisting (Fig. 92). The bending moment will be $M = \frac{ql_0^2}{11}$.

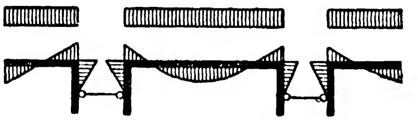


Fig. 92. Structural diagram of a ribbed panel when the joints have been grouted with concrete

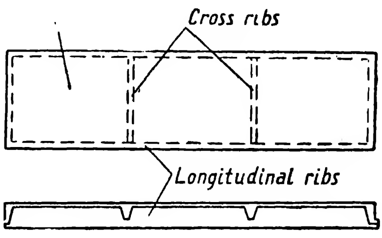


Fig. 93. Ribbed panel with cross ribs

In ribbed panels with cross ribs (Fig. 93), the bending moment is computed as for a slab supported on all sides and bending in two directions. Reinforcement of the flange is calculated according to formulae for rectangular sections and by assuming that $b = 100$ and $h = h_{fl}$.

When panels are of non-prestressed reinforced concrete, cracking in the tension zone must be taken into account when computing rigidity and deflection (see formulae in Sec. 11), in which case cavity panels are considered as I-sections with flanges in both the compression and tension zones, and a ribbed panel is considered as a T-section whose flange lies either in the compression or tension zone.

When panels are prestressed, rigidity and deflection calculations are conducted according to the formulae given in Chapter VI.

The reinforcement of panels consists of prefabricated (welded) mats and blocks (Fig. 94) made of either cold-drawn wire or intermittently deformed bars. The lower longitudinal bars constitute the main effective steel; in ribbed panels they are placed in the ribs, while in cavity panels they are located both in the ribs and underneath the cavities.

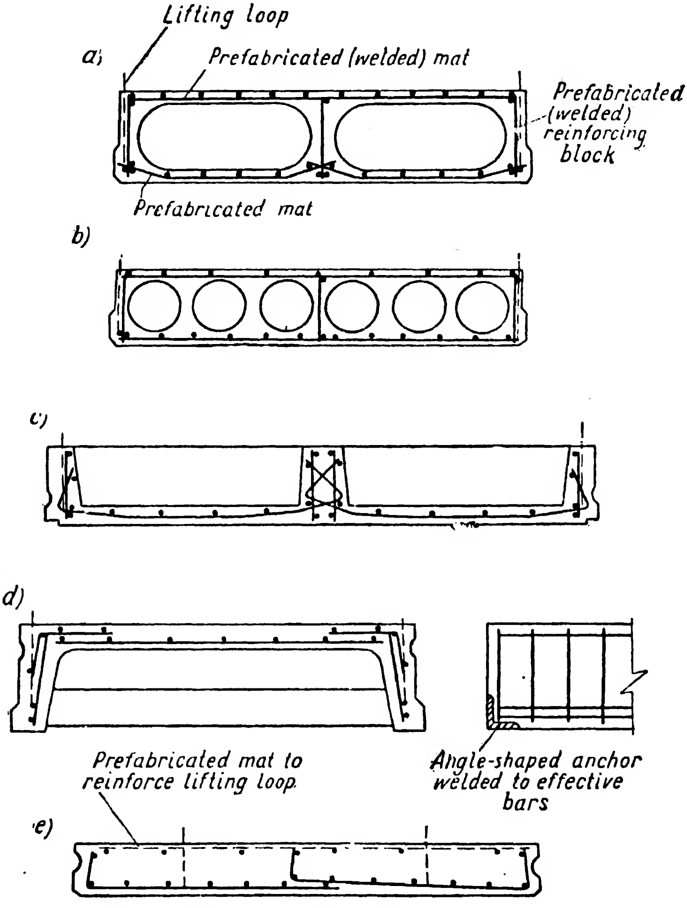


Fig. 94. Panel reinforcement

The upper longitudinal reinforcement is placed as installation bars in the panels, except when there are cross ribs, when such bars make up the effective reinforcement of the flange.

The upper lateral (horizontal) bars are determined by computation since they cope with tensile forces caused by local bending ac-

tion athwart the panel; but if the distance between ribs is insignificant, they are placed as supplementary bars.

Upright bars are subject to computation, but if

$$Q \leqslant mR_t bh_0,$$

they are considered as supplementary reinforcement. In ribbed and oval-cavity panels, flat prefabricated mats must be placed within all ribs, while in round-cavity panels they are installed in the outer ribs and in some of the intermediate ones.

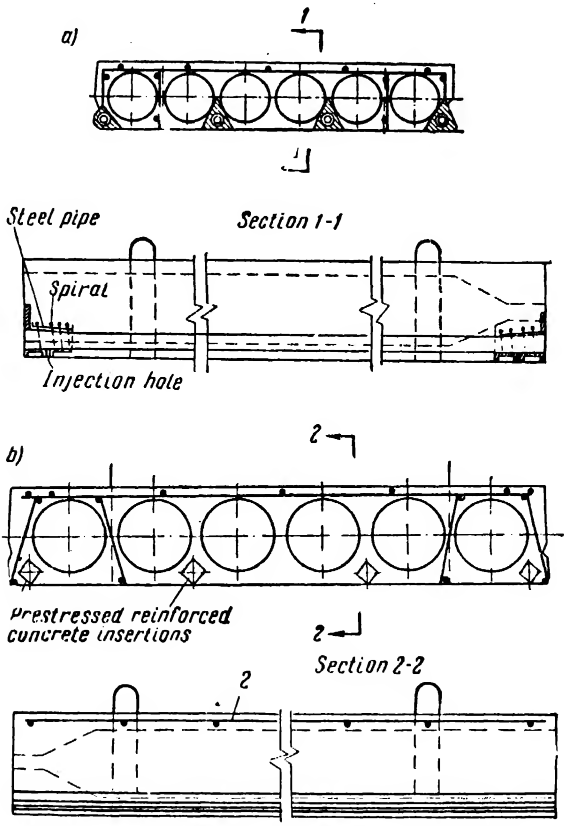


Fig. 95. Reinforcement of prestressed panels

The lower lateral (horizontal) bars of cavity and lower-rib panels are either welded to the lower longitudinal bars of the reinforcement blocks, or their ends are twisted into the meshes of the block (Fig. 94,*a* and *b*).

The longitudinal and upright bars for the ribs are woven together into flat welded latticework. The ends of the effective bars in the ribs

of ribbed panels are welded to special anchorage angles that subsequently bind the bars to the supports (Fig. 94,*d*).

Mats and latticework are all welded together into a a single skeleton, conveniently arranged for placing into the panel formwork.

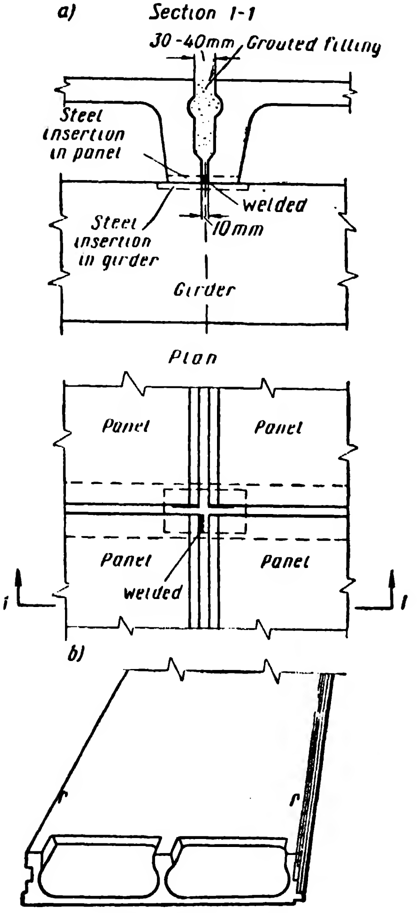


Fig. 96. Installation joints and bearing details of panels

Reinforcement in the form of pan-shaped mats are used at the bottom of solid, single- and double-ply, lightweight concrete panels.

Hoisting loops, made of round St-3 mild steel are inserted and welded to the main reinforcement on four sides of each panel. In solid panels the areas around the lugs are strengthened by means of upper meshwork (or mats) (Fig. 94,*e*).

In prestressed panels the reinforcement is made either of high-strength wire (smooth or intermittently deformed) or of regular intermittently deformed bars.

There are several ways of installing panel reinforcement: 1) tensioning by endless winding, 2) linear tensioning, either on stands or in the form before concreting, 3) tensioning within ducts in the hardened concrete (Fig. 95,*a*), and 4) incorporation of prestressed reinforced concrete inserts (Fig. 95,*b*).

The following technico-economic evaluation factors will decide which of the above methods are chosen: expenditure of concrete and bars, additional cost in prestressing, as well as stock equipment and production possibilities of the panel manufacturer.

In-situ finishing of joints for all types of panels is done by welding of inserted details and post grouting of concrete (Fig. 96,*a*), the resulting floor construction becoming a rigid horizontal diaphragm.

When panels bear on a brick wall, they are let 120 mm into the latter. If they have wide oval cavities (520 mm), their upper flanges are cut out (to the same depth as the wall nest) to prevent their being crushed by the wall (Fig. 96,*b*).

3. Design of Girders

Girders may be made either rectangular or T-shaped (with the flange on top or bottom). To lessen structural height in floor construction, the girder's flange is designed at its bottom to support the panels.

In designing a girder-column connection, the character and magnitude of the forces within it must be taken into account as well as the purpose of the building. Such a joint will undergo a bending moment and lateral stresses from the action of the load (Fig. 97, *a*). The bending moment at the support will cause tension in the upper part of the girder and compression in the lower half; this may be represented in the form of a resisting couple whose value $N = \pm \frac{M}{z}$,

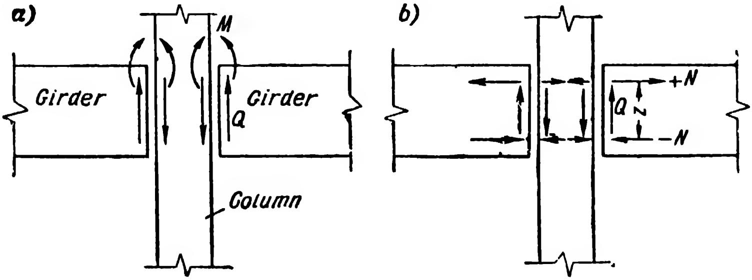


Fig. 97. Forces that act in a column-girder joint

where z—the arm of the resisting couple (Fig. 96,*b*). The forces N and Q will decide which type of joint the column-girder connection is to have.

The column may support the girder upon a bracket, either of reinforced concrete (Fig. 98, *a*, *b* and *c*) or steel (Fig. 98, *d* and *e*); a temporary steel shelf may also be used as a support during erection (Fig. 98,*f*). In all the above cases the tensile force $+N$ will be borne by the steel bars, welded during installation to the steel insertion fixed in the upper part of the girder. Such jointing bars may be concreted into the column beforehand with their ends protruding sufficiently to make a welded connection (Fig. 98,*a* and *e*), or may be inserted into an already prepared hole in the column (Fig. 98, *b*, *d* and *f*); or they may be attached to the sides of the column (Fig. 98, *c*).

There are various methods of transmitting the compressive force $-N$ and lateral stresses Q through the joint. In most instances (Fig. 98, *a* to *e*) these forces are transferred to the column's brackets by welding the latter—whether they are made of steel or of reinforced concrete with steel insertions—to the steel insertions within the girders. In the joint shown in Fig 98, *f*, compression and shear

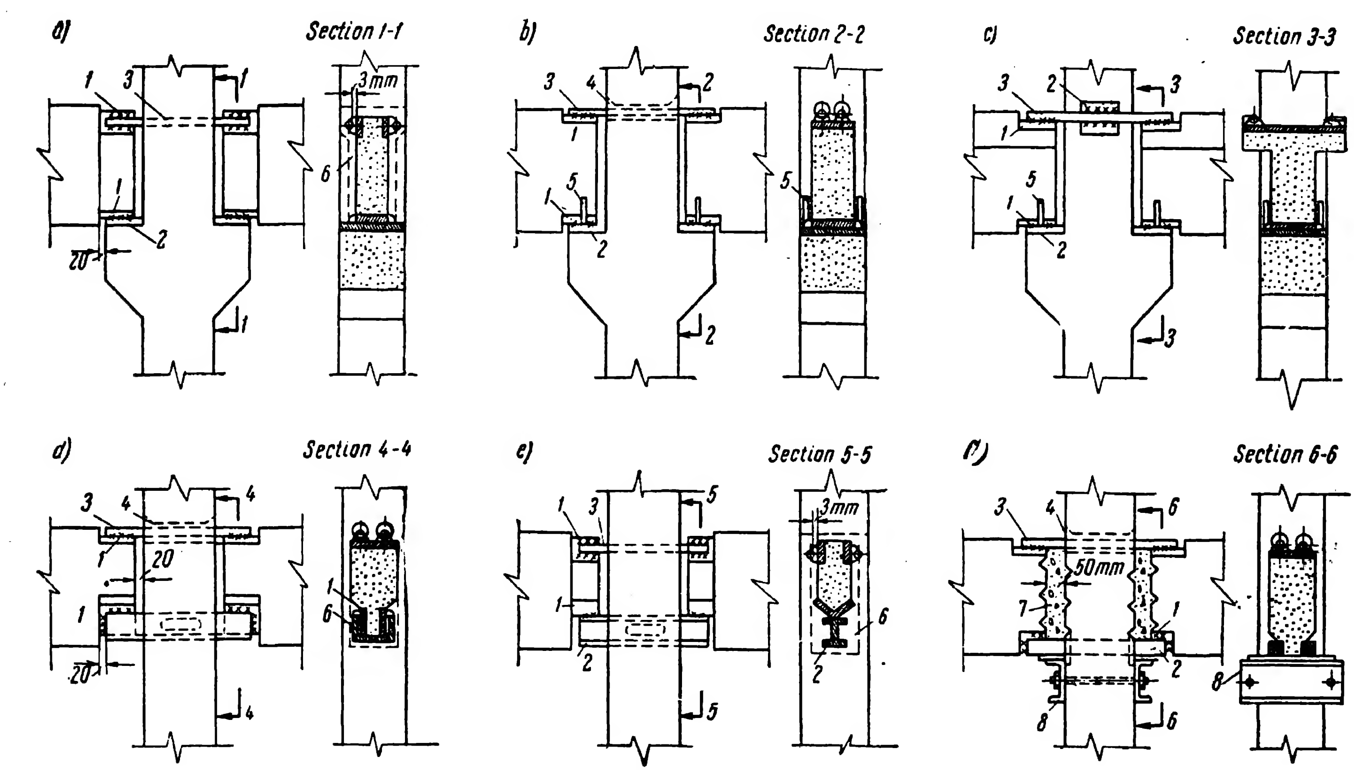


Fig. 98, a to f. Types of column-girder connections

1—girder insertions; 2—column insertions; 3—jointing bars; 4—openings for jointing bars; 5—erection pins; 6—cement covering, applied to metal lath; 7—concrete grouted in during erection; 8 - temporary erection shelf

are borne by the concrete grouted into the joint during erection. To do this, prismatic recesses are left in the column and in the butt end of the girder during their fabrication and 50 mm allowed for joint play.

There are two kinds of steel brackets for columns: 1) pan-shaped brackets formed of steel or made of welded flat steel plate from 6 to 8 mm thick (Fig. 98,*d*) and 2) I-beam brackets (Fig. 98, *e*). Steel brackets should be welded to the block reinforcement of the columns and their vertical plates perforated to afford a good bond with the column's concrete.

Reinforced concrete brackets can take comparatively large M and Q forces and are employed for floor construction in industrial buildings and also in apartment and civic buildings, where they are given a desirable architectural form.

Concealed brackets (Fig.98, *d*, *e* and *f*) are mostly adopted for apartment houses and civic buildings (where the girders are laterally placed).

Of the two kinds of steel brackets described above, the channel-shaped type is the best (Fig. 98,*d*). The I-beam type (Fig. 98,*e*), which requires that the girder's ends be cut out, weakens the ends of the girder. To compensate for this fault, additional mats and steel insertions are needed which increase expenditure of both steel and labour.

Joints with concrete pins to bear the design forces—N and Q (Fig. 98,*f*) reduce steel expenditure but demand extravagant care during erection operations in grouting a very small amount of concrete, whereas all the other aforementioned joints require only a protective layer of cement for the steel insertions, applied on a thin steel lath.

To limit displacement during erection and fix the girder into its final position, either pins or steel plates between which the girder is placed are let out of the columns.

Other designs of girder-column joints in addition to those given above, must possess not only the attribute of strength but also that of producibility; they should have rectilinear girder ends and a minimum amount of steel (see Sec. 23).

A heavy-duty girder-column joint for industrial structures (1,000 and more kg/m^2) is shown in Fig. 98,*g*. In this joint the tension carrying bars protruding from the girder and column are welded to semicircular supporting chairs, with the butts welded within a form. Compression forces are handled by the concrete subsequently grouted into the joint clearance between the girder and column. In computing girders, account should be taken of the rigidity of the column as an element of a rigid frame.

If the girder is freely supported on a wall but has equal spans and an insignificant live load ($p^s \leq 500$ kg/m^2), it is computed as a continuous beam. When the spans are equal or differ not more than 20%, the bending moments for continuous beams may be derived from the

Tables in Supplement III, in which case allowances may have to be made for moment redistribution due to plastic deformation (see Sec. 20).

The effective span of a girder is assumed as the distance between column centres; if one end of the first span begins at a wall, the effective span will be from the centre of the wall support to the centre of the column.

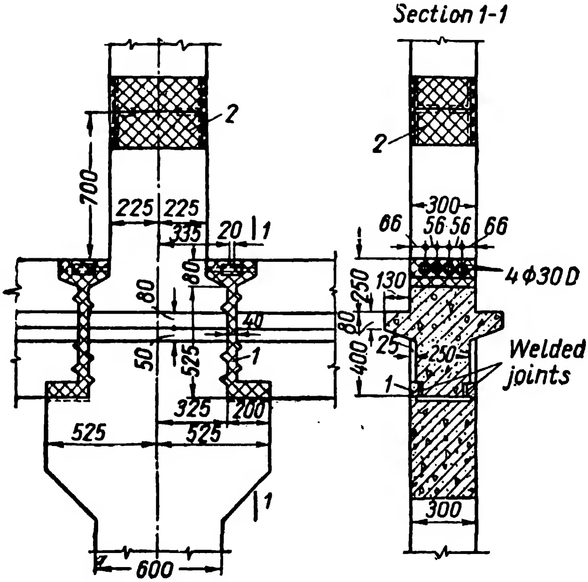


Fig. 98, g. Form-weld of jointing bars in a column-girder joint

The dead weight from panels resting on a girder may be considered either as a uniform or as concentrated loads. When there are more than four contact points of concentrated loads, load distribution will be uniform on the girder. The dead weight of the girder proper may be tentatively established by assuming that

$$h=\frac{1}{10}-\frac{1}{15}l, \quad b=0.3-0.4h.$$

Bending moments and shear forces in the girder may be evolved through the following formulae:

a) for uniform loads

$$M=(\alpha g+\beta p)\,l^2; \quad Q=(\gamma g+\delta p)\,l;$$

b) for concentrated loads

$$M=(\alpha G+\beta P)\,l; \quad Q=\gamma G+\delta P,$$

in which α, β—tabular coefficients that give the values of M for dead and live loads, respectively;

γ, δ—tabular coefficients that give the values of Q for dead and live loads, respectively.

Girder computations of moment redistribution must be based on loading diagrams that confer maximum moments to the interspan and live loads to alternating spans, as follows (Fig. 99,*a*):

1) dead loads in all spans, live loads in oddly numbered spans;
2) dead loads in all spans, live loads in evenly numbered spans.

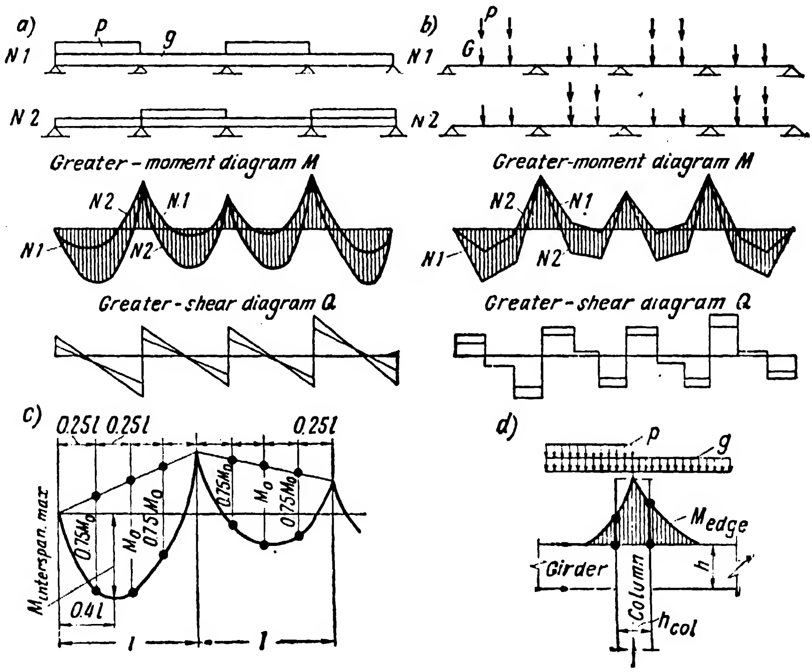


Fig. 99. Loading and moment diagrams of girders

The ordinates of the moments at the supports, derived from both the 1st and 2nd loading schemes, are measured off at the centres of the supports. Moment curves (Fig. 99,*c*) will then follow parabolas derived from the corresponding loads g or $g+p$ (depending on the given loading scheme) and will pass through the apex of the ordinates of the moments at the supports. The ordinates of the parabolas in midspan will correspondingly be

$$M_0=\frac{gl^2}{8} \text{ or } M_0=\frac{(g+p)\,l^2}{8}.$$

The other ordinates of the parabolas will be 0.75 M_0, independent of whether x is equal to $1/4\ l$ or $3/4\ l$.

That part of the curve having the greatest ordinates (shown hatched in Fig. 99) will be the greater-moment diagram M. After this the greater-moment diagram for Q is also charted.

If the girder is subject to concentrated loads, the M and Q greater-moment diagrams will take the form shown in Fig. 99,*b*, the dead weight of the girder also being assumed as a concentrated load. For example, when the contact points of the load in the span are $^1/_3$ distance from the supports, the dead weight of the girder g_{girder} (per unit length of the girder) will result in a force of

$$G_{girder}=\frac{g_{girder}l}{3}.$$

The effective cross-section of the girder is taken at the surface of the column (Fig. 99,*d*); here the bending moment

$$M_{sur}=M-Q\frac{h_{column}}{2}. \qquad (210)$$

The moment M_{sur} is greater on that side where the span carries only a dead load. Hence, the value of Q entered into formula (210) must correspond to the load of that span.

Girder reinforcement will consist of flat prefabricated frames with effective bars usually of intermittently deformed steel; 200-grade concrete is used. Reinforcement should satisfy the moment at the support M_{sur} before being calculated for the bending moment. $\alpha\leqslant 0.3$, inasmuch as moment redistribution is computed on the basis of plastic pivot formation at the support (see Sec. 19). The girder's cross-section at the support is considered as rectangular; hence,

$$h_0=2\sqrt{\frac{M_{sur}}{mbR_{be}}}.$$

When the $b\times h$ dimensions of the girder have been established, the tension bars are computed at four computation planes: in the first and middle spans, and at the first intermediate and middle supports.

The next step is to compute the action of shearing forces in the diagonal planes and determine the diameter and spacing of upright bars. Here three computation diagonals are chosen: to the right and left of the first intermediate support, and at the free end support.

Resistance computations at the girder-column joint (Fig. 100) will embrace calculations of upper bars of the joint and of the supporting bracket.

Calculating the upper bars. Design tension $N=\frac{M}{z}$, in which M—the moment of the girder at the support, and z—the arm of the internal couple, equivalent to the distance between the centres of gravity of upper and lower steel insertions, or between the centres of gravity of the welded connections of the joint.

The area of the bars will be

$$F_s=\frac{N}{mm_sR_s}\,.$$

Calculating the support bracket. The stresses in the bracket will be the result of the shearing force Q and the compressive force—N.*

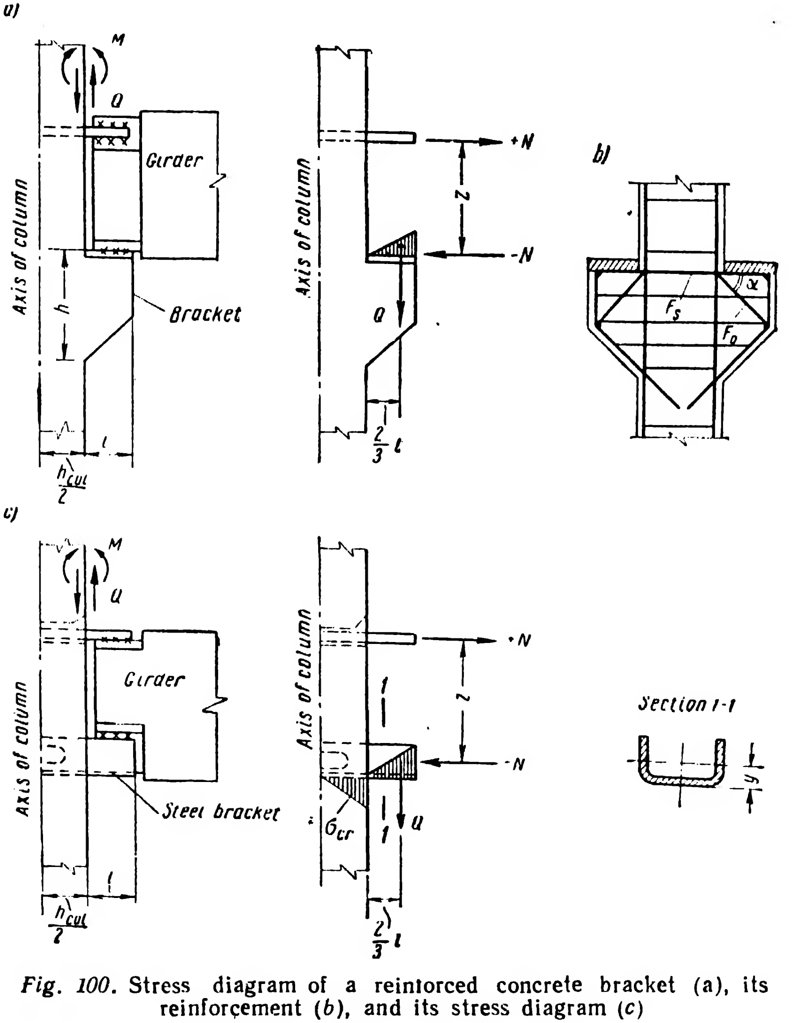


Fig. 100. Stress diagram of a reinforced concrete bracket (*a*), its reinforcement (*b*), and its stress diagram (*c*)

Assuming that the transfer of forces from the girder to the bracket can be represented by a triangular stress diagram, the shearing force is as-

* Actually, computation of the joint should be for compression with a deductioo for friction $T=Qf$. But until more knowledge is accumulated concerning jnint behaviour, the relieving effect of friction is ignored.

sumed as acting at the centre of gravity of the triangle. If l is the projection of the bracket, then the distance from the centre of gravity of the triangle to the end of the bracket will be $\frac{2}{3}l$, and the bending moment of the bracket

$$M_{br}=\frac{2}{3}lQ. \tag{211}$$

When the design forces have been determined, the resistance of the bracket may be computed.

The width of a reinforced concrete bracket is made the same as that of the column (Fig. 100,*a* and *b*). The projection of the bracket l is commonly chosen structurally from 15 to 25 cm so as to accommodate the inserted parts and the length of their weld.

The height of a reinforced concrete bracket must meet resistance requirements along diagonal planes:

$$Q\leqslant mR_tbh_0.$$

In satisfying this requirement, lateral reinforcement is considered as supplementary. If the height of the bracket is structurally restricted and the above condition cannot be met with, a reduced height may be accepted but always under the condition that $Q\leqslant\frac{1}{6}\ mR_{be}bh_0$. In this latter case all the shearing stresses in a short bracket ($l\leqslant0.9h_0$) will be borne by the bent-up bars whose area is computed as

$$F_0=\frac{Q}{2m_{di}m_sR_s\sin\alpha}. \tag{212}$$

Lateral reinforcement in long brackets ($l\geqslant0.9h_0$) is calculated just as for beams.

The longitudinal (horizontal) bars of a bracket are subject to compression, therefore their area F_s is computed according to the compressive force N (which is assumed as acting at the centre of gravity of the bars) and the bending moment M_{br} which causes tension in the bars, as follows:

$$F_s=\frac{N}{mm_sR_s}-\frac{M_{br}}{mm_sR_s\gamma_0h_0}. \tag{213}$$

Assuming that $\gamma=0.9h_0$, we obtain

$$F_s=\frac{1}{mm_sR_s}\left(N-\frac{M_{br}}{0.9h}\right). \tag{213a}$$

Within the range of the bracket height the stirrups of the column are given a closer spacing of 10-15 cm. A steel plate is welded to the upper bars of the bracket for the support of the girder. The cross-sectional area of this plate

$$F_{pl}=\frac{N}{mR}. \tag{214}$$

The steel planks welded to the upper and lower bars of the girder, at its butt end, must have the same area F_{pl}. The force borne by the weld, when round bars are welded to the plate,

$$N = m_{jo} 0.85 h_{jo} l_{jo} R_{wj}.$$

The height of the joint in cross-section is usually taken as $h_{jo} \geqslant \frac{1}{4} d_{rod}$.

Design resistance of a welded joint R_{wj}, depending upon the type of electrode, ranges from 900 to 1,200 kg/cm.

To assure reliable welded joints when the moments in the girder have been redistributed, $1.3N$ is taken as the design force in the welded joint.

The sum of all weld lengths in the round-bar and plate connections (disregarding poor weld penetration) will be

$$\Sigma l_{jo} \geqslant \frac{1.3N}{0.85 m_{io} R_{wj} h_{jo}}. \qquad (215)$$

The sum of the whole weld length in the connections of two plates (without taking poor weld penetration into account) will be

$$\Sigma l_{jo} \geqslant \frac{1.3N}{0.7 m_{jo} R_{w} h_{jo}}. \qquad (216)$$

Normal stresses in the vertical plane of a steel bracket (Fig. 100,*c*) from the action of N and M_{br}

$$\sigma = \frac{N}{F} + \frac{M_{br} y_{lf}}{J},$$

where F, J, and y_{lf}—area, moment of inertia of the cross-section, and the distance from the centre of gravity to the lower fibre, respectively.

Shear stresses in the same plane of the steel bracket (ignoring, as a reserve, the shear in the upper bars) may be taken as $\tau = \frac{Q}{F}$. The resultant of the stresses in the steel must satisfy the condition

$$\sigma_{\text{resultant stresses}} = \sqrt{\sigma^2 + 3\tau^2} \leqslant mR,$$

in which R—design resistance of steel.

Local crushing compression acting in the concrete directly beneath the steel bracket may be computed on the assumption that the stress diagram is triangular. The total amount of this crushing compression σ_{cr} and the normal stresses σ_c in the concrete of the column from the load above it must not exceed the design resistance of concrete in local crushing compression, i.e.,

$$\sigma_{cr} + \sigma_c \leqslant m\psi R_{pr}.$$

The usual manner of reinforcing a precast girder is with two flat welded frames (Fig. 101). If the load is very large, a third frame may be needed in midspan; this third component must not be prolonged to the supports as it will then complicate the attachment of the steel insertions. The area of effective bars of this reinforcement and the number of such latticed frames can be evolved from the bending moments in the computation planes at the supports and interspan. The bending moment ordinates M shorten with the distance from the said planes, with a corresponding reduction in bar area.

In order to save steel, some of the longitudinal tension bars can be interrupted, in accordance with changes of bending moment

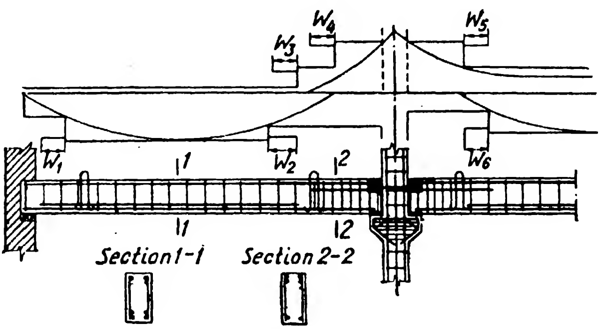


Fig. 101. Reinforcement of a precast girder

ordinates, at points extending somewhat beyond the theoretical limit; the length of this extension w is determined by means of formula (80).

It must be kept in mind that at least 50% of the tension-bar area of the girders, as computed in accordance with the maximum interspan moment, must be prolonged to the compression zone at the supports. Furthermore, since the lower part of the butt of the girder receives the force N as transmitted through the weld of the joint, the bars extended to the support at the bottom of the girder from the interspan must have at least 70% of the cross-sectional area of the bars in the upper part of the girder's end. Actually this is always provided for if the girder has been computed on the basis of redistributed moments.

A reinforcement diagram (diagram of materials) should be charted to verify the steel, both for its computed strength and effective economy. The ordinates for this diagram are plotted as moments of internal forces at the given planes of the girder:

$$M_{cs}=mm_sR_sF_sz,$$

in which F_s—cross-sectional area of tension bars at the given plane;
z—the arm of the internal couple.

Such a diagram will consist of stepped lines whose verticals will align with the theoretical cut-off points of the reinforcement. Any over-reserve of structural safety (too much tension-bar area) will be revealed when the charted line swerves considerably outwards from the larger-moment diagram M. On the other hand, a deficiency in strength will be shown where the stepped reinforcement diagram penetrates the larger-moment diagram M.

4. Sequence in Design of Precast Ribbed Slabs

The following sequence should be adhered to in designing precast ribbed floor construction:

1. *Composing the structural layout*. The column grid and the direction of girders (in plan) are chosen. Shapes and dimensions of panels are determined.

2. *Designing the panel*: 1) the design diagram and effective lengths of spans are decided upon; 2) all the loads are established; 3) the design forces M and Q are ascertained; 4) design data is gathered: design resistance of materials, service and other coefficients, etc; 5) the cross-sectional height of panels is computed; 6) the area of longitudinal bars is calculated in accordance with bending moments; 7) calculations concerning shearing forces are executed; 8) the upper panel flange is verified for local bending resistance (perpendicular to the ribs); 9) panel rigidity is checked; 10) deflection is computed and compared with allowable values; 11) panels are checked for their possible behaviour during erection operation.

3. *Designing the girder*: 1) the design diagram and effective lengths of spans are decided upon; 2) all loads are established; 3) bending moments and shearing forces are calculated and the larger-moment diagrams M and Q are charted; 4) design data is gathered; 5) the height of the girder is computed; 6) the area of longitudinal bars is calculated for the interspan and at the supports in accordance with bending moments at computation planes; 7) calculations of shearing forces are executed at computation planes to establish diameters and spacing of upright bars; 8) the column-girder joint is computed, the cross-sectional area of steel insertions evolved, and the bearing area of supporting brackets computed; 9) the girders are checked for possible behaviour during erection operations.

B. IN-SITU RIBBED SLAB CONSTRUCTION

1. Types

There are two types of in-situ floors: slabs supported by beams on two sides (one-way slabs) and slabs supported by beams on four sides (two-way slabs), the latter also being known as coffer-type floors.

In-situ floors are seldom used today, being adopted only in individual cases or when a structure is to experience considerable dynamic loading.

The rectangular proportions of a given slab establish whether it is to be made with one-way or two-way supports.

When the relation of dimensions is $l_2 : l_1 > 2$, bending will take place practically only along the short way, while the bending action along the other direction can be ignored because of its insignificance (Fig. 102,*a*).

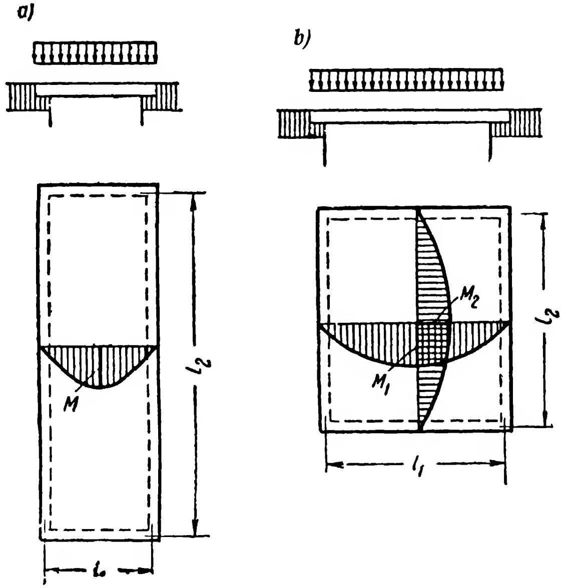


Fig. 102. Types of slabs

When the relation of dimensions is $l_2 : l_1 \leq 2$, that is, when the slab approaches a square, bending action will take place in two directions (Fig. 102,*b*).

The effective reinforcement of one-way ribbed slabs is run the short way, while the spacing bars are placed in the long direction. Two-way slabs are provided with effective reinforcement in both directions.

2. Structural Layout of a One-Way Ribbed Floor

The elements composing a one-way ribbed floor are: the slab proper, the cross beams, and the main beams (girders) (Fig. 103,*a*). All these members form a monolithic whole. The slabs are subject to bending action along the spans perpendicular to the cross beams,

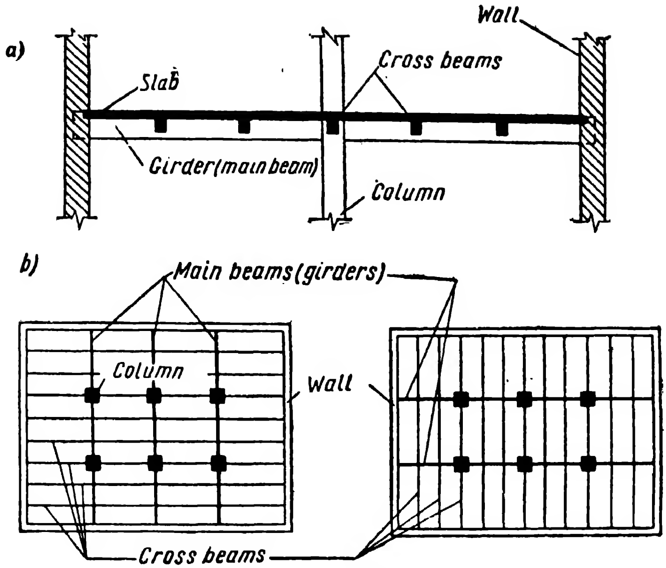


Fig. 103. One-way ribbed-slab construction

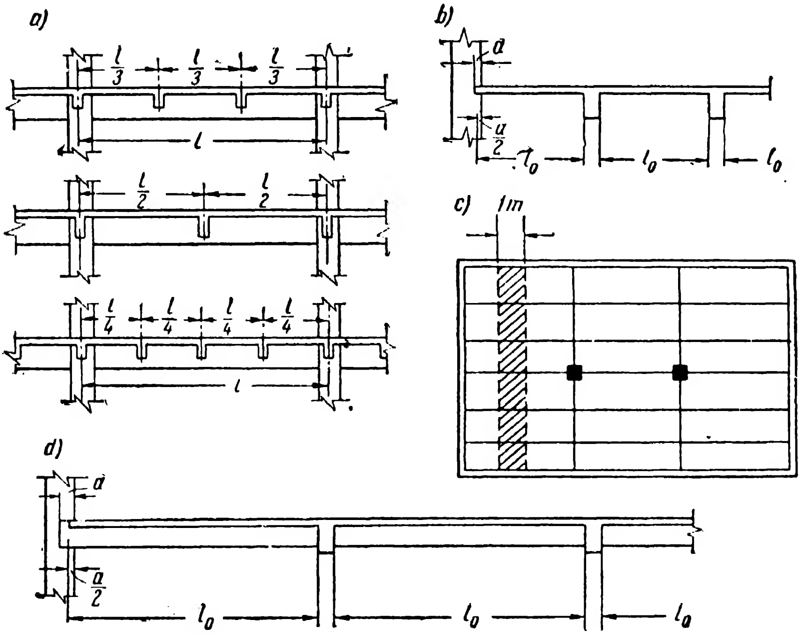


Fig. 104. Layouts of cross beams and slabs

these latter being supported by the main beams with which they form a monolithic whole. The main beams in their turn rest upon the columns or walls.

Both the cross beams and main beams act as T-sections whose flanges (the slabs) are in the compression zone. A negative moment is created at the supports of these beams, with the result that here the slabs are subject to tension. For this reason, the effective cross-section at the supports is considered a rectangle whose width b is that of the rib.

The main beams may be placed either longitudinally with the building, or athwart it (Fig. 103,*b*), with spans ranging from 5 to 8 m. The cross beams are usually arranged so that the span of the main beam is divided into equal lengths (Fig. 104,*a*). The cross beams may have a maximum span of 5-7 m and the slab spans of 1.7 to 2.7 m.

The minimum thickness of the slab, depending on the purpose of the building, is dictated by code as follows: 8 cm for construction in industrial buildings and 7 cm in civic buildings. For in-situ floors, grade 150 concrete is the usual practice, but when grade 200 is used, the above slab thicknesses can be reduced by 1 cm. If the live load is very great, a still thicker slab may be required. For example, with a live load of 1,000-1,500 kg/m^2 and a slab span of 2.2-2.7 m, the thickness will range from 9 to 10 cm, thus giving the greatest economy in reinforcing bars.

The cross-sectional height of cross beams should be from 1/12 to 1/20 l_{cb}, that of the main beams (girders) from 1/8 to 1/15 l_{gi}, while the width b of beams from 0.4 to 0.5h.

3. One-Way Ribbed Slabs

Computations of the elements of one-way ribbed slabs (the slab proper, the cross beam, and the main beam, i.e., the girder) are conducted according to moment redistribution based on plastic deformation.

Slabs. The effective span l_0 of slabs is considered as the clear distance between cross beams; if one end of the slab is supported on an exterior wall, the effective span will be from the centre of the wall support to the edge of the first rib (Fig. 104,*b*).

The load per square metre on the slab will be the sum of the dead load g (the dead weights of the slab, flooring, etc.) and the live load p.

For convenience in computation, a zone is assumed whose width $b=1$ m (Fig. 104,*c*), in which case the load allotted to 1 m^2 of slab will also denote the load for 1 linear metre and the design diagram of the slab will be represented as a continuous beam whose load $q=g+p$.

Bending moments of continuous ribbed slabs that have equal spans, or spans with maximm 20% differences, are evolved through the following formulae:

in the first span and at the first intermediate support,

$$M=\frac{(g+p)\,l_0^2}{11}; \tag{217}$$

in the intermediate spans and at intermediate supports,

$$M=\frac{(g+p)\,l_0^2}{16}. \tag{217a}$$

Shearing forces are usually not taken into account for slabs, since in most instances condition $Q \leq mR_t bh_0$ is satisfied.

The cross-sectional area of the bars, calculated in accordance with the bending moments, is determined just as for a rectangular beam whose width $b=100$ cm and height h is the thickness of the slab. Mats are used for the reinforcement, these usually being of welded cold-drawn wire.

The *cross beam*. The effective span l_0 of the cross beam is considered as the clear distance between girders; but if one end is borne by an exterior wall, the effective span will be the distance from the wall support to the edge of the girder (Fig. 104,*d*).

The slab transmits a uniform load to the beam, i.e., 1 m² of slab load multiplied by the distance between rib centres. To this must be added the dead weight of the rib.

Bending moments of continuous cross beams that have equal spans, or spans whose maximum difference is 20%, are computed by the following formulae:

in the first span and at the first intermediate support,

$$M=\frac{(g+p)\,l_0^2}{11}; \tag{218}$$

in the intermediate spans and at intermediate supports,

$$M=\frac{(g+p)\,l_0^2}{16}. \tag{218a}$$

In a continuous triple-span cross beam the bending moment in the central span will not be less than the moment in a restrained beam:

$$M=\frac{(g+p)\,l_0^2}{24}. \tag{218b}$$

The larger-moment diagram of cross beams must be charted for two loading schemes:

1) the full load $g+p$ in oddly numbered spans, and the assumed live load $g'=g+\frac{1}{4}\,p$ in the evenly numbered spans;

2) the full load $g+p$ in the evenly numbered spans, and the assumed live load g' in the oddly numbered spans.

The assumed value g' is entered into the computations so as to allow for the relieving influence of the girder when determining the negative moment in the interspan of the cross beam: the main beam forms an additional tie at the cross beam's supports and thus reduces the negative moments in insufficiently loaded spans.

In plotting the larger-moment M-diagram, the moments at the supports and the maximum interspan moments are determined by the aforementioned formulae. Minimum interspan moments are plotted along parabolas that correspond to the load g' and pass through the apex of the moment ordinates at the supports (Fig. 105). The negative

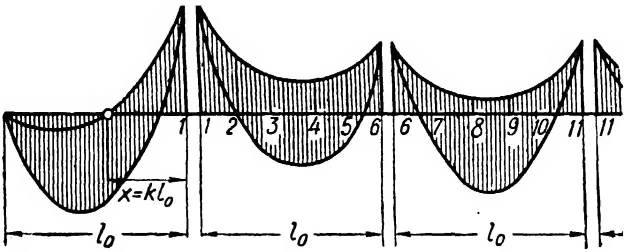


Fig. 105. Larger-moment M-diagram for a continuous beam

moments in the intermediate spans and the contraflexure point in the first span may also be taken from Table 20.

Shearing forces are calculated as follows:

at the free end support

$$Q_A = 0.4\,(g+p)\,l_0; \tag{219}$$

at the first intermediate support to the left

$$Q_B^L = 0.6\,(g+p)\,l_0; \tag{219a}$$

at the first intermediate support to the right and at all other supports

$$Q_B^R = Q_C = 0.5\,(g+p)\,l_0. \tag{219b}$$

The reinforcement for cross beams will consist of prefabricated (welded) blocks in the interspans and prefabricated mats at the supports.

In determining cross-sections according to bending moments, first the final dimensions of the cross beam are evolved from the moment at the first intermediate support. Inasmuch as the computations are based on moment redistribution, $\alpha \leqslant 0.3$. There is a negative moment at the support, hence here the slab will be in the tension zone, which requires that the beam be computed as for a rectangular shape:

$$h_0 = 2\sqrt{\frac{M}{mR_{be}b}}$$

Table 20

Values for the Coefficient β in Plotting Negative-Moment Ordinates

$$M = \beta\,(g + p)\,l_0^2$$

p/g	Points											
	1	2	3	4	5	6	7	8	9	10	11	$k = \frac{x}{l}$
0.5	—0.091	—0.025	+0.011	+0.016	—0.008	—0.0625	—0.003	+0.028	+0.028	—0.003	—0.0625	0.167
1	—0.091	—0.035	—0.005	+0.001	—0.018	—0.0625	—0.013	+0.013	+0.013	—0.013	—0.0625	0.2
1.5	—0.091	—0.041	—0.014	—0.008	—0.024	—0.0625	—0.019	+0.004	+0.004	—0.019	—0.0625	0.228
2	—0.091	—0.045	—0.02	—0.014	—0.028	—0.0625	—0.023	—0.003	—0.003	—0.023	—0.0625	0.25
2.5	—0.091	—0.048	—0.023	—0.017	—0.031	—0.0625	—0.025	—0.006	—0.006	—0.025	—0.0625	0.27
3	—0.091	—0.05	—0.027	—0.022	—0.033	—0.0625	—0.025	—0.01	—0.01	—0.028	—0.0625	0.285
3.5	—0.091	—0.052	—0.03	—0.025	—0.035	—0.0625	—0.029	—0.013	—0.013	—0.029	—0.0625	0.304
4	—0.091	—0.053	—0.032	—0.026	—0.036	—0.0625	—0.03	—0.015	—0.015	—0.03	—0.0625	0.314
4.5	—0.091	—0.054	—0.033	—0.028	—0.037	—0.0625	—0.032	—0.016	—0.016	—0.032	—0.0625	0.324
5	—0.091	—0.055	—0.035	—0.029	—0.038	—0.0625	—0.033	—0.018	—0.018	—0.033	—0.0625	0.333

Having found the final $b \times h$ dimensions of the cross-section, they are considered as valid for the whole beam length, after which the area of tension bars is computed at four computation planes: in the first and middle spans (assuming that they are T-sections) and at the first intermediate and middle supports (assuming them as rectangular shapes). When checking the negative moment in the middle span, it is done as for a rectangular shape.

The next step is to compute the shearing forces at diagonal planes and find the diameters and spacing of stirrups. Three computation planes are chosen: at the first intermediate support to the left and right (Q_B^L and Q_B^R), and at the free end support (Q_A).

The *girder*. The girder takes the dead and live concentrated loads G and P from the cross beams, these values consisting of the load of one linear metre of cross beam multiplied by the distance between girder centres. Furthermore, just as in precast slab construction, the dead weight of the girder is taken into account.

The sequence of in-situ girder computation is fully the same as already described for precast construction. The only special feature in determining its reinforcement by means of bending moments is that it is calculated for a positive moment in its interspan just as for a T-beam.

4. The Elements of One-Way Ribbed Slab Floors

The design of the elements of a floor will be considered in the already stated sequence.

The *slab proper*. Multi-span ribbed one-way slabs are reinforced with roll-type mats whose effective bars (maximum diameter 5.5 mm) are laid longitudinally; the rolls of mats are unrolled over the formwork, athwart the cross beams (Fig. 106,*a*).

The bending moments in a continuous slab provoke tension at the bottom of the slab in the interspans and at the top of the slab where it rests on the supports; therefore the mat is bent up as it is rolled over the supports, with the bent-up portion starting 0.25 l from the beam centre.

Insofar as the bending moment shows that more bars are required in the first span and over the first support than in the middle spans and over the middle supports, an additional mat is placed in the first span and over the first support and carried beyond the latter to a distance of 0.25 l (Fig. 106,*b*). However, straight-hooked individual bars may be used instead of this additional mat and then connected to the main mat. The above roll-system of reinforcement is classified as *uninterrupted*.

When stronger reinforcement is required (6 mm or more) for continuous slabs, individual mat rectangles are rolled out, with effective bars running laterally (Fig. 106,*c*) at the bottom of the slabs (the entire interspan) and others over the supports, the latter (with a width

of 0.5 l) being placed symmetrically over the cross beams. With long slab spans and great thicknesses (9-10 cm) an economy of reinforcement is achieved at the supports by using two overlapping mats, each with a width of 0.4 l (Fig. 106, *d*). If the end support is concluded with a rib (in one piece with the slab), then a supplementary mat is placed over it, 0.15 l, measured from the beam centre.

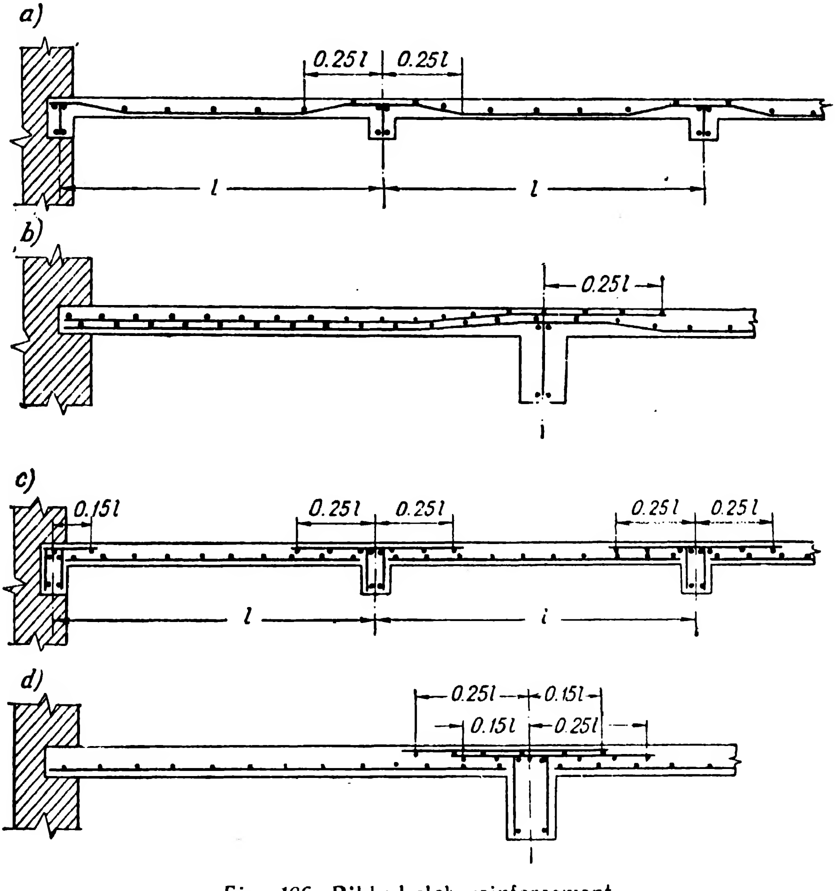


Fig. 106. Ribbed-slab reinforcement

Cross beams. The reinforcement placed in the interspans of cross beams will consist of flat latticework (frames), usually two of them, which, before being placed into the forms, are spliced into a block by means of lateral (horizontal) bars. One flat frame is sufficient if the rib is light and narrow.

Such cross-beam frames must reach the edges of the girders (Fig. 107). Mats with laterally laid effective bars are placed at the sup-

porting end of the cross beams. This is attained by rolling a mat, or placing flat mats, on top of the whole girder length. The effective area of tension bars at cross-beam supports is considered as the entire area of all effective steel of the aforementioned mats included between the centres of adjacent slab spans.

At column sides, mats overlying beam supports are interrupted and in their stead additional bars, or fractions of mats, are placed; the total area of this steel must be equal to that of the effective bars of the mats falling within the columns' widths. At end bearings, supplementary mats are superimposed over cross-beam supports.

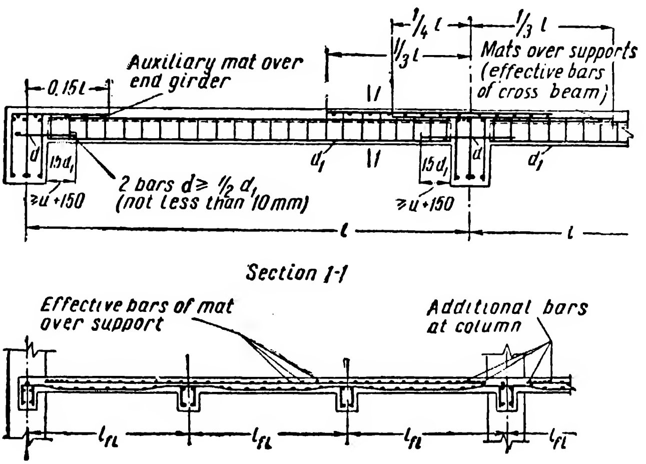


Fig. 107. Cross-beam reinforcement

To save steel when cross-beam spans are very long, two overlapping tension mats may be used over the supports.

Mats over supports are interrupted in accordance with the negative-moment diagram. If the live- and dead-load ratio $\frac{p}{g} \leqslant 3$, the cut-off point of tension steel at the supports will be as follows: 50% of the bar area is cut off at a distance of $^1/_4 l$ from the centre of the support, and 75% at a distance of $^1/_3 l$ from the same centre; interruption of the remaining 25% is then dictated by the negative moments in the interspan.

Blocks in the interspans of adjacent cross beams must be tied together at the bottoms of supports by means of jointing reinforcement let through the girders, the diameter d of such jointing rods being $\geqslant \frac{1}{2} d_1$, where d_1 is the diameter of effective bars in the cross beams (but not less than 10 mm).

The *girder*. Two or three flat frames (latticework), composing the reinforcement of a girder (Fig. 108), are tied together into a block prior to placement into the forms. Two of the frames are extended to the column's face, while the third (if there is such a one) is cut off according to the larger-value diagram. Such an interruption is also realised by cutting off separate bars in the frames. The support of the girder must be reinforced with two separate frames let through the reinforcing block of the column. The bars at the supports may

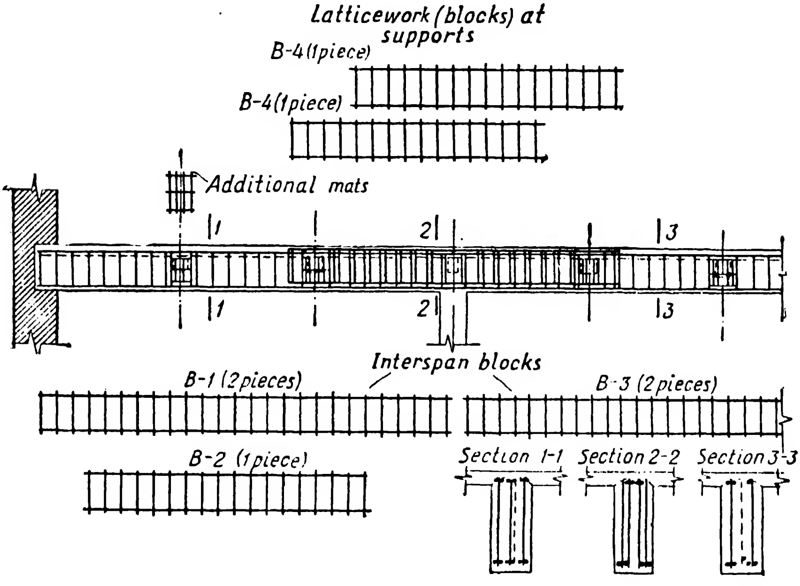


Fig. 108. Girder reinforcement

also be in the form of mats, already described for cross beams. In the latter case, the support reinforcement is placed on both sides of the column, with its overall width not more than $^1/_3$ of the distance between girders.

Cross beams may develop fissures in their tension zone at the supports and the concentrated load will then be transmitted to the girder through the compression zone of the cross beam, i.e., at mid-height of the girder (Fig. 109). This makes it necessary to install additional mats to receive the local concentrated load P (Fig. 108). The area of the upright bars, including the vertical bars of the additional mats, is computed by the formula

$$F_{lat} = \frac{P}{mm_a R_s}. \tag{220}$$

The dimension of the zone within which the upright bars will bear the concentrated load, is computed by the formula

$$s=2h_1+3b.$$

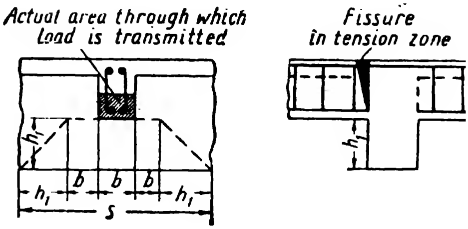


Fig. 109. Sketch indicating the transmission of a concentrated load to a girder

The cut-off line of the reinforcement frames (or of individual bars) is determined by plotting a diagram of materials.

5. Sequence in Computing Ribbed In-Situ Floors with One-Way Slabs

The following sequence should be observed in computing ribbed in-situ floors with one-way slabs:

1. *Composing the structural layout of the floor*. The column grid is determined (unless it is definitely stipulated beforehand), as well as the direction and spacing of girders and cross beams.

2. *Computing the slab*: 1) the design diagram is determined, as well as the spans and thickness of the slab; 2) all the loads are established; 3) bending moments are computed; 4) design data is gathered: design resistance of materials, service and other coefficients, etc; 5) prefabricated mats are computed for computation planes at all supports and interspans, in accordance with bending moments.

3. *Computing cross beams*: 1) the design diagram and span dimensions are determined; 2) the loads g, p, and g' are ascertained; 3) bending moments and shearing forces are determined, as well as possible negative moments in the interspans; 4) design data is gathered; 5) the cross-sections of cross beams are determined; 6) the areas of effective bars are computed for the prefabricated reinforcing frames in the interspans and the prefabricated mats at the supports; 7) computations of shearing forces are carried out to evolve diameters and spacing of upright bars in all computation planes.

4. *Girder computation*: 1) design diagrams and span dimensions are established; 2) all loads are ascertained; 3) bending moments and shearing forces are determined and the larger-value diagrams M and Q are plotted; 4) design data is gathered; 5) the cross-section of

the girder is determined; 6) areas of effective bars are computed for prefabricated reinforcing blocks in the interspans and at the supports in accordance with bending moments; 7) computation of shearing forces are carried out to evolve diameters and spacing of bars in computation planes; 8) calculations are executed to determine the upright bars in the additional mats needed at the jointing of cross beams.

6. Composing the Structural Layout of Ribbed Floors with Two-Way Slabs

The elements entering into this type of construction are: multispan slabs supported along their entire perimeters and subject to two-way bending ($l_2 : l_1 \leqslant 2$ are the dimension ratios of their sides), and girders placed in two perpendicular directions (Fig. 110, *a*). The spans of girders in this type of slab construction range from 4 to 6 m. In large rooms of civic buildings where there are no interme-

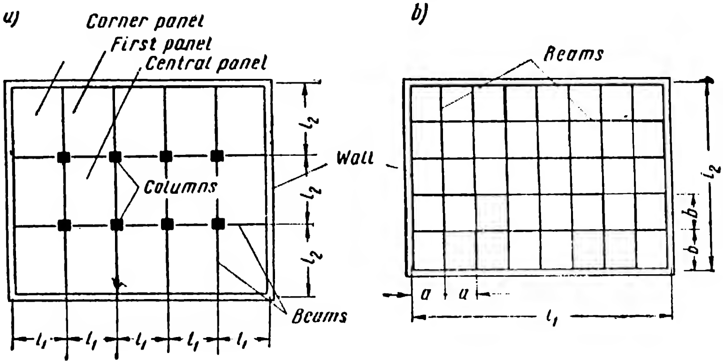


Fig. 110. Ribbed floors with two-way slabs

biate columns, use is made of closely spaced perpendicularly disposed beams of equal height and with spans up to 2 m, to form coffer-type slabs.

The thickness of slabs, depending on spans and loads, will be from 8 to 14 cm, but only 6-7 cm for spans up to 2 m.

Slabs are reinforced with effective bars placed in two directions. Tests carried out on slabs reinforced with rectangularly or diagonally placed bars (Fig. 111, *a* and *b*) have shown that in either case failure occurs at approximately the same ultimate load. However, the rectangular type of bar placement is simpler in practice and is therefore adopted for two-way slabs.

The character of failure of uniformly loaded two-way slabs is illustrated in Fig. 111, *c* and *d*; the cracks on the bottom of the slab

bisect all corners, while those on the top run parallel with the sides and are rounded at the corners. It is very important for carrying-capacity computations and reinforcement design to thus establish the character of failure in two-way reinforced concrete slabs.

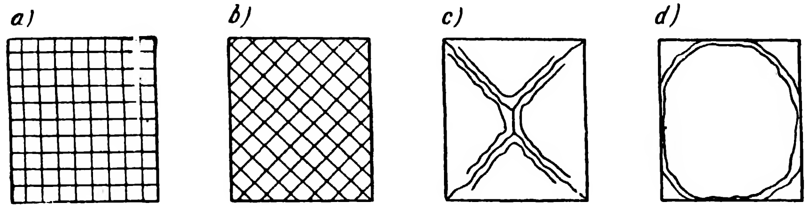


Fig. 111. Schematic layouts of two-way slab reinforcement, and types of failure

7. Computing and Designing of Slabs

Just as in one-way slabs, two-way slabs are also reinforced with prefabricated mats. If the spans are more than 2.5 m, the mats are placed separately in the interspan and at the supports; either a flat prefabricated mat or welded pieces of the roll-type, are placed on the bottom of the interspan, while mats with lateral effective bars are rolled along the beams at the supports.

The bending moment decreases towards the supports, where bar area is correspondingly reduced. To save steel, two mats can be placed one upon the other at the bottom of the slab, one prolonged to the supports and the other confined to midspan (Fig. 112, *a*). With a freely supported slab, $l_{ed}=\frac{1}{8}l_1$. 50% of the bar area is assigned to each of the two above mats.

Roll-type mats are used when slab spans are less than 2.5 m. Mats in which the longitudinal effective bars are 5.5. mm in diameter are rolled across the shorter span of the two-way slab, just as shown in Fig. 106, *a* for one-way slab reinforcement. Mats with lateral bars for resisting the long-span moment at the supports are then rolled on top of the slab and over the beams lying parallel to the latter-mentioned rolling operation. The width of the above mats is taken as $^1/_2$ of the lesser span of the slab.

There will be a greater bending moment in the slab of the first span than in the others; hence, here another mat is rolled over the main one. When the free support lies perpendicular to the way the main mats are being rolled, additional mats are laid down just as in one-way slabs (Fig. 106, *b*). But if the direction of the free support coincides with that of the rolling operation, then the additional mats are placed over the main one along the entire first strip, as

shown in Fig. 112, *b*. When both the main and additional mats have been rolled, additional bars running perpendicular to the effective bars of the additional mats are placed in the corner panels of the slab. These bars are to be extended from the first span through the first intermediate support and beyond that to a point equal to $^1/_4$ of the smaller span (Fig. 112, *c*). After this another mat, extended from the first span, is rolled over the support and on top of the bars.

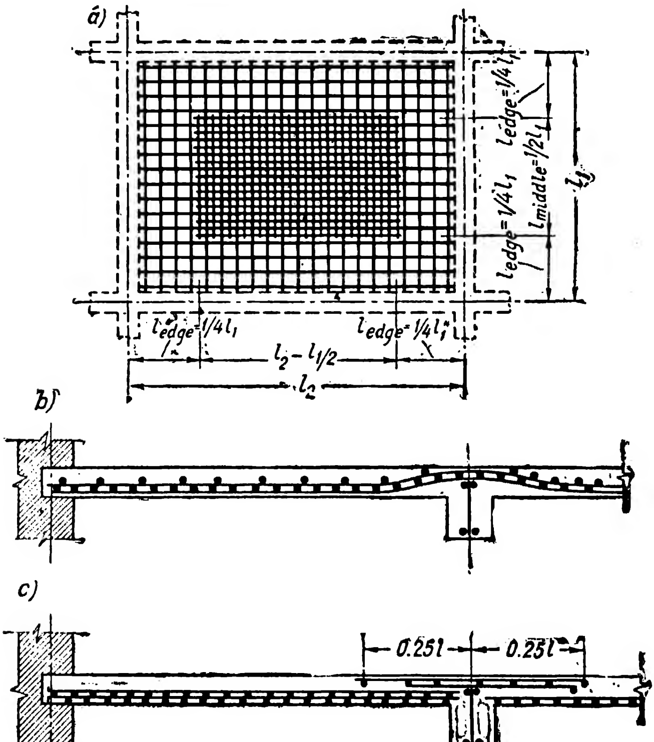


Fig. 112. Reinforcement of two-way slabs

If lateral effective bars are required in the roll mats, separate mats are used just as in one-way slabs and the mats are rolled in two directions, perpendicular to each other.

Computations of two-way slabs are conducted by the Ultimate Equilibrium Method of Professor A. A. Gvozdyev, in which the slab is considered as a train of members linked to each other at the planes of fracture of the plastic pivots created in the interspan and at the supports (Fig. 113,*a*). The values of the bending moments in

the slab will depend upon the areas of bars intersected by plastic pivots. Bending moments of uniformly distributed continuous two-way slabs are evolved from the following equations, depending upon the type of reinforcement:

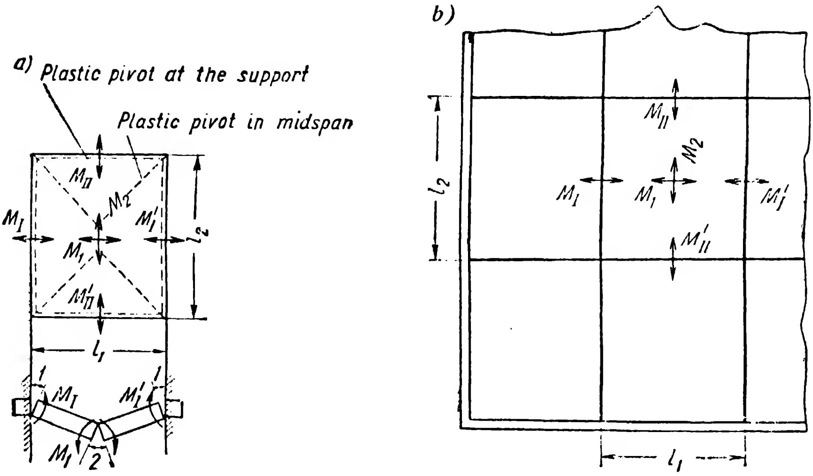


Fig. 113. Computing a two-way slab

when the reinforcement consists of flat prefabricated mats and $l_{ed}=\frac{1}{4}l_1$

$$\frac{(g+p)\,l_1^2}{12}(3l_2-l_1)=l_2(2M_1+M_\mathrm{I}+M'_\mathrm{I})+$$
$$+l_1\left(\frac{3}{2}M_2-\frac{1}{2}M_1+M_\mathrm{II}+M'_\mathrm{II}\right). \qquad (221)$$

Ditto and when $l_{ed}=\frac{1}{8}l_1$

$$\frac{(g+p)\,l_1^2}{12}(3l_2-l_1)=l_2(2M_1+M_\mathrm{I}+M'_\mathrm{I})+$$
$$+l_1\left(\frac{7}{4}M_2-\frac{1}{4}M_1+M_\mathrm{II}+M'_\mathrm{II}\right). \qquad (221a)$$

When the reinforcement consists of roll-type mats with longitudinally placed effective bars

$$\frac{(g+p)\,l_1^2}{12}(3l_2-l_1)=l_2(2M_1+M_\mathrm{I}+M'_\mathrm{I})+$$
$$+l_1(M_2-M_1+M_\mathrm{II}+M'_\mathrm{II}). \qquad (221b)$$

When the reinforcement consists of roll-type mats with laterally placed effective bars

$$\frac{(g+p)\,l_1^2}{12}\,(3l_2-l_1)=l_2\,(2M_1+M_{\rm I}+M'_{\rm I})+l_1\,(2M_2+M_{\rm II}+M'_{\rm II}). \qquad (221c)$$

The design moments per unit width of cross-section are entered into the right halves of equations (221)-(221c), i.e., two interspan moments M_1 and M_2 and four moments at the supports $M_{\rm I}$, $M'_{\rm I}$, $M_{\rm II}$, $M'_{\rm II}$ (Fig. 113, *b*). By making use of definite correlations between design moments within the limits shown in Table 21, the whole problem is brought down to solving one unknown M_1.

Table 21

Allowable Limits in Correlations Between Design Moments in Two-Way Slabs

$\frac{l_2}{l_1}$	$\frac{M_2}{M_1}$	$\frac{M_{\rm I}}{M_1}$ and $\frac{M'_{\rm I}}{M_1}$	$\frac{M_{\rm II}}{M_1}$ and $\frac{M'_{\rm II}}{M_1}$
1-1.5	0.2-1	1.3-2.5	1.3-2.5
1.5-2	0.15-0.5	1-2	0.2-0.75

The effective spans l_1 and l_2 are assumed as the clear distance between beams, or, with a free end support, the distance from the centre of the wall support to the edge of the beam.

If the slab has one or several free supports, then the corresponding moments at the supports in equations (221)-(221c) are assumed as being equal to zero.

Areas of bars are found according to formulae and tables that apply to rectangular sections. Slab thicknesses are first computed on the basis of required rigidity, as follows: not less than $^1/_{50}\,l_1$ when the support is elastic, and not less than $^1/_{45}l_1$ with a free support; when light concretes are used, the corresponding minimum ratios will be $^1/_{42}\,l_1$, and $^1/_{38}\,l_1$, in which l_1 represents the lesser span of the slab. Effective bars placed along the lesser span are installed beneath those running along the greater span. For this reason the effective height of the slab in each direction will differ by a value equal to the diameter of the bars.

8. Computation and Design of Beams

A continuous floor beam is one supported by walls and columns (Fig. 114,*a*). The load from the slab is transmitted to it either by the principle of the triangle or that of the trapezoid.

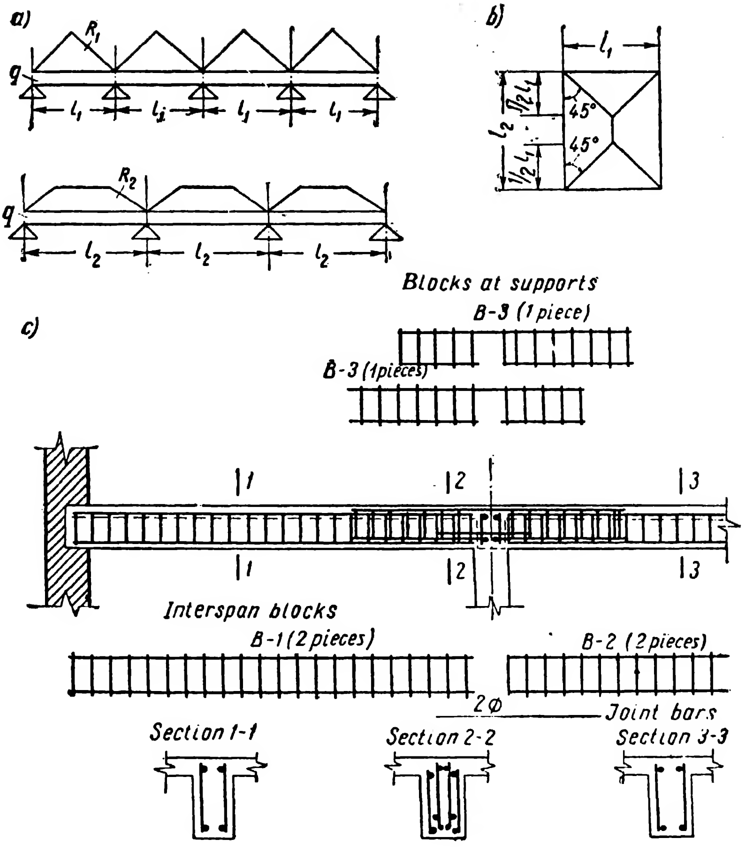


Fig. 114. Design diagrams and reinforcement delineations for beams supporting two-way slabs

To determine the amount of the load, the bisectrices of the panel (in plan) are extended to their intersections, and the corresponding triangles (or trapezoids) will give the loads needed for beam computation (Fig. 114, *b*). Multiplication of the load $g + p$ (per m^2) by the involved floor area will give the total load from both sides of the span of the beam:
if the span is l_1,

$$R_1 = \frac{(g+p)\,l_1^2}{2};$$

if the span is l_2,

$$R_2 = \frac{(g+p)\,l_1\,(2l_2 - l_1)}{2}.$$

In a freely supported beam the corresponding bending moments from such a loading will be

$$M_1=\frac{(g+p)\,l_1^2}{12}\,; \tag{222}$$

$$M_2=\frac{(g+p)\,l_1\,(3l_2^2-l_1^2)}{24}\,. \tag{222a}$$

Moreover, account must be taken of the uniform load q of the beam itself and part of the floor carrying the live load, as determined by a loaded strip with a width b.

Continuous beams are computed on the basis of moment redistribution caused by plastic deformation. The respective moments will be:
in the first span and at the first intermediate support

$$M=0.7M_0+\frac{ql^2}{11}\,;$$

in the intermediate spans and at the intermediate supports

$$M=0.5M_0+\frac{ql^2}{16}\,.$$

In the above, M_0 is obtained through formulae (222) and (222a).

The effective spans of the beams will be equal to the clear distance between columns, or between the centre of the wall support (if it is a free support) to the edge of the first column. To simplify calculations, the effective span of beams is assumed as equal to the clear slab span between ribs (which gives a slightly exaggerated beam span). In a triple-span beam the moment in the central span must be taken as not less than that in a restrained beam:

$$M=0.4M_0+\frac{ql^2}{24}\,.$$

Negative moments in the interspans are obtained by plotting a diagram that corresponds to the dead loads in all the spans and the live loads in every other span, with the curve passing through the apexes of the ordinates of design moments at the supports.

Shearing forces in continuous beams are determined by the following formulae:

$$Q_A=0.5\,(R+ql)-\frac{M_B}{l}\,,$$

$$Q_B^L=0.5\,(R+ql)+\frac{M_B}{l}\,,$$

$$Q_B^R=Q_C=0.5\,(R+ql),$$

in which M_B is the bending moment at the first intermediate support.

The sequence in determining cross-sections of beams and the principles of their reinforcement are the same as for girders of one-

way ribbed slabs. Since the support reinforcement of perpendicular beams meet a the column, the spacing bars of the support reinforcement are interrupted here (Fig. 114, *c*) and additional connecting bars placed during erection.

A simplified method is used to compute coffer-slab beams that are supported on walls without intermediate columns (Fig. 115, *a*) and whose spans are l_1 and l_2. Given a and b as the respective dis-

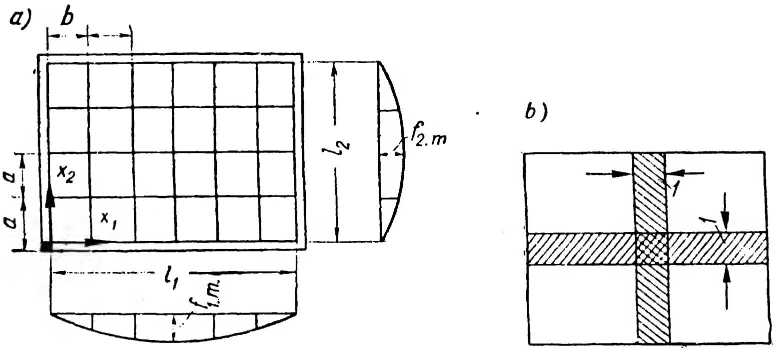


Fig. 115. Investigation of beams in coffer-type slabs

tance between beams in both directions, the bending moments for the beams in the centre of the slab will be

$$M_1 = \frac{q_1 a l_1^2}{8} \text{ and } M_2 = \frac{q_2 b l_2^2}{8},$$

where q_1 and q_2 are parts of the load q per m^2 transmitted in each direction, respectively, i.e.,

$$q = q_1 + q_2. \qquad (223)$$

The amounts of the loads q_1 and q_2 are based on the assumption that deflection will be alike for two perpendicular bands of the slab as represented by one and the same point (Fig. 115, *b*).

Deflection in mid-span of a band whose width is represented as 1 will be as follows:

$$f = \frac{5}{384} \times \frac{q_1 l_1^4}{EJ_1} = \frac{5}{384} \times \frac{q_2 l_2^4}{EJ_2}.$$

Inasmuch as with equal heights of beams the moments of inertia J_1 and J_2 of the bands in each direction of the slab will be equal to each other, then

$$q_1 l_1^4 = q_2 l_2^4. \qquad (224)$$

The mutual solution of equations (223) and (224) gives

$$q_1 = q\frac{l_2^4}{l_1^4 + l_2^4} \text{ and } q_2 = q\frac{l_1^4}{l_1^4 + l_2^4}.$$

Deflection and bending moments of beams at the margins of the system will be less than the above. Deflection in a freely supported beam at any plane situated at a distance x from the support

$$f = \frac{16}{5}(\alpha - 2\alpha^3 + \alpha^4) f_m = kf_m,$$

where f_m is the deflection of the same beam in mid-span;

$$\alpha = \frac{x}{l}.$$

By assuming that a tentative value of the moment of the periphery beam situated at a distance x from the slab's margin will be proportional to its deflection, we find that

$$M_{1x} = k_1 M_1, \text{ and } M_{2x} = k_2 M_2,$$

where

$$k_1 = \frac{16}{5}(\alpha_2 - 2\alpha_2^3 + \alpha_2^4); \quad \alpha_2 = \frac{x_2}{l_2};$$

$$k_2 = \frac{16}{5}(\alpha_1 - 2\alpha_1^3 + \alpha_1^4); \quad \alpha_1 = \frac{x_1}{l_1}.$$

This approach to the solution is also valid when no beam aligns with either of the axes of the floor in plan.

Sec. 27. AN EXAMPLE IN THE DESIGN OF A PRECAST FLOOR

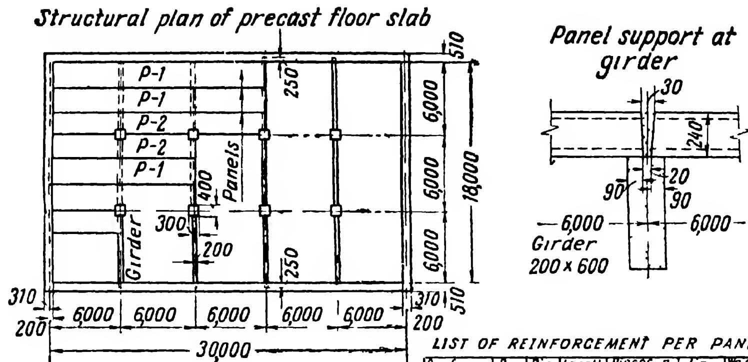
(Illustrative problem 19)

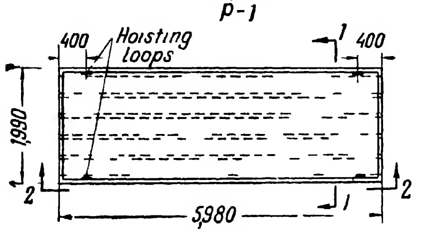
Design a precast ribbed floor for an 18×36 m room in a civic building where the architectural design dictates a 6×6 m column grid. Exterior walls are to be 51 cm thick of pre-laid brick blocks. Flooring will be of tiling and the specified live load equal to 400 kg/m^2, with a load coefficient $n = 1.4$. Ultimate weight of any precast member is established as 3 t, which is the lifting capacity of the building crane on the site.

1. *Structural diagram of the floor.* The longitudinal walls of the building are to have wide windows, on whose spandrels it is not advisable to rest floor panels or beams, hence the girders are to run laterally and the panels longitudinally (Fig. 116). Panel dimensions are established from the aforementioned dictated structural weights: cavity panels weigh 250 kg/m^2, therefore a 3-ton panel 6 metres long will have a width of $\frac{3{,}000}{250 \times 6} = 2$ m.

Hence, each panel, 200 cm wide, will have five oval cavities, each one 335 mm wide. In order to achieve only one size of panel width, lateral grid spacing is to be centred from interior wall surfaces.

2. *Panels.* The panels will rest on the tops of the girders. To establish the panel's effective span, a tentative girder cross-section is assumed as $h = {}^1/_{10} \times 600 = 60$ cm, and $b = 20$ cm. Then the effective span of the panels $l_0 = 6 - \frac{0.2}{2} = 5.9$ m. The design diagram of the panel will be denoted as a uniformly loaded single-span beam.





LIST OF REINFORCEMENT PER PANEL

Reinforcement	Bar No	Dia, mm	Length, mm	Pieces in: Per unit	Pieces in: Total	l in m	Weight per m	Total weight, kg
B 1 6 pieces	1	Ø18D	5,960	1	6	35 76	1 998	71 2
	2	Ø8	5,960	1	6	35 76	0 395	14 1
	3	Ø6	220	32	192	42 24	0 222	9 4
Mat 1 2 pcs	4	Ø3	5,950	8	16	95 20	0 055	5 2
	5	Ø3	960	40	87	156 80	0 055	8 6
Separate bars	6	Ø12	1970	—	2	3 94	0 888	3 5
	7	Ø10	800		4	3 20	0 617	2 0

Total steel 116 kg

Concrete grade 200 Concrete volume 1 24 m3

Panel weight 3 100 kg Metal content K $\frac{116}{1.24}$ 93 5 kg/m

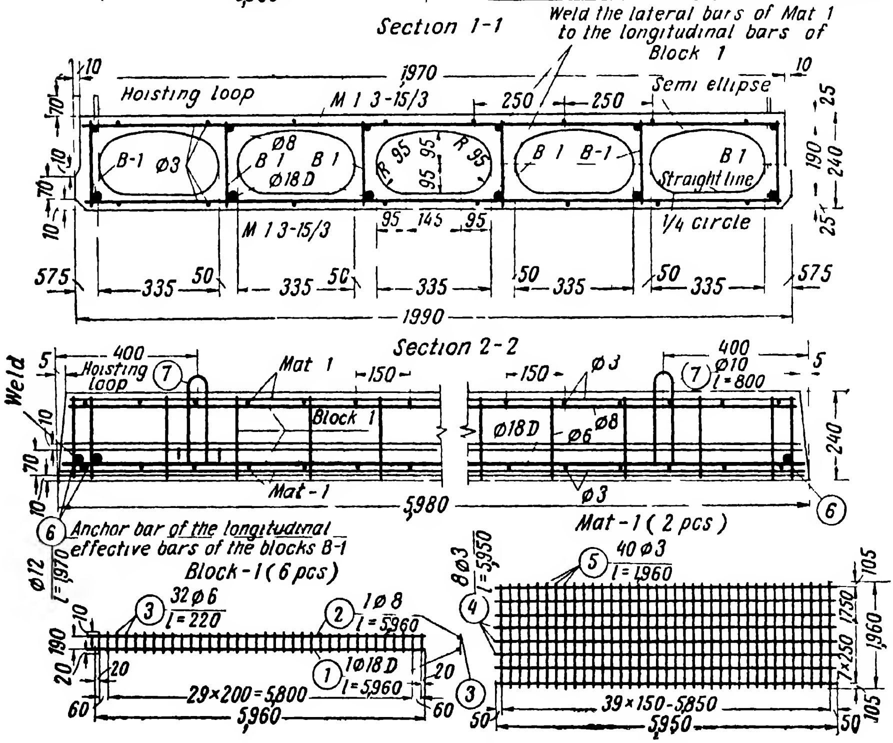


Fig. 116. Illustrating problem 19 plan of a precast floor and details of the panel

Ascertaining loads

cified dead load per m^2 of the slab:

Dead weight of the panels		250 kg/m²
Cement bed beneath tile flooring . .	$0.025 \times 2{,}200 = 55$	kg/m²
Tile flooring		35 kg/m²
	Total	340 kg/m²
Design dead load	$340 \times 1.1 = 374$	kg/m²
Design live load	$400 \times 1.4 = 560$	"
Full design load		935 kg/m²

Design load on 1 linear metre of the panels, 2 m wide:

$$q = 2 \times 935 = 1{,}870 \text{ kg/linear m}.$$

Specified load per linear metre of panel for computation of deflection:

Constant load . . . . . $g^s = 2 \times 340 = 680$ kg/linear m
Momentary load . . . . $p^s = 2 \times 400 = 800$ " " "

$$q^s = 1{,}480 \text{ kg/linear m}$$

2) *Design forces*

$$M = \frac{1{,}870 \times 5.9^2}{8} = 8{,}200 \text{ kg/m};$$

$$Q = \frac{1{,}870 \times 5.9}{2} = 5{,}500 \text{ kg}.$$

3) *Gathering design data.* Service coefficient of the panel $m = 1.1$. Grade 200-B concrete, $R_{be} = 100$ kg/cm²; $R_t = 6.4$ kg/cm², specified modulus of elasticity $E_c^s = 290{,}000$ kg/cm².

Bars: prefabricated frames (latticework) with St-5 intermittently deformed effective bars, $R_s = 2{,}400$ kg/cm², $m_s = 1$; lateral and spacing bars of round St-3 steel, $R_s = 2{,}100$ kg/cm², $m_s = 1$; $m_{dl} = 0.8$, mats of cold-drawn wire $d \leq 5.5$ mm, $R_s = 4{,}500$ kg/cm², $m_s = 0.65$.

Coefficient of rigidity reduction due to constant loading of cavity-type anels $\Theta = 2$; coefficient allowance for good rigidity of cavity-type panels $= 1.2$; aximum panel deflection $f \leq 1/200\ l$.

4) *Tentative height of panel cross-section*

From formula (208)

$$h = 22 \times 590 \frac{1.1 \times 1 \times 2{,}400}{2.1 \times 10^6} \times \frac{680 \times 2 + 800}{1{,}480} = 23{,}4 \text{ cm}.$$

Assume that $h = 24$ cm, $h_0 = 21$ cm, thickness of upper and lower flanges, 2.5 cm each. Absolute width of ribs $b = 200 - 5 \times 33.5 = 32$ cm.

The transformed cross-section of rectangular cavities will be

$$0.95h_1 = 0.95 \times 19 = 18 \text{ cm}; \quad 0.95b_1 = 0.95 \times 33.5 = 32 \text{ cm}.$$

The width of the rib of the design T-section $b = 200 - 5 \times 32 = 40$ cm; width of flange $b_{fl} = 200$ cm; thickness of compression flange $h_{fl} = 2.5 + (19 - 18) = 3.5$ cm; thickness of tension flange $= 2.5$ cm.

5) *Deriving longitudinal bar area by means of the bending moment.*

The relationship of $\frac{h_{fl}}{h}=\frac{3.5}{24}=0.145>0.1$; therefore the entire width of the flange ($b_{fl}=200$ cm) is entered into the computations;

$$A_0=\frac{820{,}000}{1.1\times 100\times 200\times 21^2}=0.085;$$

from Table 10: $\alpha=0.9$, $\gamma_0=0.96$; $x=0.09\times 21=1.9$ cm $<h_{fl}=3.5$ cm;

$$F_s=\frac{820{,}000}{1.1\times 1\times 2{,}400\times 0.96\times 21}=15.4 \text{ cm}^2,$$

which is satisfied by 6ø18 D ($F_s=15.27$ cm²) (—0.9%).

6. *Resistance to shear at diagonal planes*

$$mR_tbh_0=1.1\times 6.4\times 32\times 21=4{,}700<Q=5{,}500 \text{ kg};$$

hence, stirrups are required.

The required value

$$q_{st}=\frac{\left(\frac{5{,}500}{1.1}\right)^2}{0.6\times 100\times 32\times 21^2}=30.5 \text{ kg/linear cm}.$$

A block with 18 mm effective bars on only one face is chosen for each rib (6 in all); according to Table 4, 6 mm must be the minimum diameter of the stirrups, the structural spacing of which $u=20$ cm. Then,

$$q_{st}=\frac{0.8\times 1\times 2{,}100\times 6\times 0.5}{20}=250>30.5 \text{ kg/linear cm}.$$

7) *Computing deflection of the upper flange perpendicular to the ribs*

The span $l_0=0.335$ m, and the bending moment

$$M=\frac{935\times 0.335^2}{11}=9.6 \text{ kg/m}.$$

$$h_0=1.2 \text{ cm}.$$

$$A_0=\frac{960}{1.1\times 100\times 100\times 1.2^2}=0.06.$$

From Table 10 $\gamma_0=0.97$; then

$$F_s=\frac{960}{1.1\times 0.65\times 4.500\times 0.97\times 1.2}=0.26 \text{ cm}^2$$

which is satisfied by a mat of bars $d=3$ mm, spaced 250 mm ($F_s=0.29$ cm²).

8) *Computing panel rigidity*

$$n=\frac{2{,}100{,}000}{290{,}000}=7.25; \qquad \mu=\frac{15.27}{40\times 21}=0.0182;$$

$$\alpha=3\times 0.0182\times 7.25=0.4; \qquad \gamma'=\frac{(200-40)\ 3.5}{40\times 21}=0.67;$$

$$\gamma_1=\frac{(200-40)\ 2.5}{40\times 21}=0.48.$$

From Table 13 $\eta=0.92$; $c=0.65$.

The bending moment from the specified load

$$M^s=\frac{1{,}480\times 5.9^2}{8}=6{,}500 \text{ kg/m}.$$

Stresses in the bars

$$\sigma_s=\frac{650{,}000}{0.91\times 21\times 15.27}=2{,}200 \text{ kg/cm}^2,$$

From Table 12 $\psi=0.88$, and the rigidity

$$\frac{2.1\times10^6}{0.88}\,0.65\times15.27\times21^2=12.6\times10^9\ \text{kg/cm}^2;$$

while with a constant load

$$B=12.6\times10^9\,\frac{1{,}480}{680\times2+800}=8.6\times10^9\ \text{kg/cm}^2.$$

9) *Panel deflection*

$$f=\frac{5\times14.8\times590^3}{384\times8.6\times10^9}=\frac{1}{218}\,l<1/200l.$$

10) *Checking panel behaviour during erection handling*

Four hoisting loops (eyes) made of round St-3 steel are inserted at a distance of 0.7 m from the panel ends. Considering a dynamic-load coefficient of 1.5 (from the panel's dead weight), then

$$q=1.5\times1.1\times250\times2=825\ \text{kg/linear m}.$$

The negative moment of the overhang $M=825\,\dfrac{0.7^2}{2}=207$ kg/m.

This negative moment is borne by the longitudinal spacing bars of the reinforcement blocks, while the upper horizontal mats function as temporary spacing steel.

Assuming that $\gamma_0\approx0.9$, the required bar area

$$F_s=\frac{20{,}700}{2{,}100\times0.9\times21}=0.52\ \text{cm}^2,$$

which is less than the actual area of the embedded steel, the latter being $6\,\text{ø}\,6\,(F_s=1.68\ \text{cm}^2)$.

If the panel should be raised by means of only two loops, then the force on one loop

$$N=\frac{825\times6}{2}=2{,}475\ \text{kg},$$

and the force N will be taken by only one leg of this loop, which means that

$$F_s=\frac{2{,}475}{1\times2{,}100}=1.18\ \text{cm}^2,$$

which is satisfied by a rod, $d=12$ mm, whose area (of one leg) $F_s=1.13\ \text{cm}^2$ (—4%).

The working drawing of the panel is shown in Fig. 116.

3. Girder Computation

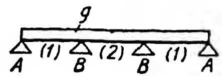
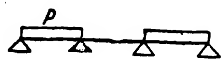
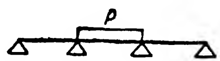
The girder consists of a triple-span continuous member with its ends freely supported by walls and its load uniformly received from the slab panels. The central span $l_2=6$ m between column centres; the end span, from the centre of the wall support to the centre of the column, $l_1=6+\dfrac{0.25}{2}=6.13$ m. The difference in span dimensions is $2\%<20\%$.

1) *Determining the load per linear metre of girder*

Specified dead loads:

from slab panels and flooring	$340\times6=2{,}040$ kg/linear m
from dead weight of girder	$0.2\times0.6\times2{,}500=300$ kg/linear m
	$g^s\approx2{,}350$ kg/linear m

Table 22

Scheme No.	Loading scheme under consideration	Bending moment M_1	Bending moment M_2	Bending moment M_B
1		$0.08\times2,600\times \times6.13^2=7,800$	$0.025\times2,600\times \times6^2=2,350$	$-0.1\times2,600\times \times\left(\frac{6.13+6}{2}\right)= =-9,550$
2		$0.101\times3,360\times \times6.13^2=12,700$	$-0.05\times3,360\times \times6^2=-6,050$	$-0.05\times3,360\times \times\left(\frac{6.13+6}{2}\right)^2= =-6,200$
3		$-0.025\times3,360\times \times6.13^2=-3,150$	$0.075\times3,360\times \times6^2=9,100$	$-0.05\times3,360\times \times\left(\frac{6.13+6}{2}\right)^2= =-6,200$
4		—	—	$-0.117\times3,360\times \times\left(\frac{6.13+6}{2}\right)^2 =-14,500$
5	The worst combination	$7,800+12,700= =20,500$	$2,350+9,100= =11,450$ $2,350-6,050= =-3,700$	$-9,550-14,500= =-24,050$

Dead design load	$g=2{,}350\times1.1=2{,}600$ kg/linear m
Specified live load	$p^s=400\times6=2{,}400$ " " "
Live design load	$p=2{,}400\times1.4=3{,}360$ " " "
Full design load	$q=2{,}600+3{,}360=5{,}960$ " " "

2) *Computing bending moments and shearing forces*

Bending moments and shearing forces are determined according to data given in Supplement II. Bending moments for various loading schemes may be established through Table 22.

The girder is calculated on the basis of moment redistribution due to plastic deformation. The equalising design moment-diagrams correspond to the lines M of loading schemes 1, 2 and 1, 3 and give the greatest interspan moments. The equalised design moment at the support

$$M=-9{,}550-6{,}200=-15{,}750\ \text{kg/m},$$

in which case

$$\frac{M}{M_{\text{H}}}=\frac{-15{,}750}{-24{,}050}=0.66<0.7.$$

Shearing forces for design loading schemes:
for Schemes 1 and 2

$$Q_A=(0.4\times2{,}600+0.45\times3{,}360)\,6.13=15{,}600\ \text{kg};$$

$$Q_B^L=-(0.6\times2{,}600+0.55\times3{,}360)\,6.13=-21{,}000\ \text{kg};$$

$$Q_B^R=0.5\times2{,}600\times6=7{,}800\ \text{kg};$$

for Schemes 1 and 3

$$Q_A=(0.4\times2{,}600-0.05\times3{,}360)\,6.13=5{,}300\ \text{kg};$$

$$Q_B^L=-(0.6\times2{,}600+0.05\times3{,}360)\,6.13=-10{,}700\ \text{kg};$$

$$Q_B^R=(0.5\times2{,}600+0.5\times3{,}360)\,6=17{,}900\ \text{kg}.$$

Assuming that $h_{\text{col}}=40$ cm, the moment at the edge of the column (support moment)

$$M_{\text{edge}}=-15{,}750+7{,}800\,\frac{0.4}{2}=-14{,}200\ \text{kg/m}.$$

3) *Gathering working data.*

Service coefficient $m=1.1$.
Concrete: grade 200-B; $R_{\text{be}}=100$ kg/cm²;

$$R_t=6.4\ \text{kg/cm}^2;\ R_{\text{pr}}=80\ \text{kg/cm}^2.$$

Bars: prefabricated blocks with intermittently deformed St-5 effective steel, $R_s=2{,}400$ kg/cm², $m_s=1$; upright and spacing bars of round St-3 steel, $R_s=2{,}100$ kg/cm², $m_s=1$, $m_{\text{dl}}=0.8$.

4) *Determining girder height.*

Cross-sectional height is determined through the moment at the support, considering that $\alpha=0.3$ (inasmuch as plastic deformation enters the calculations) and assuming that the width $b=20$ cm:

$$h_0=2\sqrt{\frac{1{,}420{,}000}{1.1\times20\times100}}=51\ \text{cm};$$

$a=5$ cm, $h=51+5=56$ cm, which is therefore fixed as 60 cm and in which case $h_0=55$ cm.

5) *Computing longitudinal bar area through bending moments.*

The area in the first span

$$A_0=\frac{2,050,000}{1.1\times100\times20\times55^2}=0.31;\ \gamma_0=0.85;$$

$$F_s=\frac{2,050,000}{1.1\times1\times2,400\times0.85\times55}=16.6\ \text{cm}^2,$$

which is satisfied by 2ø20 D + 2ø25 D ($F_s=16.1$ cm², or 3% less than required), prefabricated (welded) into 2 flat blocks.

The area in the central span

$$A_0=\frac{1,145,000}{1.1\times100\times20\times55^2}=0.17;\ \gamma_0=0.9;$$

$$F_s=\frac{1,145,000}{1.1\times1\times2,400\times0.9\times55}=8.8\ \text{cm}^2,$$

which is satisfied by 2ø20 D + 2ø14 D ($F_s=9.36$ cm², or 6.5% more than required), also prefabricated into two blocks.

The area to gratify the negative moment

$$A_0=\frac{370,000}{1.1\times100\times20\times55}=0.055;\ \gamma_0=0.97;$$

$$F_s=\frac{370,000}{1.1\times1\times2,400\times0.97\times55}=2,6\ \text{cm}^2,$$

which is satisfied by supplementary steel, 2ø18 D ($F_s=5,09$ cm²).

The area at the support

$$A_0=\frac{1,420,000}{1.1\times100\times20\times55^2}=0.214;\ \gamma_0=0.88;$$

$$F_s=\frac{1,420,000}{1.1\times1\times2,400\times0.88\times55}=11.2\ \text{cm}^2,$$

which is satisfied by 2ø20 D + 2ø18 D ($F_s=11.37$ cm², or 1.5% in excess of the computed area), prefabricated into two blocks.

6) *Shearing forces at the diagonal planes.*

At the left side of the first intermediate support, $mR_tbh_0=1.1\times6.4\times20\times\times55=7,800$ kg$<Q_B^l=21,000$ kg; hence computations show that upright bars are required; therefore

$$q_{st}=\frac{\left(\frac{21,000}{1.1}\right)^2}{0.6\times100\times20\times55^2}=101\ \text{kg/linear cm}.$$

With a unilateral placement of 25 mm effective bars in the frame, the upright bars must be 8 mm in diameter in accordance with Table 4 (area $f_{st}=0.5$ cm²).

Spacing of upright bars

$$u=\frac{0.8\times1\times2,100\times2\times0.5}{101}=16.6\ \text{cm};$$

$$u=\frac{1.1\times0.1\times100\times20\times55^2}{2,100}=32\ \text{cm};$$

$$u=\frac{60}{2}=30\ \text{cm}.$$

The spacing taken for the upright bars at the support $u=15$ cm.

At a distance of one metre from the support, $Q=21{,}000-1\times5{,}960=$ $=15{,}040$ kg, therefore the required spacing

$$u=16.6\left(\frac{21{,}000}{15{,}040}\right)^2=32.4 \text{ cm},$$

in which case it is fixed at $u=30$ cm.

At the last support $Q_A=15{,}600$ kg$>mR_t bh_0=7{,}800$ kg, which indicates that here also upright bars are required, and

$$q_{st}=\frac{\left(\frac{15{,}600}{1.1}\right)^2}{0.6\times100\times20\times55^2}=55 \text{ kg/linear cm}.$$

When the upright-bar spacing $u=30$ cm, the required bar area

$$f_{st}=\frac{30\times55}{0.8\times1\times2{,}100\times2}=0.49 \text{ cm}^2.$$

$$\text{Actually, } f_{st}=0.5 \text{ cm}^2>0.49 \text{ cm}^2.$$

The protrusion of the 25 mm effective bars beyond the face of the support, when the concrete is grade 200 and the bars are intermittently-deformed, $l_{pro}=10\times2.5=25$ cm, which is satisfactory.

At the right side of the first intermediate support $Q_B^R=17{,}900$ kg, and

$$q=\frac{\left(\frac{17\,900}{1.1}\right)^2}{0.6\times100\times20\times55^2}=72 \text{ kg/linear cm}.$$

With a unilateral placement of 20 mm effective bars in the reinforcement frame, the upright bars must be 8 mm in diameter in accordance with Table 4 (area $f_{st}=0.5$ cm^2).

Spacing of upright bars

$$\frac{0.8\times1\times2{,}100\times2\times0.5}{72}=23.3 \text{ cm}.$$

$$u=\frac{1.1\times0.1\times100\times20\times55^2}{17\,900}=37.6 \text{ cm};$$

$$u=\frac{60}{2}=30 \text{ cm}$$

the final spacing being established as $u=25$ cm ≈ 23.3 cm.

7) *The column-girder joint.*

A reinforced concrete bracket on the column is proposed as a support for the girder. The vertical distance between the centres of gravity of the steel insertions

$$z=60-7=53 \text{ cm}.$$

Tensile forces

$$+N=\frac{14\,200}{0.53}=26{,}800 \text{ kg}.$$

Area of the upper bars placed in the column

$$F_s=\frac{26{,}800}{1.1\times1\times2{,}400}=10.2 \text{ cm}^2,$$

which is satisfied by 2ø25 D ($F_s=9.82$ cm^2, which is 3.7% less than required).

To weld these bars, a weld joint is assumed as $h_{jo}=\frac{1}{4}\times25\approx6$ mm; from formula (215) it is found that

$$\Sigma l_{jo}=\frac{1.3\times26{,}800}{0.85\times1\times0.6\times1{,}400}=49 \text{ cm}.$$

Each of two bars is fixed by a double-sided weld, therefore $l_{jo}=\frac{49}{2\times2}=$ $=12.3$ cm, which. if possible poor weld penetration be allowed for, is increased to $l_{jo}=13.3\approx14$ cm. The protrusion of the bars, with an allowance for joint space between the column and girder, will be $14+1.5\approx16$ cm.

The overhang (projection) of the bracket is fixed structurally as 20 cm, and the width, both of column and bracket, $b=30$ cm.

The height of the supporting bracket

$$h_0=\frac{21{,}000}{\frac{1.1}{6}100\times30}=38 \text{ cm};$$

$h=38+3.5=41.5$ cm. The final $h=45$ cm, in which case $h_0=41.5$ cm.

Since the bracket's overhang $l=20<0.9\times41{,}5=37{,}5$ cm. the bracket falls into the *short* category. When $\alpha=45°$, the required area of inclined bars

$$F_0=\frac{21{,}000}{2\times1.1\times0.8\times1\times2{,}400\times0.71}=7 \text{ cm}^2,$$

which is satisfied by 2ø22 D ($F_s=7.6$ cm²).

The bending moment in the bracket

$$M_{br}=\frac{2}{3}\,0.2\times21{,}000\doteq2{,}800 \text{ kg/m}.$$

Area of longitudinal horizontal bars in the bracket,

$$F_s=\frac{1}{1.1\times2{,}400}\left(26{,}800-\frac{280{,}000}{0.9\times41.5}\right)=7.3 \text{ cm}^2,$$

which is satisfied by 2ø22 D ($F_s=7.6$ cm²).

Area of bearing plate of St-3 steel

$$F_{pl}=\frac{26{,}800}{1.1\times2{,}100}=11.6 \text{ cm}^2.$$

The connecting plate, welded to the butt end of the girder's longitudinal bars, must have the same area as the above plate.

The welded joint for fixing the girder's connecting plate to the bearing plate, when $h_{jo}=12$ mm, must have a total length

$$\Sigma l_{jo}=\frac{1.3\times26{,}800}{0.7\times1\times1.2\times1{,}400}=29.5 \text{ cm}.$$

The length of each of the two welded joints, with an addition for possible poor penetration,

$$l_{jo}=\frac{29.5}{2}+1\approx16 \text{ cm}.$$

For structural reasons the thickness of the bearing plate is taken as $\frac{12}{1.5}=8$ mm, in which case its cross-sectional area

$$F_{pl}=0.8\times20=16>11.6 \text{ cm}^2.$$

Fig. 117 shows the working drawing of the girder.

8) *Behaviour of girder during erection handling.*
Checking the middle span against a possible negative moment after the welding of the steel insertions at the supports have created girder continuity and the slab panels have been placed only on the end spans of the girder.

Design load in the end spans will be,
dead weight of girder

$$0.2\times0.6\times2{,}500\times1.1=330 \text{ kg/linear m};$$

ditto, slab panels

$$250\times6\times1.1=1{,}650 \text{ kg/linear m};$$

live load during erection, including a load coefficient of 1.3 and a lowered coefficient of 0.9 in case of additional load combinations

$$100\times1.3\times0.9\times6=700 \text{ kg/linear m}.$$

Total: 2,680 kg/linear m

Design load in the central span:
dead weight of girder 330 kg/linear m.
Negative moment in the central span

$$M=(-0.05\times2{,}680+0.075\times330)\,6^2=-3{,}900 \text{ kg/m}.$$

The upper bars in the girder's central span are 2⌀18 D (F_s=5.09 cm²);

$$\alpha=\frac{5.09}{20\times55}\times\frac{1\times2{,}400}{100}=0.11; \quad \gamma_0=0.95;$$

$$M_{cs}=1.1\times1\times2{,}400\times5.09\times0.95\times55=700{,}000 \text{ kg/cm}=7{,}000 \text{ kg/m}>3{,}900 \text{ kg/m}.$$

Sec. 27a. ALTERNATE DESIGN: AN IN-SITU FLOOR

1. **Structural Layout of the Floor.** The lateral orientation of girders (Fig. 117a) is chosen on the same basis as for the precast floor. The cross beams are centered at the columns and at the third's of girder spans. Slab spans l_{slab} are between rib centres and are equal to $\frac{6.0}{3}=2$ m. This 2-metre centering is repeated for the slab at the walls, whose grid centering is measured from the walls' interior surfaces.

The dimensional proportions of the slabs in plan $l_2:l_1=\frac{6}{2}=3>2$, hence we are dealing with a ribbed slab.

2. **Slab Design**: 1) *Design diagram of the slab.*
The continuous ribbed slab is composed of nine spans. A 5-span diagram is to be used for the computations. A tentative cross-section of the cross beams will be: $h=1/15\times600=40$ cm; $b=18$ cm. In the intermediate spans of the slab $l_0=2.00-0.18=1.82$ m; in the first span, if the depth of the bearing is taken as 12 cm, $l_0=2.0-0.2+\frac{0.12}{2}-\frac{0.18}{2}=1.77$ m. Slab thickness will be considered as 7 cm.

2) The *loads.*

Load per 1 m² of floor:

Specified dead load:	
Dead weight of slab, 0.07×2,500	175 kg/m²
Cement layer beneath tiling 0.025×2,220	55 „
Tile flooring	35 „
Total:	265 kg/m²

Design dead load $g = 265 \times 1.1 = 290$ kg/m²
Design live load $p = 400 \times 1.4 = 560$ „
Full design load $q = g + p = 850$ „

Assuming the design width of a strip of slab $b = 1.0$ m, then the load per linear metre $q = 850$ kg/linear m.

3) *Computing bending moments.*

In the first span and at the first intermediate support

$$M = \frac{850 \times 1.77^2}{11} = 245 \text{ kg/m.}$$

In the intermediate spans and at the intermediate supports

$$M = \frac{850 \times 1.82^2}{16} = 177 \text{ kg/m.}$$

4) *Gathering design data.*

Concrete of grade 150-A, $R_{\mathrm{be}} = 85$ kg/cm². Reinforcement of prefabricated $\leqslant 5.6$ mm cold-drawn wire mats, $R_s = 4{,}500$ kg/cm², $m_s = 0.65$ m.

For the intermediate spans $m = 1.25$; for the first span $m = 1.0$ since $\frac{l_{\mathrm{end}}}{l} = 3 > 2$ and there are no beams bordering the slab.

5) *Determining the area of longitudinal bars by means of bending moments.*

In the intermediate spans and at the intermediate supports when $h = 7$ cm and $h_0 = 5.7$ cm,

$$A_n = \frac{17{,}700}{1.25 \times 85 \times 100 \times 5.7^2} = 0.051; \quad \gamma = 0.97;$$

$$F_s = \frac{17{,}700}{1.25 \times 0.65 \times 4{,}500 \times 0.97 \times 5.7} = 0.88 \text{ cm}^2,$$

which is satisfied by roll-type mats, grade 4-15/3, 2,300 mm in width, having longitudinal effective bars whose total area $F_s = 2.14 \text{ cm}^2 > 0.88 \times 2.3 = 2.02 \text{ cm}^2$.

The slabs of the intermediate spans at the end of the room have no beams around their border and here $m = 1.0$; therefore

$$F_s = 1.25 \times 0.88 = 1.1 \text{ cm}^2,$$

which is satisfied by grade 5-20/4 mats, 2,300 mm in width ($F_s = 2.55 \text{ cm}^2 > > 1.1 \times 2.3 = 2.53 \text{ cm}^2$).

In the first span and at the first intermediate support

$$A_n = \frac{24{,}500}{1 \times 85 \times 100 \times 5.7^2} = 0.089; \quad \gamma = 0.95;$$

$$F_s = \frac{24{,}500}{1 \times 0.65 \times 4{,}500 \times 0.95 \times 5.7} = 1.56 \text{ cm}^2,$$

which is satisfied by grade 4-15/3 main mats and grade 4-20/3 additional mats, 2,300 mm in width ($F_s = 2.14 + 1.64 = 3.78 > 1.56 \times 2.3 = 3.58 \text{ cm}^2$).

3. Design of Cross Beams: 1) *Design diagram.* The cross beam forms 6 continuous spans. A 5-span diagram is to be used for the computations. A tentative cross-section of the girder will be: $h = 1/10 \times 600 = 60$ cm; $b = 25$ cm.

In the middle span $l_0 = 6.0 - 0.25 = 5.75$ m; in the first span, if the depth of the wall bearing is taken as 25 cm, $l_0 = 6.0 - 0.2 + \frac{0.25}{2} - \frac{0.25}{2} = 5.8$ m.

2) *The loads*

Specified dead load:	
Slab and flooring, 265×2	530 kg/linear m
Dead weight of cross beam, 0.18×(0.40 – 0.07) 2,500	150 „ „ „
Total:	680 kg/linear m
Design dead load $g=680\times1.1$	750 kg/linear m
Design live load $p=400\times1.4\times2$	1,120 „ „ „

Full design load $q=g+p=1.870$ kg/linear m

3) *Bending moments and shearing forces.*

Bending moments:
in the first span and at the first intermediate support

$$M=\frac{1{,}870\times5.8^2}{11}=5{,}700 \text{ kg/m}$$

in the intermediate spans and at the intermediate supports

$$M=\frac{1{,}870\times5.75^2}{16}=3{,}900 \text{ kg/m}.$$

In accordance with Table 22, when $\frac{p}{g}=\frac{1{,}120}{750}=1.5$, the negative moments in the planes of the middle span are:

$$M_2=-0.041\times1{,}870\times5.75^2=-2{,}550 \text{ kg/m},$$
$$M_3=-0.014\times1{,}870\times5.75^2=-870 \text{ kg/m},$$
$$M_4=-0.008\times1{,}870\times5.75^2=-500 \text{ kg/m},$$
$$M_5=-0.024\times1{,}870\times5.75^2=-1{,}480 \text{ kg/m}.$$

In the first span the distance a from the support B to the point of contraflexure is equal to $0.228l_0=0.228\times5.8=1.32$ m.

Lateral forces:

$$Q_A=0.4\times1{,}870\times5.8=4{,}300 \text{ kg};$$

$$Q_L^B=0.6\times1{,}870\times5.8=6{,}500 \text{ kg};$$

$$Q_B^{\text{interspan}}=0.5\times1{,}870\times5.75=5{,}400 \text{ kg}.$$

4) *Gathering design data.* Grade 150-A concrete, $R_{bc}=85$ kg/cm², $R_t=5.8$ kg/cm².

Reinforcement: in the interspans, prefabricated blocks with effective bars of intermittently deformed St-5 steel, $R_s=2{,}400$ kg/cm², $m_s=1.0$; stirrups and spacing bars in the blocks will be of round St-3 bars, $R_s=2{,}100$ kg/cm², $m_s=1.0$, $m_{di}=0.8$; at the supports, prefabricated mats of $\leqslant5.5$ mm cold-drawn wire, $R_s=4{,}500$ kg/cm², $m_s=0.65$. Service coefficient $m=1.0$.

5) *Computing beam height.* The support moment at the first intermediate support will be considered as $\alpha=0.3$ (with due consideration for plastic deformation) and, assuming that $b=18$ cm, then

$h_0=2\sqrt{\frac{570{,}000}{1\times85\times18}}=38.7$ cm, in which case h is fixed as 40 cm and $h_0=36.5$ cm.

6) *Determining the area of longitudinal bars in accordance with bending moments.*

The cross-section in the first span behaves as a T-beam:

$$\frac{h_{\text{interspan}}}{h} = \frac{7}{40} = 0.175 > 0.1;$$

he full width $b_{\text{interspan}} = 200$ cm and enters the calculations:

$$A_0 = \frac{570{,}000}{1 \times 85 \times 200 \times 36.5^2} = 0.025;$$

$$\gamma_0 = 0.99; \quad \alpha = 0.025; \quad x = 0.025 \times 36.5 = 0.9 \text{ cm} < 7 \text{ cm};$$

$$F_s = \frac{570{,}000}{1 \times 1 \times 2{,}400 \times 0.99 \times 36.5} = 6.55 \text{ cm}^2,$$

which is satisfied by 2ø20 D ($F_s = 6.28$ cm², or 4% less than calculated) to be contained in two flat blocks.

The cross-section in the intermediate spans is considered the same as for the first span:

$$F_s = \frac{390{,}000}{1 \times 1 \times 2{,}400 \times 0.99 \times 36.5} = 4.5 \text{ cm}^2,$$

which is satisfied by 2ø18 D ($F_s = 5.09$ cm², or 13% more than calculated) to be contained in two flat blocks.

The behaviour of the cross-section when effected by a negative moment is that of a rectangular shape because its flange is in the tension zone. In the cross-section situated $0.4l$ from the left support $M_s = 870$ kg/m;

$$A_0 = \frac{87{,}000}{1 \times 85 \times 88 \times 36.5^2} = 0.043; \quad \gamma_0 = 0.98;$$

$$F_s = \frac{87{,}000}{1 \times 1 \times 2{,}100 \times 0.98 \times 36.5} = 1.16 \text{ cm}^2,$$

for which 2ø10 spacing bars have already been provided ($F_s = 1.57 \text{ cm}^2 > > 1.16 \text{ cm}^2$).

The cross-section at the intermediate support behaves as a rectangular shape:

$$A_0 = \frac{570{,}000}{1 \times 85 \times 36.5^2} = 0.28; \quad \gamma_0 = 0.83;$$

$$F_s = \frac{570{,}000}{1 \times 0.65 \times 4{,}500 \times 0.83 \times 36.5} = 6.43 \text{ cm}^2.$$

$\frac{6.43}{2} = 3.22$ cm² are required for each mat when two overlapping mats are used. Required bar area per linear metre of mat width (and 2-metre beam spacing) will be $\frac{3.22}{2} = 1.61$ cm². This is satisfied by a roll-type mat, grade 4/5.5-15, whose lateral bars are also effective ($F_s = 1.58$ cm², which is 2% less than calculated).

The cross-section at the middle supports is considered the same as at the first intermediate support:

$$A_0 = \frac{390{,}000}{1 \times 85 \times 18 \times 36.5^2} = 0.192; \quad \gamma_0 = 0.89;$$

$$F_s = \frac{390{,}000}{1 \times 0.65 \times 4{,}500 \times 0.89 \times 36.5} = 4.1 \text{ cm}^2.$$

$\frac{4.1}{2} = 2.05$ cm² are required for each mat when two overlapping mats are used at the support. Required bar area per linear m of mat width (and 2-metre

beam spacing) will be $\frac{2.05}{2}=1.03$ cm², which is satisfied by the roll-type mat, grade 4/5-20, whose lateral bars are also effective ($F_s=0.98$ cm², which is 5% less than calculated).

7) *Computation of diagonal planes to resist shearing forces.*

At the first intermediate support to the left

$$mR_t bh_0^2 = 1\times 5.8\times 18\times 36.5 = 3,800 \text{ kg};$$

$Q_B^L = 6,500$ kg$>mR_t bh_0 = 3,800$ kg; this means that upright bars are required;

$$q_{st}=\frac{6,500^2}{0.6\times 85\times 18\times 36.5^2}=36 \text{ kg/linear cm}.$$

According to Table 4, with a unilateral placement of 20 mm effective bars, an 8 mm diameter is required for the upright bars ($f_{st}=0.5$ cm²).

Deriving the spacing of upright bars:

$$u=\frac{0.8\times 1\times 2,100\times 2\times 0.5}{35}=48 \text{ cm};$$

$$u=\frac{0.1\times 85\times 18\times 36.5^2}{6,500}=31 \text{ cm};$$

$$u=\frac{40}{2}=20 \text{ cm}.$$

20 cm is chosen as the final spacing of upright bars.

At the end support

$Q_A=4,300$ kg$>mR_t bh_0^2=3,800$ kg. This means that upright bars are required;

$$q_{st}=\frac{4,300^2}{0.6\times 85\times 18\times 36.5^2}=15.2 \text{ kg/linear cm}.$$

By establishing the spacing u of upright bars as 20 cm, their area

$$f_{st}=\frac{20\times 15.2}{0.8\times 1\times 2,100\times 2}=0.09 \text{ cm}^2.$$

When the concrete is grade 150 and the bars are of intermittently deformed steel, the required protrusion (l_{tl}) of the 20 mm effective reinforcement beyond the edge of the support will be $15\times 2.0=30$ cm. Since the depth of the bearing is 25 cm, the above protrusion cannot be carried out, instead of which it will be necessary to increase by 50% the calculated area of the upright bars: $f_{st}=1.5\times 0.09=0.135$ cm²; actually, provisions have been made for a diameter of 8 mm, i.e., $f_{st}=0.5$ cm² >0.135 cm².

At the first intermediate support to the right

$$Q_B^{\text{int.sup}}=5,400 \text{ kg};\quad q_{st}=\frac{5,400^2}{0.6\times 85\times 18\times 36.5^2}=24.2 \text{ kg/linear cm}.$$

According to Table 4, with a unilateral placement of 18 mm effective bars a 6 mm diameter is required for the upright bars ($f_{st}=0.28$ cm²).

Deriving the spacing of upright bars:

$$u=\frac{0.8\times 1\times 2,100\times 2\times 0.28}{24.2}=39 \text{ cm};$$

$$u=\frac{0.1\times 85\times 18\times 36.5^2}{5,400}=37 \text{ cm};$$

$$u=\frac{40}{2}=20,$$

and a final spacing $u=20$ is chosen for the upright bars. Fig. 117a illustrates the construction of the slab and beams.

1. The Structural Layout

The in-situ flat-slab floor is supported directly on the columns on a widened area known as the *capital* (Fig. 118,*a*).

The capital serves the following structural purposes:

1) sufficient rigidity is provided in the slab-to-column connection;

2) resistance of the slab to shear is assured;

3) the effective span of the slab is reduced; furthermore, a better distribution of moments is effected along the slab width.

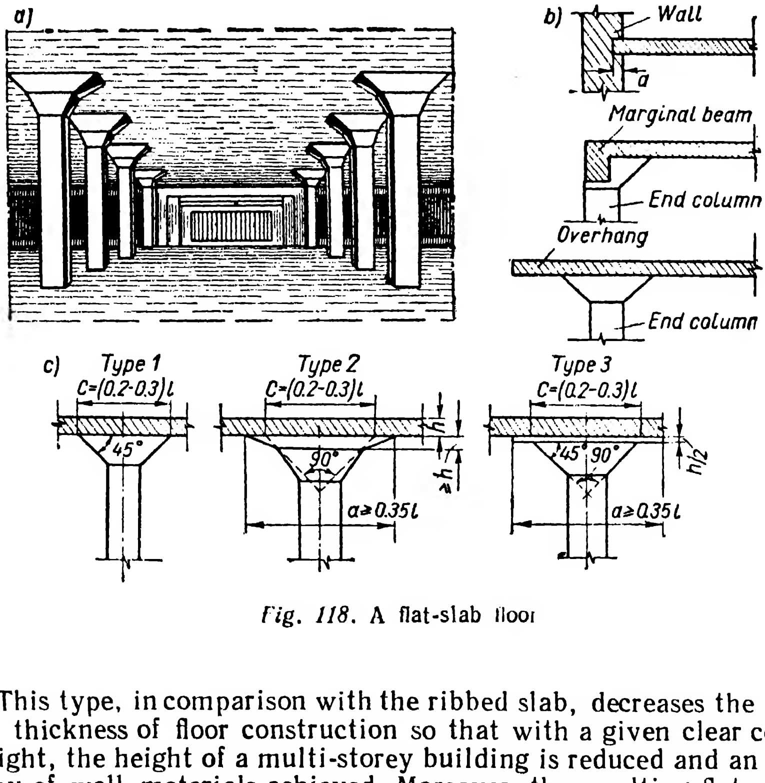


Fig. 118. A flat-slab floor

This type, in comparison with the ribbed slab, decreases the overall thickness of floor construction so that with a given clear ceiling height, the height of a multi-storey building is reduced and an economy of wall materials achieved. Moreover, the resulting flat ceiling simplifies formwork when concrete is poured in-situ, and improves interior ventilation and illumination.

Flat slabs are more economical than the ribbed type in the carrying of large live loads—1,000 kg/m^2 and more—and are almost always chosen for such structures as multi-storey warehouses, refrigerating plants, etc.

Either square or rectangular column grids are employed with flat slabs; for rectangular grids the span is limited within a range of $\frac{l_2}{l_1} \leqslant 1.5$, but the most rational is the square grid with 6×6 m as the usual column spacing. The edge of the slab may rest directly on outer walls or on marginal beams therein. It is also possible to have a slab overhang that projects beyond the outer line of columns (Fig. 118,*b*).

Three types of capitals are at the designer's disposal (Fig. 118,*c*): Type I for light loads, and Types II and III for heavy loads. All three have the same dimensions between the intersections of the bevels with the lower surface of the slab, $c = (0.2\text{-}0.3)l$, because the load from the slab is borne downward through the concrete at an angle of 45°

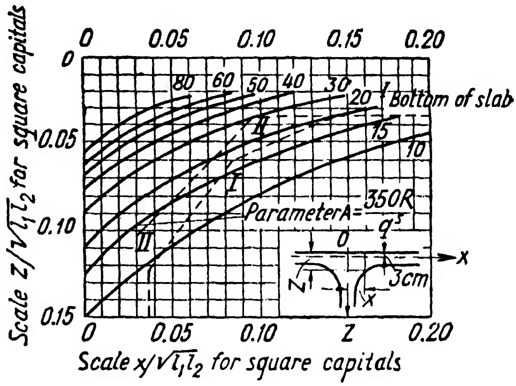


Fig. 119. Diagram for the computation of capitals in flat-slab construction

Dimensions and outlines of the capital must be computed against shearing stresses in the slab. The diagram shown in Fig. 119 will serve for checking the given dimensions of a capital to atisfy the aforementioned condition. The limiting curve that corresponds to the quantity

$$A = \frac{350R}{q^s},$$

and in which R—grade of concrete in kg/cm²,

and q^s—full specified load in kg/m²,

must be entirely contained within the outline of the capital as plotted on the diagram in accordance with the coordinates shown in the lower right side of the figure.

The thickness of the slab is selected on the basis of sufficient rigidity, that is,

$$h = \text{from } \frac{1}{32} \text{ to } \frac{1}{35} l_2,$$

where l_2 is the dimension of the larger span if the column grid is rectangular.

If the slab is of lightweight concrete, the thickness of the slab

$$h = \text{from } \frac{1}{27} \text{ to } \frac{1}{30} l_2.$$

The character of failure of a flat slab, when the load is uniformly distributed, is shown in Fig. 120; on the underside of the slab the cracks run parallel with, and halfway between, the columns, whereas

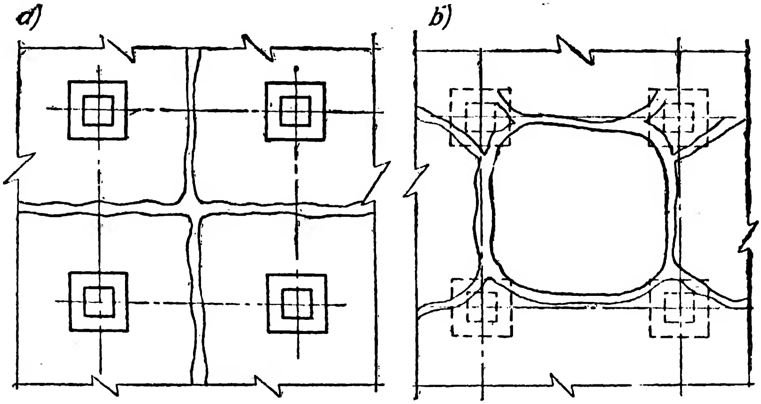


Fig. 120. Failure cracks in a flat slab

the upper side is cracked along column centres with diagonal cracks near the corners. Such an analysis of failure is needed for correct computations of carrying capacity and placement of supplementary reinforcement.

2. Computation of Flat Slabs

Flat-slab computations have been perfected through tests carried out in laboratories both in this country and in the United States. The reactions at column supports are based on triangular load distribution, while the effective span is considered as the distance between the centres of gravity of the said triangles (Fig. 121,*a*):

$$l_0 = l - \frac{2}{3}c.$$

With a free support on the wall, the effective span of the first panel

$$l_0 = l - \frac{c}{3} + \frac{a}{2},$$

in which a represents the depth of the bearing in the wall (Fig. 118,*b*).

The full load $q=g+p$ must be calculated for both directions of the spans l_1 and l_2, without consideration of the unfavourable live load. The load per linear metre ql_2 and the beam moment of the slab panel is determined by assuming the panel as a beam with a span of

l_0 and a width equal to the distance between column centres in a perpendicular direction:

$$M_0=\frac{ql_2}{8}\left(l_1-\frac{2c_1}{3}\right)^2=0.125ql_2l_1^2\left(1-\frac{2c_1}{3l_1}\right)^2.$$

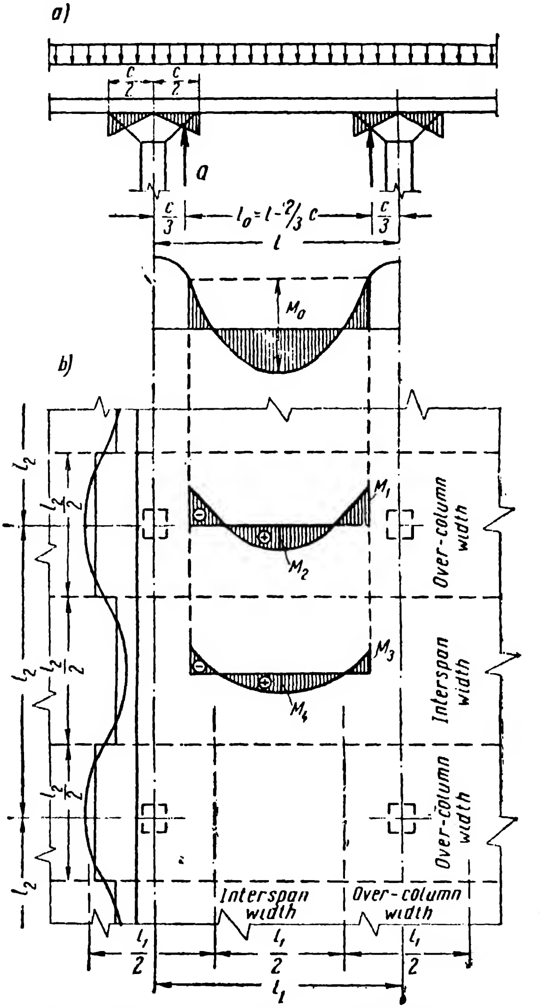


Fig. 121. Design and bending-moment diagrams for a flat slab

By considering that $P=ql_1l_2$ is the full load of the panel, we derive

$$M_0=0.125Pl_1\left(1-\frac{2c_1}{3l_1}\right)^2. \tag{225}$$

The beam moment of the panel for its span l_2 will be

$$M_0=0.125Pl_2\left(1-\frac{2c_2}{3l_2}\right)^2 \tag{225a}$$

On the other hand, if the column grid is square,

$$M_0=0.125Pl\left(1-\frac{2c}{3l}\right)^2. \tag{225b}$$

For determining the moments in the computation planes and arriving at a proper reinforcement design, the plan of the flat slab is divided into over-column strips and interspan strips whose widths are equal to half the distance between column centres in each direction (Fig. 121,*b*). Positive and negative moments will occur in each strip, with a larger moment in the over-column strip than in the interspan strip. Within the bounds of the width of the strips the changes in bending moments will follow a curved line; but for practical calculations a stepped diagram is adopted with the steps aligning with the borders of the strips on the assumption that the moments are equally distributed along the width of each strip.

Since there is a possibility of moment redistribution as an effect of plastic deformation, the moments at four computation planes are selected so that their sum is equal to the beam moment M_0. The following distributions of moments are adopted for the intermediate panel of a flat slab:

the negative moment at the supports in the over-column strip	. . . . . $M_1=0.5\ M_0$
the positive moment of the interspan in the over-column strip	. . . . $M_2=0.2\ M_0$
the negative moment at the supports of the interspan strip	. . $M_3=0.15\ M_0$
the positive moment of the interspan in the interspan strip . .	$M_4=0.15\ M_0$
	Total: $M_1+M_2+M_3+M_4=M_0$

The restraining effect of either the outer columns or the marginal beam in the outer walls must be taken into account in the final redistribution of moments along the spans l_1 and l_2 within the bounds of the entire flat slab (Fig. 122,*a*).

The design moment in the first span of the slab is evolved through a corresponding moment of the interspan by multiplying it by a coefficient given in Table 23.

The coefficients α, β, and γ are determined in accordance with the diagram given in Fig. 122, *b* and will depend upon the ratio between the sum of linear rigidity of the upper and lower columns and the linear rigidity of the slab (acting as a main beam), that is, will depend upon the ratio

$$\frac{i_{\text{upper}}+i_{\text{lower}}}{i_{\text{slab}}}=\left(\frac{J_{\text{upper}}}{l_{\text{upper}}}+\frac{J_{\text{lower}}}{l_{\text{lower}}}\right):\frac{J_{\text{slab}}}{l_{\text{slab}}}.$$

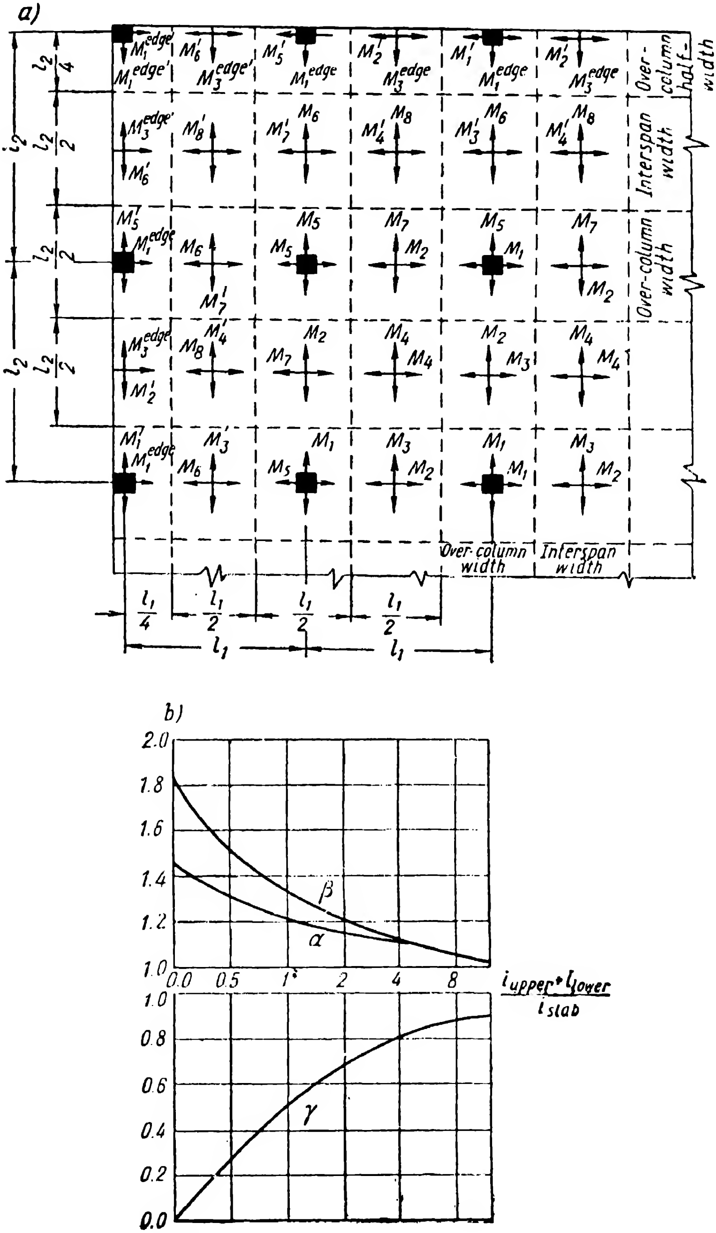


Fig. 122. Investigation of a flat slab

a—distribution of bending moments throughout the slab; *b*—diagram of the coefficients α, β and γ for determining moments in the first span of the slab

Table 23

Moments	Strips	
	Over-column	Interspan
Negative moment at the first intermediate support	$M_5 = \alpha M_1$	$M_7 = \alpha M_3$
Positive moment in the first span . . .	$M_6 = \beta M_2$	$M_8 = \beta M_4$
Negative moment at the marginal support	$M_1^{marg} = \gamma M_1$	$M_3^{marg} = \gamma M_3$

The moment of inertia of the slab (main beam) will be equal to

$$J_{slab} = \frac{l_2 h^3}{12}$$

The moment of inertia of the column is considered uniform along its whole height and equal to the moment of inertia of the column's concrete cross-section beyond the bounds of the capital.

The bending moments for a one-metre strip adjacent to the wall will be

$$M_1' = 0.5M_1; \quad M_2' = 0.5M_2; \quad M_3' = 0.8M_3; \quad M_4' = 0.8M_4.$$

In the above formulae the moments M_1, M_2, M_3, and M_4 are also taken for a one-metre width of the strip.

Bar area for each computation plane is evolved from corresponding design moments through the formula

$$F_s = \frac{0.7M}{mm_s R_s \gamma_0 h_0} = \frac{0.8M}{mm_s R_s h_0}$$

in which 0.7 is an empirical coefficient based upon the bracing effect of the construction as a whole and other factors that reduce the bending moment, and γ_0 is considered as approximately equal to 0.9.

The effective height of the slab's cross-section in relation to the first and second layer of bars will differ by the value of bar diameters, but practically this height may be considered unchanged and equal to $h_0 = h - d - 1.5$ cm. Above the columns, the effective height of the slab is assumed as follows: equal to the effective height of the slab for Type I capital; and $h_0 = 1.5h - d - 1.5$ cm for Type II or III.

The height of marginal beams running around the outside of the flat slab must be at least $2.5h$. The flat slab is assumed to transmit its load to these beams along a triangle or trapezium just as in a two-way (coffer-type) slab.

3. Designing a Horizontal Flat Slab

Just as in continuous-slab construction, the reinforcement for a flat slab is arranged in separate strips over the columns and in the interspans in accordance with the bending moments and with the use of either welded roll-type or flat mats (Figs. 123, *a* and 123, *b*).

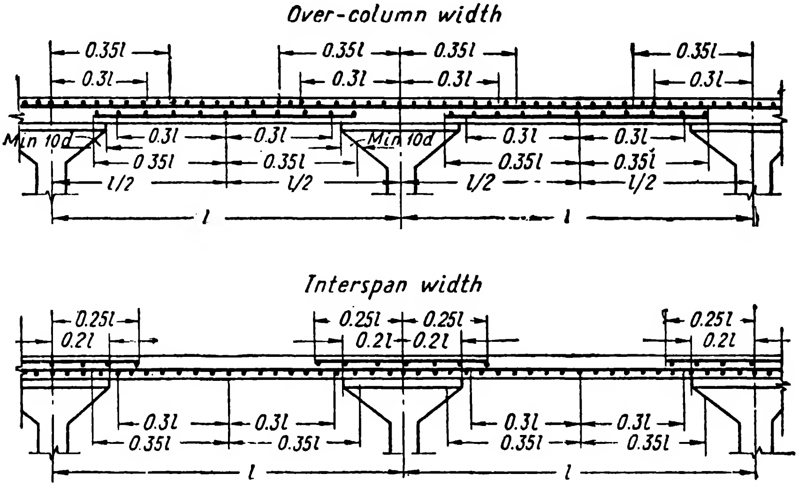


Fig. 123a. Welded mats used as reinforcement for a flat slab

The matting serving reciprocally perpendicular spans must be connected together. The mats over the supports are interrupted at the columns, and the capitals are provided with auxiliary reinforcement (Fig. 123,*c*).

4. Precast Flat Slabs

The precast flat slab (Fig. 124) is composed of three elements: 1) the capital, which is made either separately or cast in one piece with the column, 2) the over-column panel, and 3) the interspan panel. To reduce dead weights, panels are made either with cavities or with ribs. The entire dead weight of elements included in a 6 ×6 m column grid should not be more than 5 tons.

Over-column panels will behave as a beam the span of which is the distance between capitals. To provide continuity of over-column panels their upper ends and the upper part of the capitals are furnished with inserted steel parts which are welded together

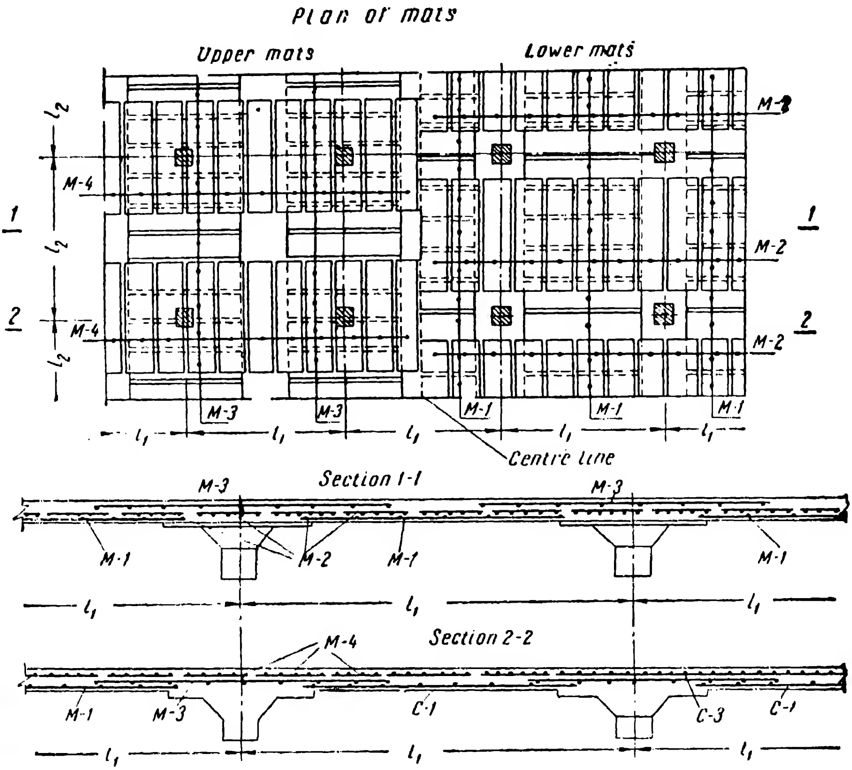


Fig. 123b. Using narrow mats as reinforcement for flat slabs

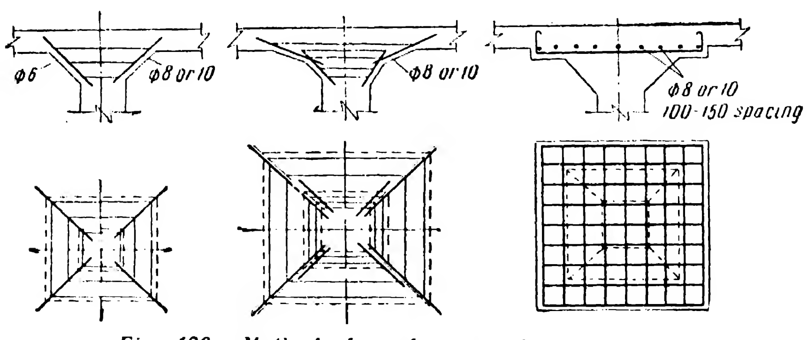


Fig. 123c. Method of reinforcing column capitals

in-situ. The floor panel is supported at its four sides by the over-column panels (which latter are provided with shelves for this purpose) and bends in two directions just as a two-way slab.

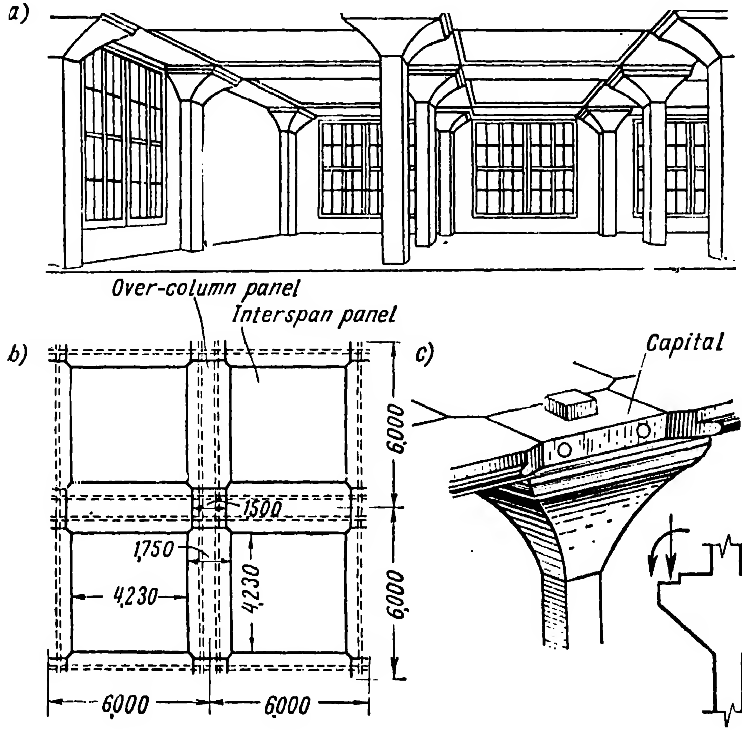


Fig. 124. A precast flat slab
a — interior view; *b* — plan; *c* — detail of capital

The precast type of flat slab bears only an outer resemblance to its cast-in-place counterpart, inasmuch as it differs in its static behaviour from the latter.

CHAPTER X

DESIGN OF ROOF GIRDERS, TRUSSES, AND ARCHES

Sec. 29. GENERAL REMARKS

The principal bearing members entering the construction of a roof consist of various kinds of girders, trusses, and arches. In a girder the web is a solid system, while in a truss it is an open framework. All of the above spanning units are, as a rule, made of prestressed reinforced concrete.

The usual spacing of girders, trusses, and arches is 6 or 12 metres. They carry all the roof members, which bear upon them via steel plates fixed to their tops. In subsequent erection, either giant panels or purlins are welded to these steel plates. The upper chord of trusses and the upper flange of girders are built with a pitch of 1/12 in accordance with atmospheric precipitation run-off requirements for the roll-type of roofing.

The ends of standard girders and trusses are made 800 mm high to assure interchangeability and connection standardisation.

Sec. 30. GIRDERS

Girders are built with either a single or double pitch, depending upon the roof profile, and with spans of 18 m and over (Fig. 125). Economy in concrete is achieved by adopting I- or T-shapes, with the thickness of web reduced to a minimum (from 6 to 8 cm). The main requirements of such reduced thicknesses are the disposition of the bars and resistance to both diagonal tension and crack formation.

The girder is widened at its ends by the addition of rigidity ribs to strengthen it against vertical reactions and horizontal thrust at the columns.

Concrete of 400- to 500-grades are used for prestressed girders.

For practical considerations (stability during transportation and erection), the width of the upper flange must be 1/40-1/50 of the length. As a rule, such a width is also sufficient as a bearing for roof members. The width of the lower flange is made from 20 to 25 cm to accommodate the tendons.

The height h of single-pitch girders (or the height in the middle of the span if it is double-pitch) is taken as $1/10$-$1/15\ l$.

Reinforcement classifications of girders differentiate them as those tensioned between abutments prior to concreting (pretensioning), and those tensioned against the hardened concrete (post tensioning).

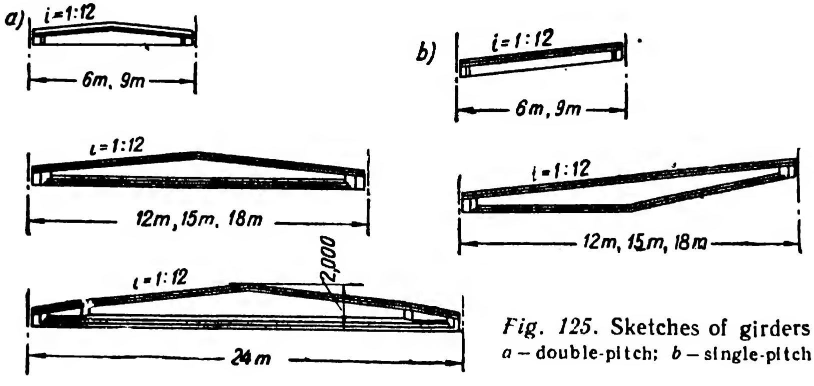


Fig. 125. Sketches of girders
a — double-pitch; *b* — single-pitch

Lower costs and reduced labour expenditure are attained when tensioning is done between abutments in vertical linear-type stands, for which reason this method is accepted as the standard in mass production.

The tendons of such beams may be of high-strength wire or of grade 30ХГ2С hot-rolled intermittently deformed bars.

The reinforcement of webs is composed of one or two welded frames whose upright bars provide resistance along diagonal planes. The girder's upper flange is reinforced in a similar manner.

When the tensioned tendons are released against the concrete, the girder's upper zone may be subject to tension with the appearance of hair-line fissures. Tests have shown that these fissures close when the girder receives its service load and carrying capacity remains uneffected.

Still, to limit such fissures in a member containing a great number of tendons (when service loads are very large) it is supplied with additional tendons on the level of the upper edge of the ends of the girder (Fig. 126); the resultant of the forces of the tendons will then be brought closer to the girder's axis, and when the tendons are released against the concrete the bending moment responsible for tension in the upper zone will be minimised.

Fig 127, *a* presents a standard prestressed girder containing bar and wire reinforcement. The high-strength reinforcing wire is disposed in paired horizontal groups for convenience in pouring the concrete.

Another example of a girder with bars tensioned between abutments is one with endless reinforcement; here high-strength wire tendons are wound against pegs protruding from the tray, a method which makes it possible to compress the concrete both longitudinally and laterally and thus considerably heighten fissure resistance.

When tensioning the tendons on a revolving table the length of the beam cannot exceed 9 m. A 12 or 18 m girder must therefore be made up of two elements connected (welded) in mid-span, which is a drawback in such a member, a component of which is shown in Fig. 127,*b*. To simplify winding of the wire for this girder, its bearing end is strengthened with laterally placed prestressed reinforced concrete insertions.

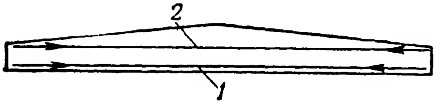


Fig. 126. Scheme showing the disposition of tendons in a double-pitch girder
1—the main tendons; *2*—additional tendons

In case of tensioning against the hardened concrete, the girder can be fabricated of separate blocks, each 3 m in length (Fig. 127,*c*) with tensioning operation accomplished in the final stage of assembly. Ducts for the tendons are left in the lower flange of each block and also in the web of the end blocks. During assembly, quick hardening mortar is grouted into the 10-15 mm joints between the blocks, while the upper flange is converted into an integral system by means of welded steel plates. When the grout has hardened, the tendons, either of bunched wire or of bars, are stretched with the aid of jacks, anchored, and either cement paste or mortar injected into the ducts.

The above method has its shortcomings: it requires much labour, and the final assembly and tensioning operations on the site become rather complicated. Neither is this block-assembly method rational when fabrication is done in a factory, for then there are facilities for one-piece manufacture of the girder and its subsequent transportation to the site.

Roof girders with ordinary reinforcement (Fig. 127,*d*) are economical only for spans of 6 to 9 m, and are made in T-shapes with 10 cm webs and with vertical ribs at their ends. The effective bars in their tension zone are of bunched intermittently deformed bars placed directly one upon the other and arc-welded together at intervals of 1-1.5 m. Some of the bars are cut off in the interspan in accordance with the moment diagram, while the rest are continued to the supports and anchored by means of short lengths of welded angles. The web is reinforced with prefabricated mats doubled about the tension bars.

Apart from the usual cladding loads, all types of roof girders may be subject to loads from suspended cranes (usually in pairs of 3 tons each), in which case the amount of reinforcement may increase by 20-30%.

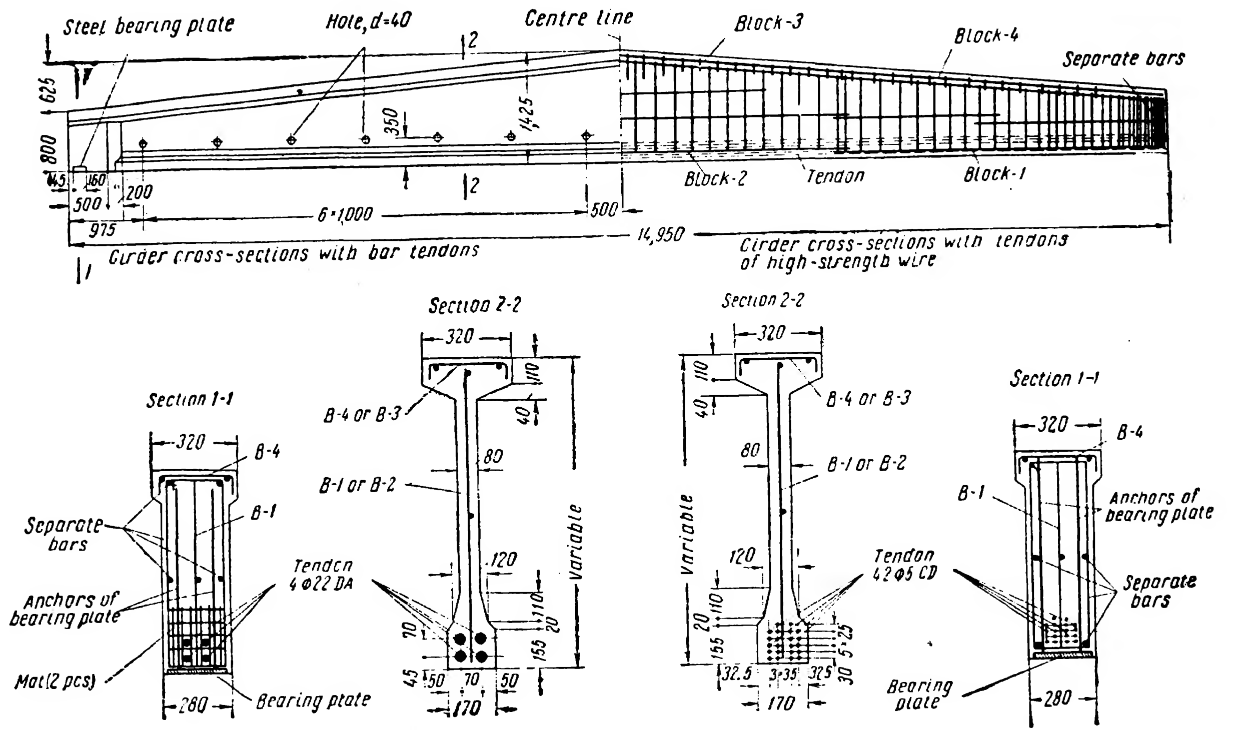


Fig. 127a. A prestressed girder with tendons tensioned against abutments

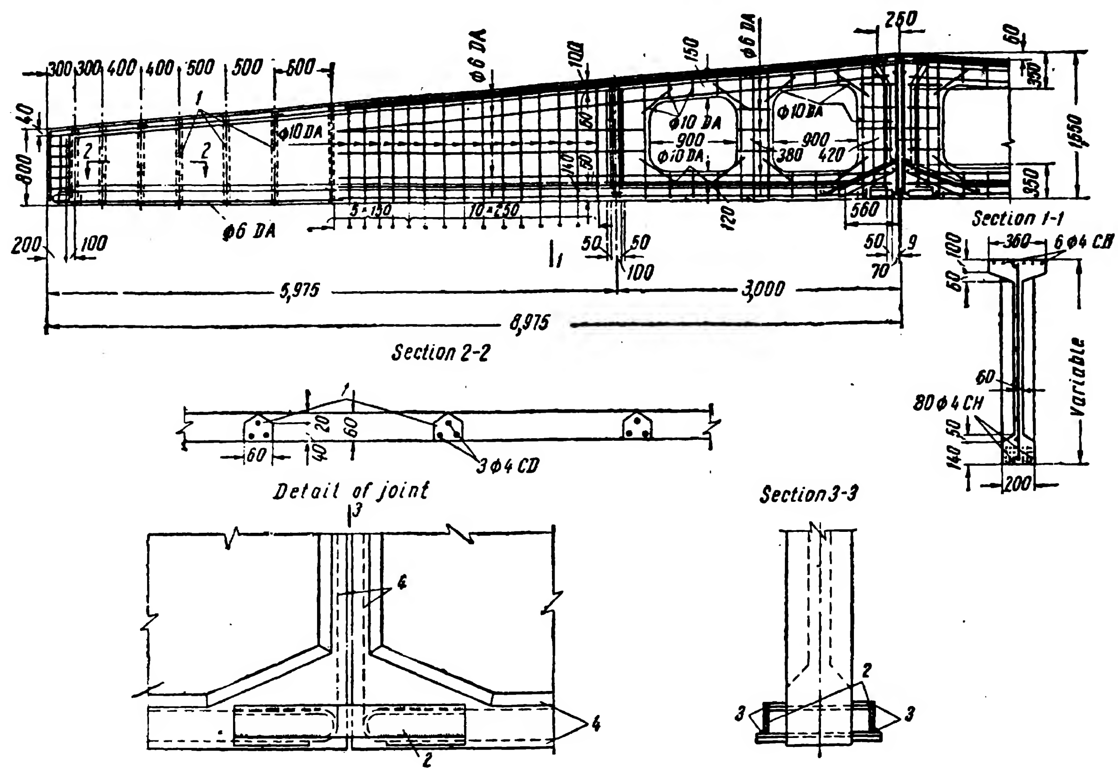


Fig. 127b. A prestressed girder with endless tendons

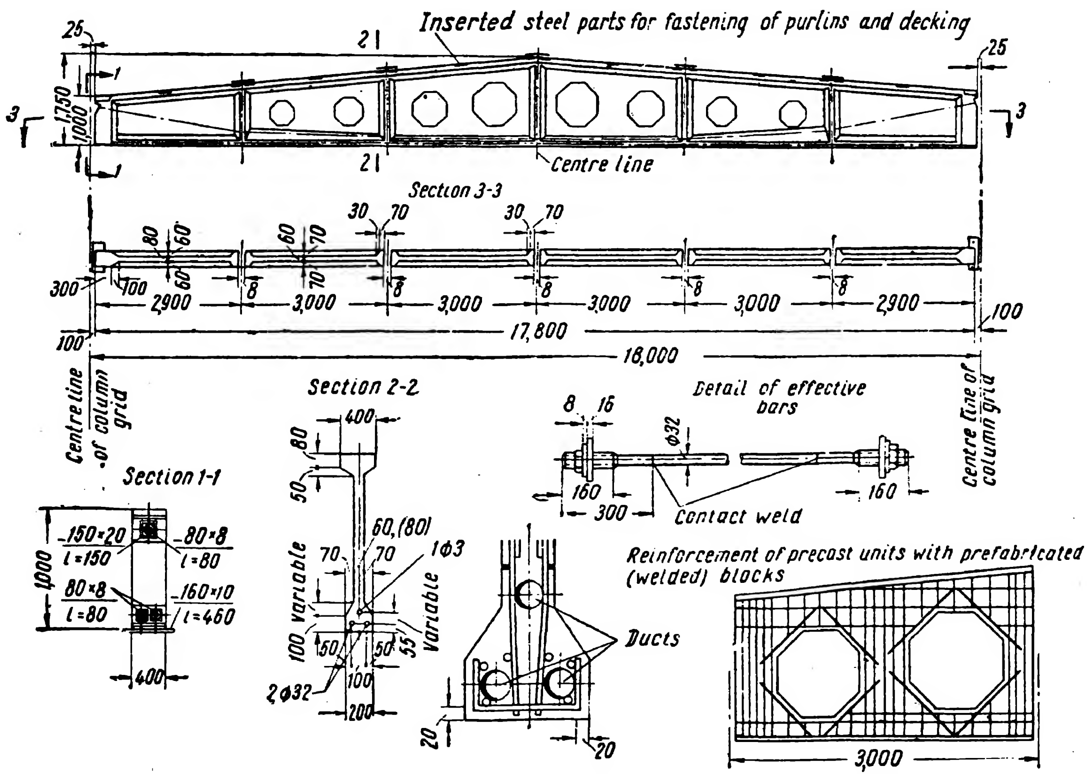


Fig. 127c. A girder made up of blocks tensioned against the hardened concrete

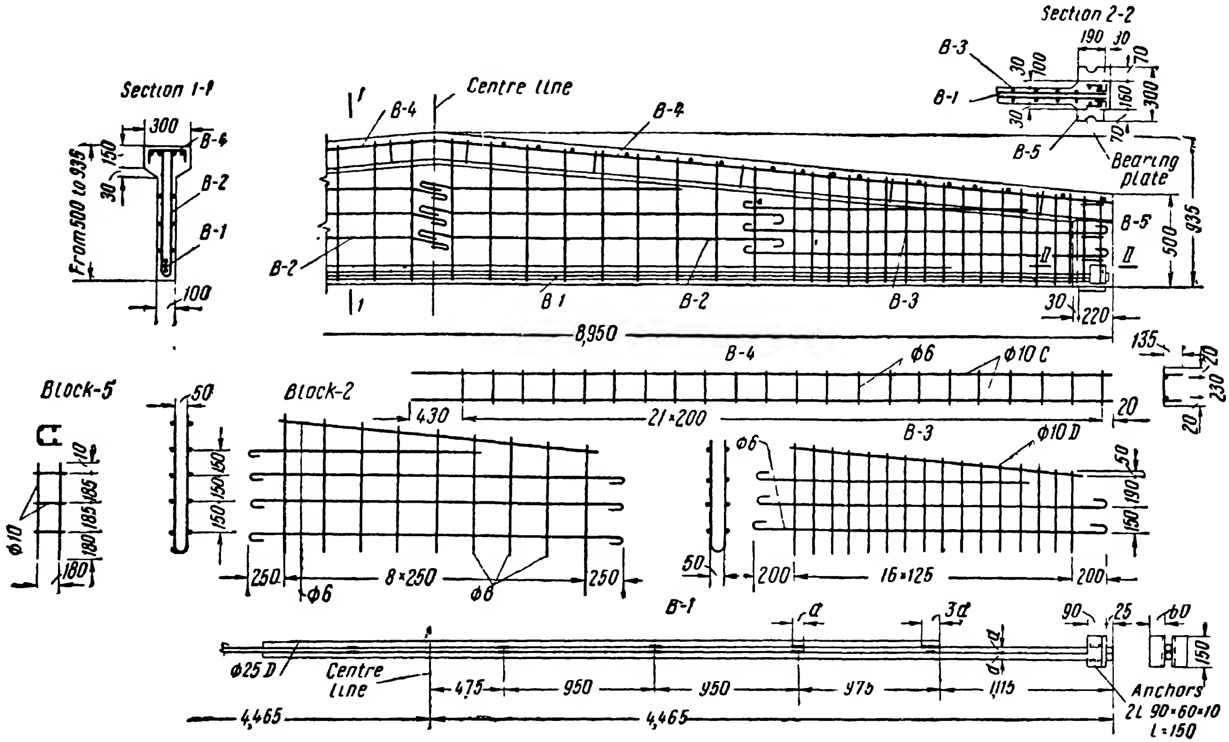


Fig 127d. An ordinary reinforced concrete T-girder

From 7 to 8 tons is the weight of an 18 m reinforced concrete girder, and from 11 to 12 tons when the span is 24 m.

Table 24 presents technico-economic evaluation factors for roof girders with spans of 12-24 metres, spaced 6 m apart.

Table 24

Comparative Technico-Economic Evaluation Factors for Precast Two-Pitch Roof Girders and Trusses Having 12-30 m Spans, 6 m Spacing and Design Loads from Roof of 350-550 kg/m²

Girder, truss (span)	Dead weight in tons	Concrete grade	Expenditure of materials	
			Concrete in cu m	Steel in kg
Prestressed girder with bar tendons:				
12 m	4.1	400	1.65	127-153
15 m	5.5 -5.9	400	2.2 -2.35	219-301
18 m	7.1 -7.5	400	2.84-2.98	341-474
24 m	11.7-12	400-500	4.67-4.78	604-884
Prestressed girder with wire tendons:				
12 m	4.1	400	1.65	87-108
15 m	5.5 -5.9	400	2.2 -2.35	145-210
18 m	7.1 -7.7	400	2.84-3.07	230-358
24 m	11.7-12	400-500	4.67-4.78	396-564
Prestressed bow truss with bunched wire tendons:				
18 m	4.3 -4.8	300	1.72-1.9	338-433
24 m	8.8-10	300-400	3.5 -4	621-689
30 m	15.2-17	300-400	6.08-6.8	1,041-1,219
Prestressed arch truss with bunched wire tendons:				
18 m	5.2- 5.9	300-400	2.07-2.36	313-395
24 m	9.2-10	400	3.68-4	564-732
30 m	14 -15.8	400	5.6 -6.32	920-1,281

The table indicates that the least expenditure of steel is achieved in girders with high-strength wire reinforcement, which gives an economy of approximately 35% in comparison to bar tendons.

Bending moments and shearing forces are computed just as for a freely supported beam. The roof loads are transmitted to the girder via the panel ribs, i.e., in the form of frequently spaced concentrated loads. When such concentrated points are more than four in number, the load may be considered as uniformly distributed. If the roof has a monitor, or if there is a crane suspended from beneath, either are considered as additional concentrated loads.

To determine the area of effective bars by means of bending moments, several planes are selected along the length of the girder. In a uniformly loaded two-pitch girder, the design cross-section will be found at a distance of 0.3-$0.4l$ from the supports. But if there is a

monitor on the roof, the design cross-section may occur directly beneath the monitor's posts.

Upright bars are determined by resistance computations of several diagonal planes near the supports. Computations of fissure resistance (or widths of fissures, depending on the type of structure) and beam deflection are also required.

Sec. 31. TRUSSES

Truss outlines are chosen in accordance with the kind of roof given, positions and types of monitors, and the general layout of the roof. The best structural type of upper chord is obtained with the bow truss (Fig. 128,*a* and *b*), whose upper outline makes either a broken

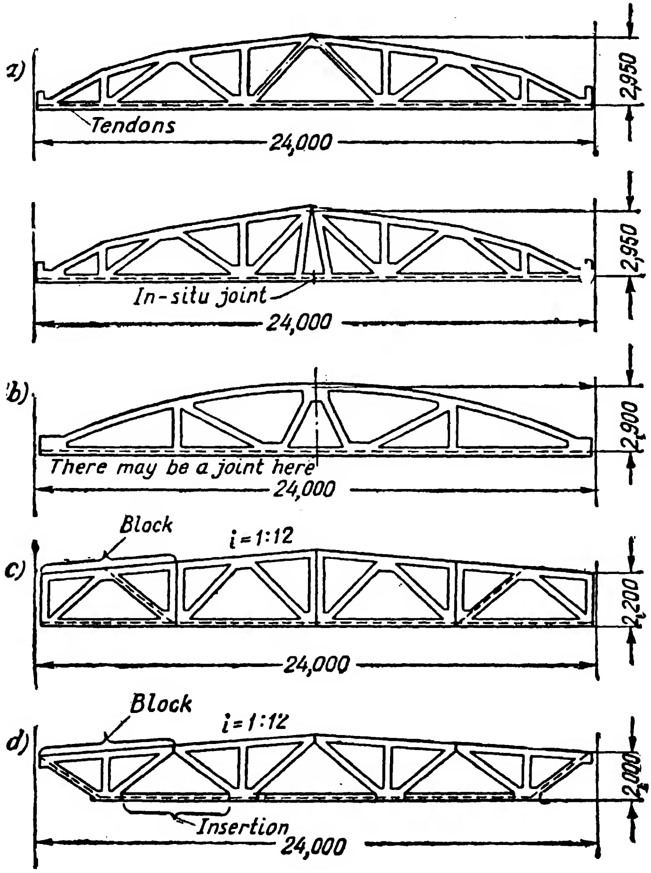


Fig. 128. Sketches of reinforced concrete trusses

line or a curve (in the latter case it is called an arch truss). The web of bow trusses forms a series of triangles and its members are subject to insignificant stresses. The rise of the truss at its end is small, thus keeping its weight within rational bounds and also lowering the height of a building's walls. All this has led to its adoption as a standard type for industrial structures.

The polygon truss can be built either with raised (Fig. 128,*c*) or dropped struts at its ends (Fig. 128,*d*). In the latter instance the truss's centre of gravity is disposed lower than the support, a factor which makes the truss very stable during erection without the use of special props. However, with this type it becomes necessary to increase the height of columns.

The height of a reinforced concrete truss in mid-span is equal to 1/7-1/9 of its length. Truss spans range from 18 to 30 metres and more.

The chords and struts of trusses are made alike in their width (from 200 to 240 mm) for convenience in fabricating in a horizontal position. Grades of concrete range from 300 to 500 and with a large percentage of reinforcement in order to reduce dead weight.

The upper (broken-line) chord of bow trusses is made in 3-metre panels to exclude bending between joints when carrying 3-metre roof slabs.

In arch trusses, where the upper chord is curved, bending moments from loads borne otherwise than at the joints are reduced because the eccentricity of the longitudinal forces creates a negative moment (Fig. 129). This allows greater panel lengths of the upper chord and a sparser layout of struts.

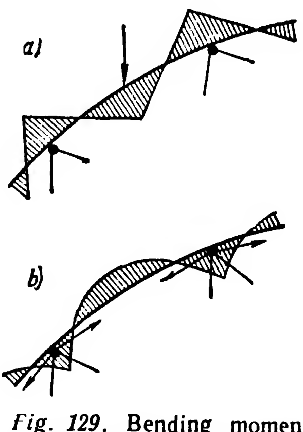
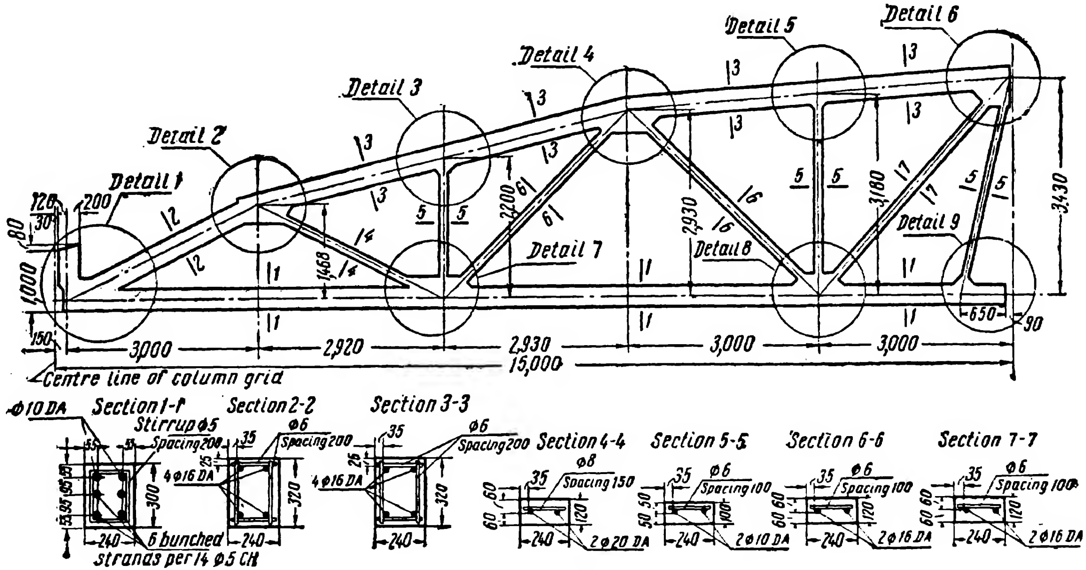


Fig. 129. Bending moments in the upper chord of an arch truss

a—moment from load between joints; *b*—the result of eccentricity of longitudinal forces

The lower tension chord of bow trusses is prestressed with the use of bunched high-strength wire, tensioned against the hardened concrete (post tensioning). When the span is from 18 to 24 m these trusses are made in one piece, but when spans run from 24 to 30 m they are made in two pieces with the joint in mid-span. Table 24 presents technico-economic evaluation factors for this type of truss with spans of 18-30 m and a spacing of 6 m.

The erection joint of the lower, tension chord can be either welded, or drawn together by tensioning the tendons against the whole length of the chord, the choice depending upon transportation and site facilities, etc. It must be kept in



Detail 1
Detail 2
Detail 3
Detail 4
Detail 5
Detail 6
Detail 7
Detail 8
Detail 9
3,000
2,920
2,930
3,000
3,000
15,000
3,430
Centre line of column grid
Section 1-1
Section 2-2
Section 3-3
Section 4-4
Section 5-5
Section 6-6
Section 7-7
Φ10 DA
Stirrup Φ5
Spacing 200
4 Φ16 DA
6 bunched strands per 14 Φ5 CH
Φ8
Spacing 150
2 Φ20 DA
Spacing 100
2 Φ10 DA
2 Φ16 DA

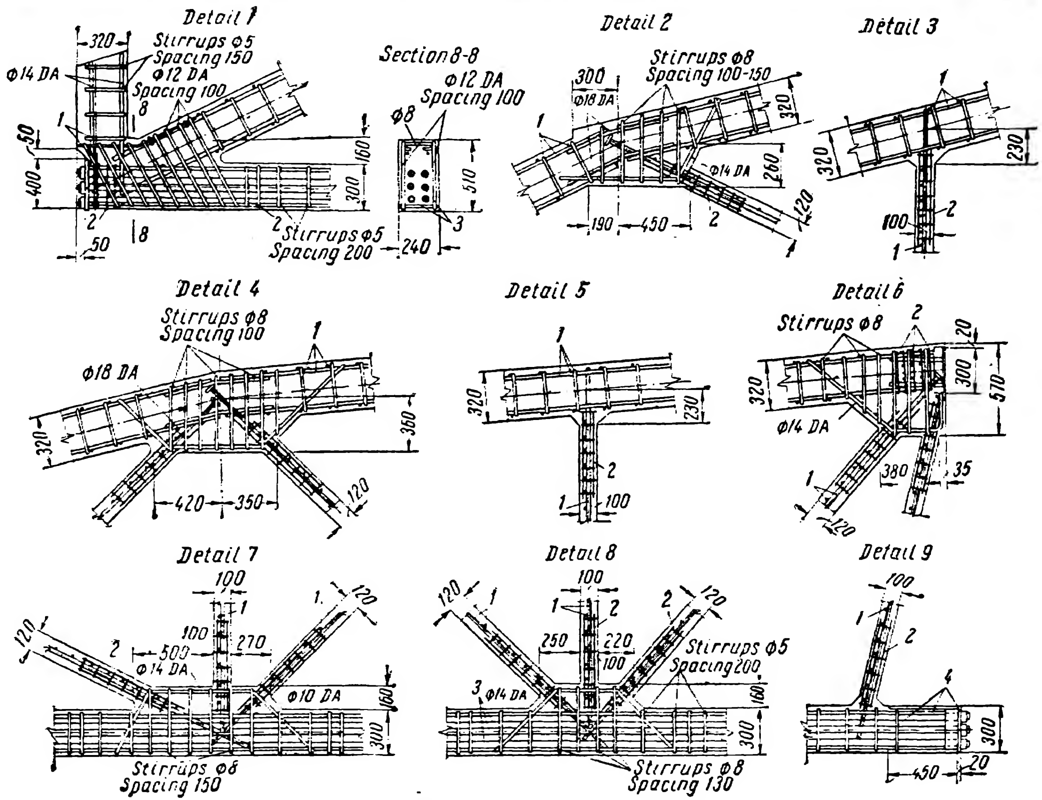


Fig. 130a. A 30-metre bow truss

1 — prefabricated blocks; 2 — prefabricated mats; 3 — bunched tendons; 4 — inserted parts

mind, however, that the latter method increases the work at the site. Furthermore, transportation of the halves of the truss with its untensioned lower chord may result in crack formation.

Fig. 130,*a* presents an example of a 30 m bow truss, showing its reinforcement and weld-type of lower-chord joint. The joint plates are welded to inserted parts of the chord during final assembly. Ducts for the bunched tendons are left in the bottom chord when the concrete is poured. The upper chord and the struts of the web are reinforced with prefabricated mats.

Gussets (widened wings) are arranged at the joints and reinforced with additional bent bars. These gussets provide the necessary anchorage for the bars of the struts.

To increase the producibility of bow trusses, an alternate production method consists in assembling them of separate straight members (Fig. 130, *b*) the concrete of which is compacted by means

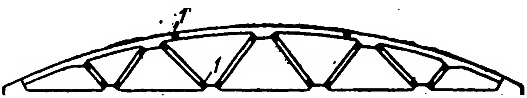


Fig. 130b. Sketch of a bow truss built up of separate, straight pieces
1—joint made by welded protruding bars, subsequently grouted with concrete; *2*—prestressed lower chord

of vibration equipment. These straight elements are connected at the truss joints by welding the protruding bars, and the joint subsequently grouted with concrete. The lower chord could be made in one piece, prestressed during fabrication. Assembly of such trusses must be done in a factory.

Polygon trusses having upward-inclined struts at the supports could be made of 6-metre blocks, or of half-trusses with 3-metre panels. The great stresses borne by the struts of this type of truss require not only prestressing of the lower chord, but also of those struts undergoing tension, especially those at the supports. All this considerably complicates fabrication.

Polygon trusses with dropped struts at their supports could be assembled out of triangular units, with prestressed lower chord and end struts.

Polygon trusses are less economical than the bow type in their expenditure of both materials and labour.

10-12 mm steel bearing plates, anchored into the truss ends when the concrete is poured, are used for all types of trusses beneath their bearings at columns.

Aside from roof loads, trusses of industrial buildings are liable to loads from suspended cranes—usually three or four of them, each with a capacity of 3 to 5 tons—the additional forces of which increase the percentage of reinforcement in the trusses by 20-30%.

Reinforced concrete trusses are computed by usual methods. The rigidity of joints has very little effect on the forces acting upon the struts, hence the joints are assumed as hinged in the design diagram.

In ascertaining of forces, the most disadvantageous loading scheme is taken—that of the snow load applied to one-half of the truss, and the loads from suspended cranes (if there are such). The roof load and the dead weight of the truss proper is considered as applied to the joints of the upper chord, while the loads from suspended cranes are applied to the joints of the lower chord. If any loads are transmitted otherwise than via the centres of joints, bending moments are computed on the assumption that the chords are continuous beams whose spans are equal to the distance between joints. If it is an arch truss, then the relieving effect of eccentric longitudinal forces is taken into account if the loads are applied between the joints of the chords.

The cross-section of the upper chord is computed on the basis of axial compression; but with any load applied between the joints, eccentric compression is assumed. The cross-sections of the struts and lower chord are calculated as axially compressed and axially tensioned members. If suspended cranes (or a suspended ceiling) are hung between the joints of the lower chord, the latter is computed by eccentric tension formulae. Fissure formation must be checked for all tension members. Furthermore, the truss as a whole must be checked for erection behaviour.

When computing longitudinal bending in the plane of the truss, the effective lengths of its elements are taken as follows: the distances between joint centres for the upper chord and struts at the support; the distances between joint centres multiplied by a coefficient of 0.8 for the rest of the struts. When calculating longitudinal bending not in the plane of the truss, the effective lengths of the upper chord are taken as the distance between joints fixed against longitudinal displacement, while the effective lengths of struts is considered as the distance between joints.

Sec. 32. ARCHES

Arches may be triple-hinged, double-hinged, and fixed (completely restrained) (Fig. 131,*a*). The thrust of an arch may be borne either by a tie or by buttressing construction such as columns or a foundation. If precast construction is used, a tied double-hinged arch is used in most cases.

Ordinarily, arches are made in one piece, but long and heavy arches are divided into elements put together during their final assembly (Fig. 131,*b*).

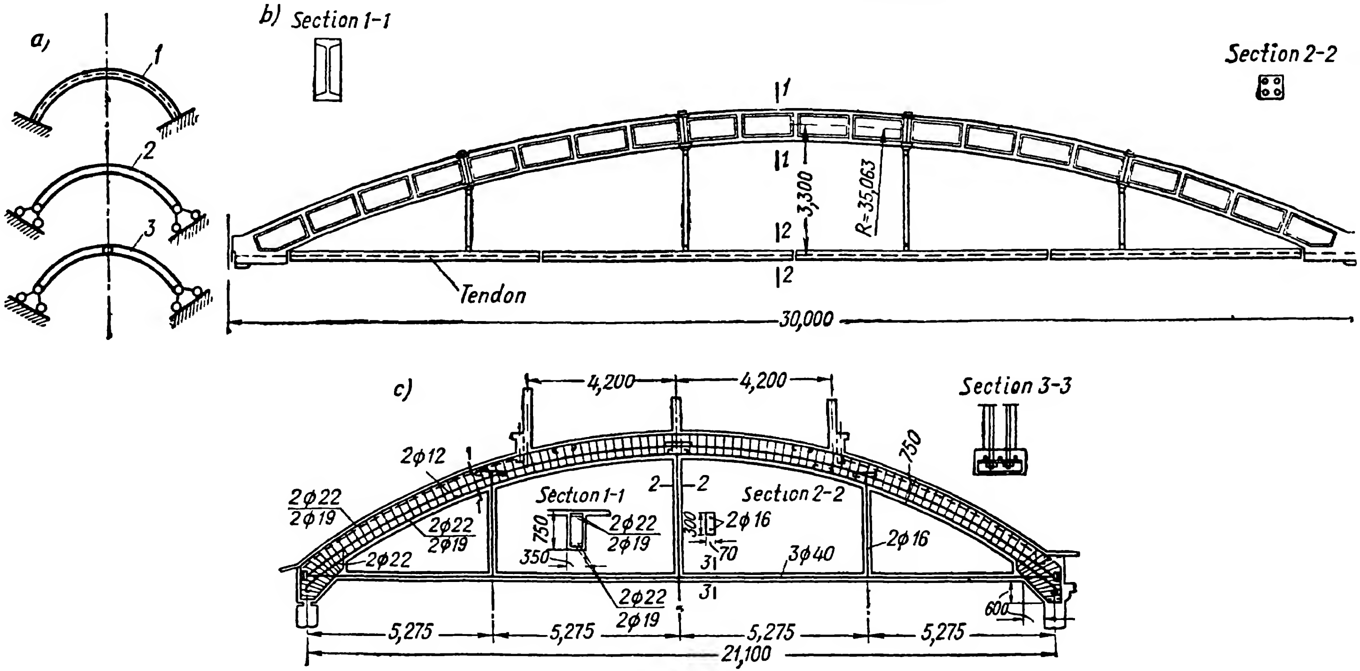


Fig. 131. Reinforced concrete arches

a—arch diagrams; *b*—a precast arch; *c*—reinforcement for an in-situ arch

Trusses are easier to erect than arches (and in some cases their technico-economic design-evaluation factors are better); but with very long spans (over 36 m), arches become more economical than trusses.

The curves of arches are usually kept low with rises f = from $^1/_5$ to $^1/_7 l$. The height of the cross-section h is made from $^1/_{30}$ to $^1/_{40} l$. I-sections are used for precast arches, and T-sections or rectangular shapes for in-situ arches. Prefabricated reinforcement blocks are used with the bars arranged symmetrically because of possible reversal of moment signs. Prestressed ties are employed with tendons consisting of bunched high-strength wire anchored at the supports.

The tendons of the tie in in-situ arches usually consist of very thick bars with anchorage at the supports implemented through nuts and washers (Fig. 131,*c*). To keep the tie from sagging, it is hung from the arch at intervals of 5-6 m.

Arches are computed on the basis of symmetric loads from the roof cladding and unsymmetrical snow loads (on one half of the roof). Unsymmetrical erection loads must also be considered. The thrust H and the forces M, N, and Q are determined either through the principles of Strength of Materials or with the aid of tabular data.

Ties are calculated by means of axial tension formulae, while the arches themselves are computed on the basis of eccentric compression accompanied by bending. The effective length of an arch is assumed as $0.58s$ when it is triple-hinged, $0.54s$ when it is double-hinged, and $0.36s$ when it is entirely restrained (s—the developed length of the arch's axis).

CHAPTER XI

CANOPY ROOF SHELLS

Sec. 33. GENERAL REMARKS

The canopy shell recommends itself by its low material consumption and its capability of roofing great spans, both in industrial and civic buildings and also of parts of structures.

A number of types of this roof are known: long and short vaulting, plicated shells, domes, flat hipped vaulting, and two-way-curvature shells. The types mostly in use are the long vault, the short vault, and the plicated shell. All types of canopy shells may be either of in-situ or precast construction, and of flat or curved components.

Sec. 34. LONG VAULTING

Long vaults are those whose proportions $\frac{l_1}{l_2} \geqslant 1$. This type is composed of a thin curved slab (Fig. 132,*a*) bordered by longitudinal springer curbing and by lateral diaphragms at the ends. The length between diaphragms is known as the *vault span* l_1, while the distance between springer curbing l_2 is called the *throw of the vault's arch*. The ratio $\frac{l_1}{l_2}$ may reach a value of 3-4. The vault rise f (including the longitudinal springer curbing) must be at least $1/10\ l_1$ and $1/6 l_2$. The arch's throw is usually not more than 20 m, but the longitudinal span may reach a length of 30 m or more.

The canopy shell of the vault together with its bordering springer curbs form a troughlike girder that possesses a great resisting moment and transmits its bearing pressure to the rigid diaphragms. In contrast to an ordinary girder, the vault is calculated against lateral deformation and for vaulting action in both directions. Vault stresses are composed of longitudinal compressive and tensile forces and lateral bending moments.

The number of arches classify it as a *single-* or a *multi-arch* vault (Fig. 132,*b*), the latter being composed of several parallel single-arch vaults connected by common springer curbing. The system may be

made up of single or multiple spans (Fig. 132,*c*); in the latter case the vaulting is supported by intermediate and end diaphragms.

The vault slab may be either plain or ribbed, with ribs running laterally and longitudinally on the underside of the shell (Fig. 132,*d*). Ribbed vaults are best made in precast reinforced concrete with the

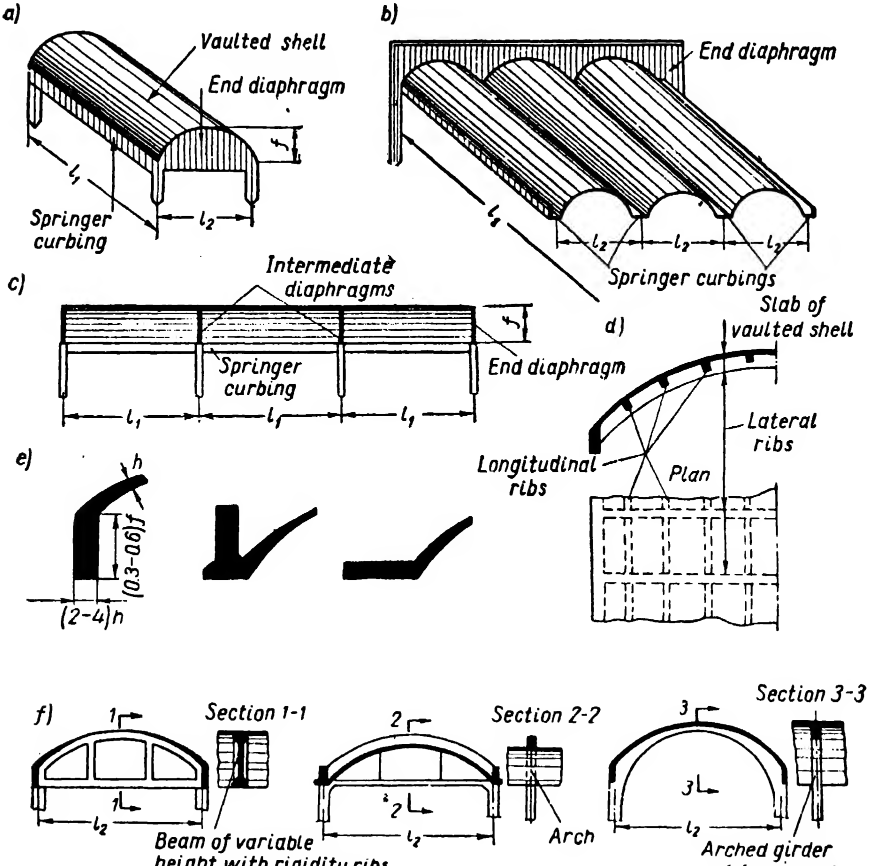


Fig. 132. Long vaulting

a—a single-span single-arch vault; *b*—a multi-arched vault; *c*—a multi-span vault; *d*—a ribbed vault; *e*—types of longitudinal springer curbing; *f*—types of diaphragms

ribs dividing the shell into curved panels, joined during final assembly by tensioning of the reinforcement (tendons) placed in the ducts of the ribs.

The shell attains a cylindrical appearance, with its cross-section forming a segment. This is the simplest type to erect, although elliptical and other cross-sections are sometimes met with.

The longitudinal springer curbing (Fig. 132, *e*) is made in a number of shapes: dropped, raised, and horizontal. The heights of curbs range from 0.3 to 0.6 *f*, and widths from 2 to 4 *h*, in which *h*—the thickness of the shell.

There are several types of diaphragms (Fig. 132, *f*): the solid girder of variable height divided by stiffening ribs, the tied arch, the curved girder, etc.

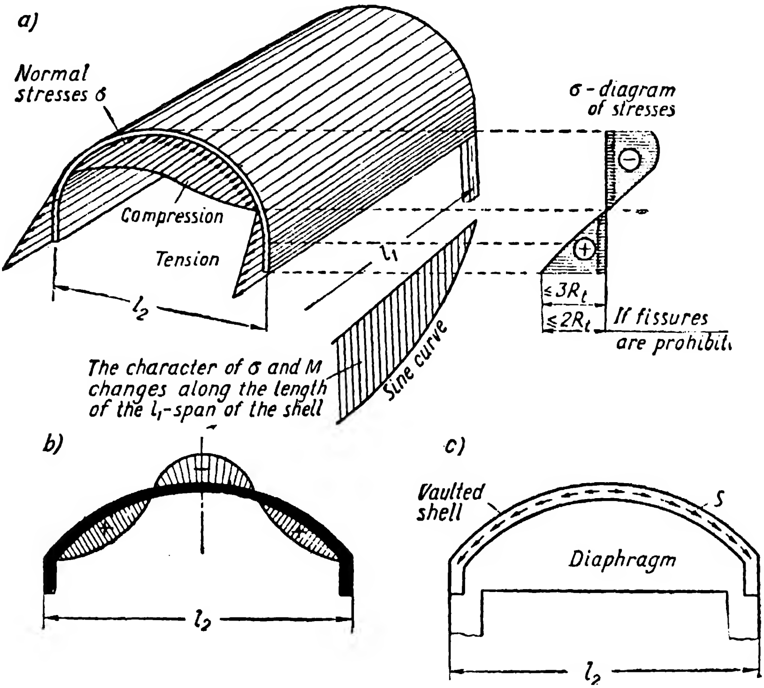


Fig. 133. Stresses in long vaults

a—diagram of normal stresses; *b*—diagram of lateral bending moments; *c*—scheme showing transmission of forces from the vault to the diaphragm

Computations of single-arch vaults and side half-arches of multi-arch vaults are done in accordance with the method of V. Z. Vlasov, based on mutual action of longitudinal forces and lateral bending moments. Fig. 133,*a* and *b* show diagrams of longitudinal (normal) stresses σ, and lateral bending moments *M* in a mid-span plane of a single-span, single-arch vault. The ordinates of both diagrams follow a sine-curve and decrease along the span to a zero value at the ends of the vault. The diagram illustrates that longitudinal tension is created in the spring of the shell and compression in the crown.

The central vault is considered as a girder with a troughlike section (undeformed) and with a span l_1. Cross-sectional dimensions

and bar areas are computed on the basis of longitudinal stresses σ, diagonal tensile stresses τ and lateral bending moments M.

The longitudinal tensile stresses in the spring of the shell and adjacent curbing are limited to $3R_t$ when the width of fissures is to be restricted, and to—$2R_t$ when fissures are entirely forbidden.

The principal longitudinal tension-bearing steel is disposed in the curbing and springs of the shell. In the crown the longitudinal steel is placed only as supplementary reinforcement, inasmuch as compressive stresses there are borne by the concrete. Lateral bars take the tensile forces of lateral bending moments and also play the role of stirrups at the supports where they bear diagonal tensile stresses.

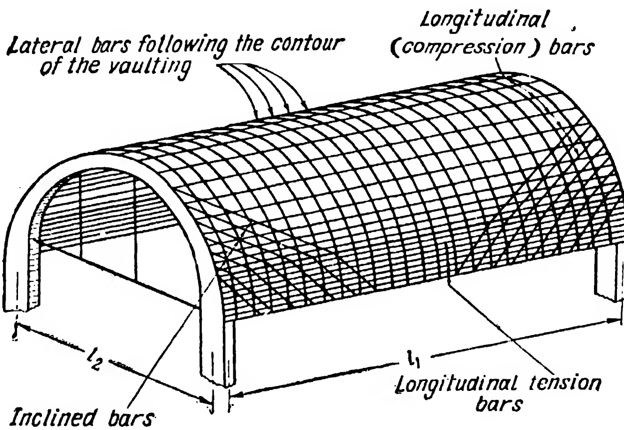


Fig. 134. Scheme of reinforcement of a long vault

The shell of the long vault can be reinforced with welded mats (Fig. 134) and with welded blocks placed in the springer curbing and diaphragms. Additional mats must be disposed in the corners of the shell where the greatest diagonal tensile stresses occur. All mats must be properly jointed in both directions.

The diaphragms take bearing compression from the shell through the thrust S, tangential to the crown (Fig. 133,*c*). The forces M, N, and Q, acting in a series of the diaphragm's planes, are determined by static computations. The cross-sections of the diaphragm undergo eccentric tension and the areas of longitudinal bars are derived through eccentric tension formulae.

Sec. 35. SHORT VAULTS

Vaults are considered as short when $\frac{l_1}{l_2} < 1$. Short vaults (Fig. 135,*a*) consist of a thin curved shell, longitudinal springer curbing, and intermediate diaphragms spaced at intervals of 6 or 12 m. The

rise of the vault $f \geqslant {}^1/_8\, l_2$, while the throw of the arch l_2 may be 30 m and more. The short vault is more economical in materials than its long counterpart.

The principal stress occurring in short vaults is compression, which attains an insignificant value. When the maximum dimension of the span l_2 of a monolithic vault is 30 m, empirical thicknesses h are established for the shell, depending upon the vault span l_1: when $l_1 = 6$ m, $h = 5$ cm.

The shell of the vault is reinforced with a minimum amount of supplementary bars, inasmuch as the role of the latter consists in

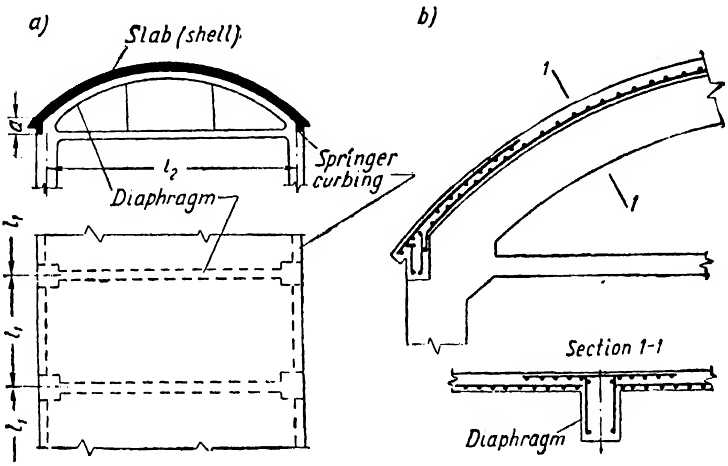


Fig. 135. A short vault
a—Its structural elements; *b*—its reinforcement

bearing shrinkage and temperature and other forces not provided for by computation. Welded mats are used for this reinforcement (Fig. 135,*b*).

Springer curbs of short vaults are made in the form of rectangular beams whose height $a = 1/10\text{-}1/15 l_1$. Welded reinforcement blocks are placed therein to bear tensile stresses which are computed by considering the short vault as a troughlike girder with a span of l_1 and resting upon the diaphragms.

The types of diaphragms introduced into short vaults are (Fig. 132,*f*) tied arches, curved girders of frames (bents), and other forms. The thrust S in the shell is transmitted to the diaphragms in the same manner as in long vaults. Approximate methods are used in the computations of diaphragms, based on the combined action of the vertical load gathered along the bearing area and normal tensile forces.

Sec. 36. PLICATED SHELLS

The folded type of roof construction forming a series of plicated shells makes up a rigid system composed of flat slabs (Fig. 136). Just as in other canopy shell types, the plications are supported by diaphragms whose spacing l_1 is known as the *plication span*. Springer curbing may be either the dropped or the raised type. Plicated shells may be of single- or multi-plication, and single- or multi-spanned.

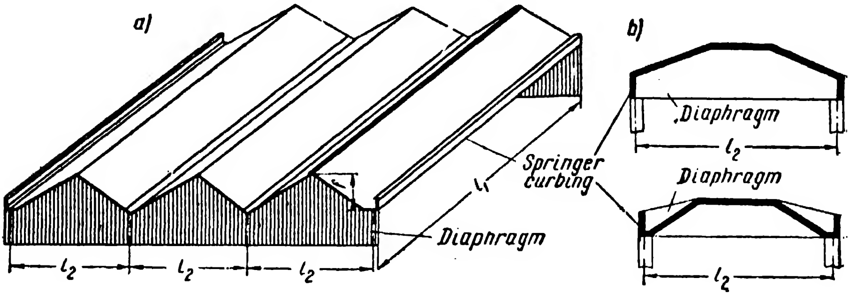


Fig. 136. Plicated shell roof
a—multiple plication; *b*—single plication

The static behaviour of a loaded plicated shell is very similar to that of long vaulting and can therefore be built in spans of 20 and more metres. An additional feature in its behaviour is lateral bending, a factor which limits the width of the facets to 3-3.5 m. This in its turn limits the throw of the plication l_2 to 10-12 m. The rise of the plication $f \geqslant 1/10 l_1$.

Dimensions of springer curbing are made the same as for the vault type of shell.

The facets (slabs) may be reinforced with welded mats properly joined in both directions, while welded reinforcement blocks are incorporated in the springer curbing and the diaphragms.

Longitudinal forces, lateral bending moments and diagonal tension are all to be included in the computations of cross-sections.

CHAPTER XII

REINFORCED CONCRETE FOUNDATIONS

Sec. 37. THE DESIGN OF COLUMN FOOTINGS AND THEIR BEHAVIOUR

Column loads are transmitted to the bearing soil by the footings.

Stepped footings are the usual type of column foundations (Fig. 137). For heavy loads and weak bearing soils, column foundations are built for whole rows of columns in the form of continuous footings (Fig. 138, *a*) or as continuous cross-footings (Fig. 138, *b*), and in special cases as a solid flat or ribbed slab (Fig. 138, *c* and *d*).

Footings for precast columns are furnished with a recess, known as a *pocket*, into which the column is inserted (Fig. 137, *b*). The joint spaces between the column and the walls of the pocket are grouted with concrete (for which very fine aggregate is used); sometimes a hole is provided in the side of the pocket in order to pack the grout from beneath.

Foundations for in-situ columns (stepped footings, continuous footings, or slab foundations) are likewise poured in-situ with the reinforcement left protruding for connection to the column reinforcement. Pocket-type stepped footings for precast columns may also be poured in-situ or brought to the site ready-fabricated. If pocket-type stepped footings are too large or too heavy to handle in one piece, they may be composed of separate thick slabs and a shoe (Fig. 137, *c*).

Footings for precast columns should be designed so that all work below the cardinal level be completed before column erection commences, i.e., so that column placing starts only after the completion of all foundations, underground utilities, backfill, and underfloor layers. To do this, the upper level of all footings should be finished at —0.15 m (in other words, 15 cm below the finished ground floor). If such a level should cause any difficulty in arranging the lowest line of footings, then either an underlayer of concrete can be provided beneath them, or the upper step of the footing raised that bears the pocket of the column.

The load transmitted from the column to the footing results in a reaction in the bearing soil which bends the footing (Fig. 139).

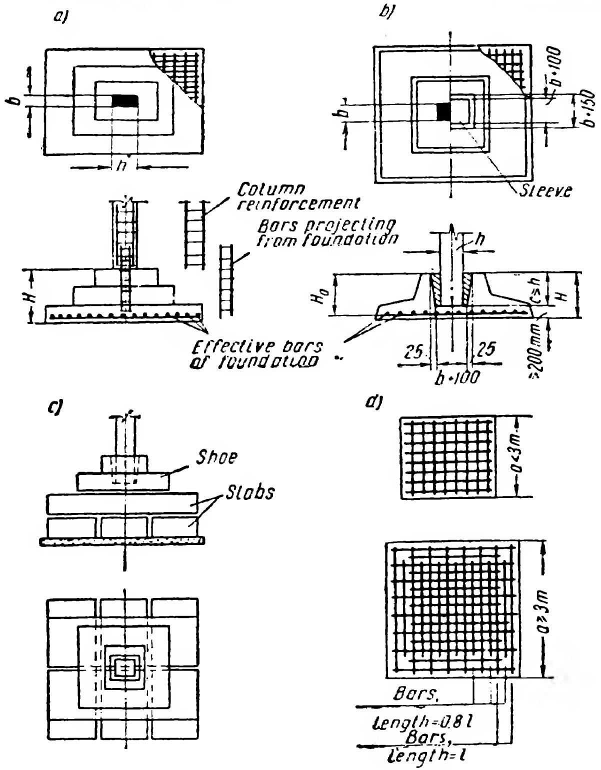


Fig. 137. Stepped footings for columns

a—in-situ reinforced concrete; *b*—precast reinforced concrete; *c*—footing made of individual precast pieces; *d*—reinforcement

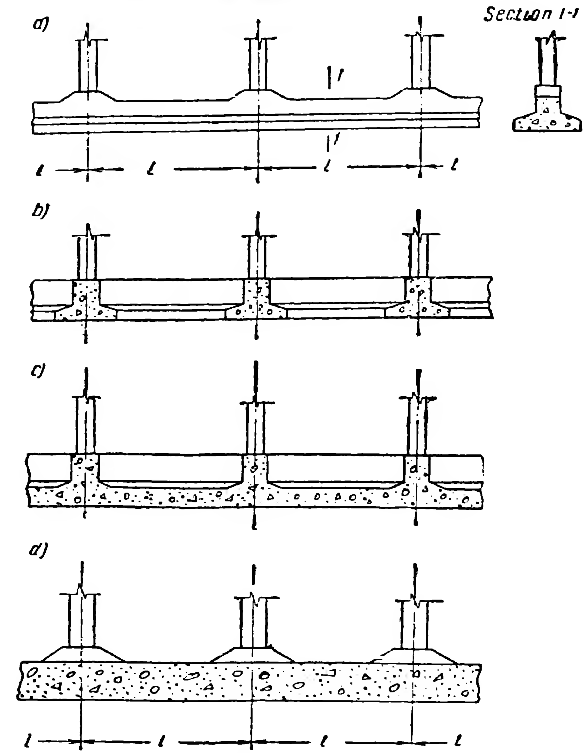


Fig. 138. Column foundations

a—continuous footing; *b*—continuous crossed footings; *c*—ribbed-slab foundation; *d*—flat-slab foundation

The pressure stresses (reaction) in the bearing soil will depend upon the rigidity of the footing and the physico-mechanical properties of the soil (its elasticity). In computing separate axially compressed stepped footings it is assumed that the pressure (reaction) is equally distributed along the bottom of the footing. If the stepped footing is eccentrically compressed, a straight-line distribution of the pressure is likewise assumed. In either continuous or slab-type foundations the reaction diagram will depend upon bearing soil elasticity and foundation rigidity. Reaction stresses in the bearing soil for a continuous foundation are illustrated in Fig. 140.

When a stepped footing bends against the reaction, the tension zone will be at the bottom (Fig. 139) for which reason the reinforcement, in the form of a mat with effective bars running in two directions, is placed in the lower part. Because of the comparative great height of a stepped foundation, diagonal tensile stresses created by bending action are borne by the concrete and the necessity of upright bars is excluded.

The reaction beneath a continuous foundation causes it to behave as a continuous beam (Fig. 140) and it is therefore reinforced with longitudinal and vertical effective steel, the longitudinal bars being disposed in the lower part at the columns, and on top in the span between columns.

The reactions beneath solid flat-slab foundations cause them to behave just as a flat-slab floor and they are therefore reinforced with mats, both above and below. When such slab foundations are very thick, they are furnished with intermediate mats to resist shrinkage and temperature stresses; all mats in slab foundations must be united by means of vertical reinforcement blocks or individual bars.

Foundations in the form of solid ribbed slabs act just as two-way ribbed floor slabs.

Foundations are usually poured directly upon a tamped layer of soil, but if the site is very wet, a plastic consistency of broken stone and mortar is first put down. From 3 to 5 cm is the established thickness of the protective covering for the bottom bars of foundations.

Sec. 38. COMPUTATION AND DESIGN OF STEPPED FOOTINGS

1. Footings for Axially Loaded Columns

Computations of a footing consist in determining its contour in plan, its height, and areas of effective reinforcement. Its load will be the sum of the longitudinal force N transmitted through the column and the dead weight G of the foundation, the latter also including the weight of the soil carried by the shelving of the footing.

Tentatively, G is assumed as 0.08-0.12 N.

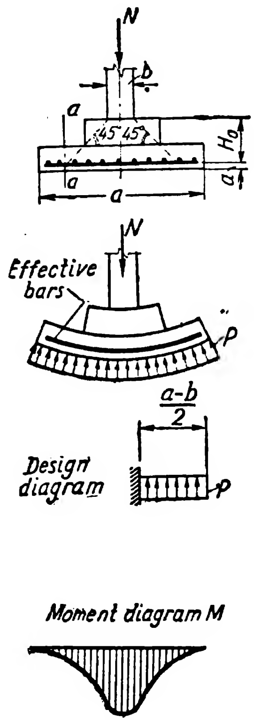


Fig. 139. Behaviour of a loaded stepped footing

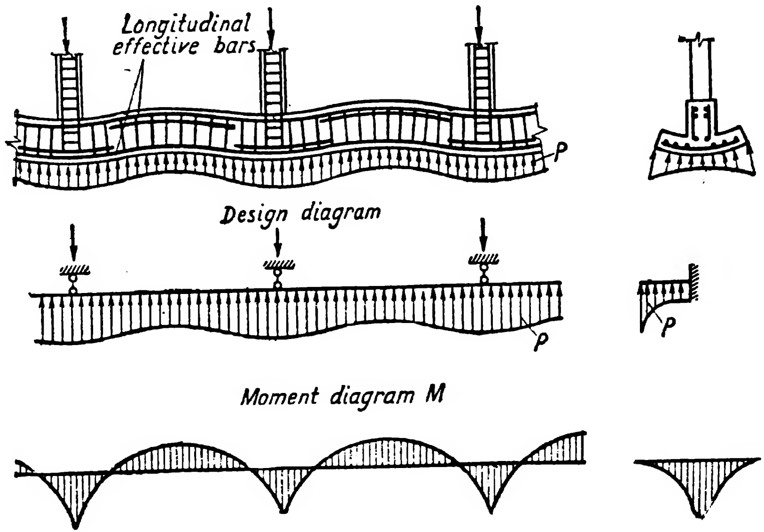


Fig. 140. Behaviour of a loaded continuous footing

The contour of the footing in plan is determined by the design resistance of the bearing soil R_{soil} as reacting against the specified loads:

$$\frac{N^s + G^s}{F} \leqslant R_{soil}, \tag{226}$$

where F—area of the footing.

In the given case, specified loads are chosen because the value of R_{soil} is established by codes on the basis of settling deformation of bearing soils.

From formula (226)

$$F = \frac{N^s + G^s}{R_{soil}}. \tag{227}$$

Stepped footings for axially loaded columns are, as a rule, made square in plan and the dimension of each side

$$a = \sqrt{F}. \tag{228}$$

When computing bending action from the soil reaction, it is approximately assumed that the footing shelving protruding beyond the edge of the column will behave as four independent brackets whose widths are the dimensions perpendicular to the widths of the footing in plan.

The reaction p beneath the footing is determined from the longitudinal *design* force N transmitted by the column. The loads from the dead weight of the footing and the soil bearing upon its shelves are not taken into account since they are evenly spread over the whole footing area and are balanced by a corresponding up-thrust of the soil and hence do not cause bending of the footing:

$$p = \frac{N}{F}. \tag{229}$$

The height H of the footing is determined by the following conditions.

1) Resistance of the concrete to shear along the perimeter of the column from the longitudinal design force N, i.e.,

$$H \geqslant \frac{N}{2(b+h) R_{sh}}, \tag{230}$$

where b and h—cross-sectional dimensions of the column,

R_{sh}—design shear resistance of the footing's concrete, which can be derived from formula (5) by assuming a uniformity coefficient $k_c = 0.5$.

For 100-grade concrete $R_{sh} \approx 10$ kg/cm², for grade 150 $R_{sh} \approx 15$ kg/cm², and $R_{sh} \approx 17$ kg/cm² for 200-grade concrete.

2) Resistance of the footing along diagonal planes when there is no vertical reinforcement, i.e.,

$$H_0 \geqslant \frac{Q}{mR_t b_1}, \tag{231}$$

where b_1—upper width of footing,

Q—resultant of soil reaction upon the bracket.

With a square foundation whose side dimension is a and a square column whose cross-sectional dimension is b (Fig. 139)

$$Q = pa\frac{(a-b)}{2}. \tag{232}$$

When the column is precast, the depth of the pocket in the footing may also be a factor in deciding the footing height H. To set the column properly into the pocket, its depth c must not be less than 1-1.5 h, in which h—the larger face of the column. The thickness of the shoe beneath the pocket h_{shoe} must be at least 20 cm; this thickness must also be checked for concrete shear, a factor which may be met with during erection of the column, when the grout in the pocket has not had time to harden.

Hence,

$$H \geqslant c + h_{shoe}.$$

The dimensions of the pocket in plan are made greater than those of the column (15 cm more at the top and 10 cm at the bottom) for convenience in erection and grouting.

The minimum height of footings for in-situ columns must be such as to allow proper insertion of the column's longitudinal bars (or the bars protruding from the footing and having the same diameters as those in the column):

for foundations carrying axially loaded columns

$$H \geqslant 20d_{bars};$$

for foundations carrying eccentrically loaded columns

$$H \geqslant 30d_{bars}.$$

In individual cases the height of footings will be dictated by structural considerations concerning footing profile, special requirements of foundation depths, level of ground beams, etc.

The footing is divided into 2 or 3 steps, from 30 to 50 cm each in height, with the width of steps arranged so that they align with a 45° angle carried down from the lower edges of the column (Fig. 139). Resistance along diagonal planes will thus be assured without the addition of vertical reinforcement. If the lower step should project beyond this angle, then diagonal plane resistance at these points

(plane a-a, Fig. 139) is warranted when the height of the lower step

$$h_0 \geqslant \frac{Q_1}{mR_t a}, \tag{231a}$$

in which Q_1—the resultant of soil reaction upon the sector lying outside the plane a-a,

a—width of the lower step (foundation width).

To derive bar area, it is required to determine the bending moment in the bracket from the edge of the column:

$$M = Qe, \tag{233}$$

in which e is the distance from the line of the force Q to the edge of the column,

Q is derived through formula (232).

With a square footing whose side dimension is a and a square column whose cross-sectional dimension is b

$$e = \frac{a-b}{4}.$$

By entering the values of Q and e into formula (233) and carrying out the necessary conversion, we get

$$M = \frac{pa\,(a-b)^2}{8}. \tag{234}$$

The area of the bars F_s is computed on the basis of bending resistance of the footing:

$$M \leqslant mm_sR_sF_sz.$$

Assuming that $z \approx 0.9H_0$ and $m=1$, we get

$$F_s = \frac{M}{0.9m_sR_sH_0}. \tag{235}$$

The effective bars in the footing are determined according to this derived area (the bars being made alike in both directions). Bar diameters must be at least 8 mm (usually they are 10-16 mm), and their spacing 10-20 cm. If the footing is very large ($a > 3$ m), only 50% of the bars are carried to its edge and the rest cut off at a distance of $0.1a$ from the edge. In this case the reinforcement mat will be made up of bars with lengths of a and $0.8a$, laid alternately.

2. Footings for Eccentrically Loaded Columns

Eccentrically loaded columns transmit to their footings the longitudinal force N, the bending moment M, and the shearing force Q (Fig. 141). The height H of footings is determined as already described in Item 1.

The moment at the level of the footing's shoe

$$M^{s}_{soil}=M^{s}+Q^{s}H.$$

The eccentricity of the longitudinal force at the level of the footing's shoe

$$e_0=\frac{M_{soil}}{N^{s}+G^{s}}$$

The reaction at the edge of the footing is found through the well-known formula in Strength of Materials:

$$p=\frac{N^{s}+G^{s}}{b_1h_1}\left(1\pm\frac{6e_0}{h_1}\right), \qquad (236)$$

in which b_1 and h_1 are the dimensions of the footing in plan.

With a plus-sign in formula (236), p_{max} is obtained ($p_{max}\leq 1.2R_{soil}$); with a minus-sign, the result will be p_{min}.

The dimensions of the footing in plan are obtained by approximations. In the first approximation the required footing area is derived through the formula

$$F=(1.5\text{-}2)\,\frac{N^{s}+G^{s}}{R_{soil}} \qquad (237)$$

The outlines of the reaction diagram will depend upon the amount of eccentricity (Fig. 141); when $e_0<\frac{h_1}{6}$, the diagram will be a trapezium; when $e_0=\frac{h_1}{6}$, it will take the form of a triangle; and when $e_0>\frac{h_1}{6}$, the base of the triangle will be less than h_1, that is, only part of the footing area will bear upon the soil (the remaining footing area will be separated from the underlying soil). The value of p_{max} in the latter instance is computed by considering the centre of gravity of the triangular reaction diagram as being directed along the line of the eccentrically placed force:

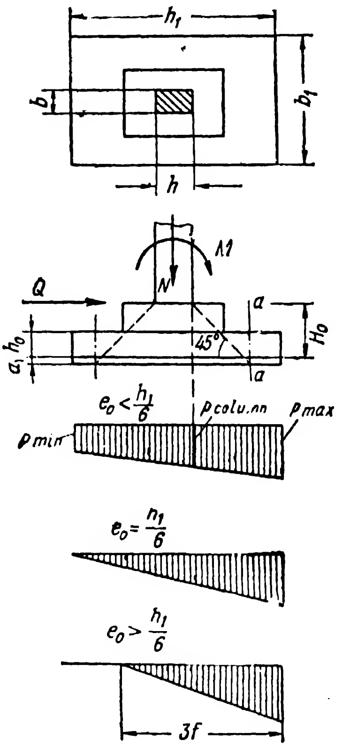


Fig. 141. Investigation of an eccentrically loaded stepped footing

$$p_{max}=\frac{2\,(N^{s}+G^{s})}{3b_1f} \qquad (238)$$

in which

$$f=\frac{h_1}{2}-e_0.$$

The dimensions of the footing must satisfy the condition that

$$3f\geqslant 0.75h.$$

Footings whose reaction diagram is triangular can be used only for columns that have no crane loads. If the column loads include that of a crane, the reaction diagram beneath the shoe of the footing must be in the form of a trapezium and satisfy the condition that

$$p_{min}\geqslant 0.25p_{max}.$$

Eccentricity of the longitudinal force may be reduced by displacing the centre of the foundation, as related to the centre of the column, towards the line of eccentricity. If the amount of this displacement $c=e_0$, then the reaction diagram will become rectangular and the problem will become one of axial compression. However, usually the amount of this displacement $c=\frac{e_0}{2}$.

If the foundation is displaced, then the load on the bearing soil

$$p=\frac{N^s+G^s}{b_1h_1}\left[1\pm\frac{6(e_0-c)}{h_1}\right] \tag{239}$$

To compute bar area, it will be necessary to determine the reaction beneath the footing from the design forces N, M, and Q. The quantity first determined is

$$M_{soil}=M+QH.$$

With a displaced foundation

$$M_{soil}=M+QH-Nc.$$

The next step to be determined is

$$e_0=\frac{M_{soil}}{N}.$$

If it is found that $e_0\leqslant\frac{h_1}{6}$, then the reaction is derived through the formula

$$p=\frac{N}{b_1h_1}\left(1\pm\frac{6e_0}{h_1}\right). \tag{240}$$

But if $e_0>\frac{h_1}{6}$, then

$$p=\frac{2N}{3b_1f}. \tag{241}$$

The reaction diagram will first reveal the pressure opposite the edge of the column p_{column}; the mean pressure will then be

$$p' = \frac{p_{max} + p_{column}}{2}.$$

Bending moments are to be computed in two directions; the moment at the edge of the column (in the plane of the moment)

$$M_I = \frac{p' b_1 (h_1 - h)^2}{8}, \quad (242)$$

while the moment in the other direction

$$M_{II} = \frac{p_0 h_1 (b_1 - b)^2}{8}, \quad (242a)$$

in which

$$p_0 = \frac{N}{b_1 h_1} \quad (243)$$

After finding M_I and M_{II}, formula (235) will determine the bar areas for each direction.

Illustrative problem 20. Design a square footing for a precast axially loaded 30×30 cm column. Given: $N^s = 74$ tons; $N = 90$ tons; $R_{soil} = 2$ kg/cm^2 $= 20$ t/m^2; grade 100-Б concrete; $R_t = 4$ kg/cm^2; $R_{sh} = 10$ kg/cm^2. Reinforcement to be of round St-3 steel bars; $R_s = 2{,}100$ kg/cm^2.

Solution.

Assume that $G^s = 0.08N^s = 0.08 \times 74 = 6$ t.
Through formula (227) the area of the footing

$$F = \frac{74 \times 6}{20} = 4 \text{ m}^2.$$

The side dimension of the footing

$$a = \sqrt{4} = 2 \text{ m}.$$

The reaction that causes bending of the footing is computed through formula (229):

$$p = \frac{90{,}000}{200 \times 200} = 2.25 \text{ kg/cm}^2.$$

The resultant of the reaction upon the bracket, according to formula (232),

$$Q = 2.25 \times 200 \, \frac{200 - 30}{2} = 19{,}100 \text{ kg}.$$

Shear requirements, according to formula (230), demand that the footing's height

$$H = \frac{90{,}000}{4 \times 30 \times 10} = 75 \text{ cm}.$$

In accordance with formula (231), when there are no vertical bars and when the upper width of the footing $b = 40 + 30 + 40 = 110$ cm, the requirements of

diagonal plane resistance dictate that the footing height

$$H_0 = \frac{19,100}{1 \times 4 \times 110} = 43.5 \text{ cm.}$$

When $a = 4.5$ cm, $H = 43.5 + 4.5 + 48$ cm.

Requirements of sufficient pocket depth for the column dictate that

$$H = 30 + 20 = 50 \text{ cm,}$$

all of which indicates the final height to be: $H = 75$ cm, and $H_0 = 70.5$ cm (see Fig. 142).

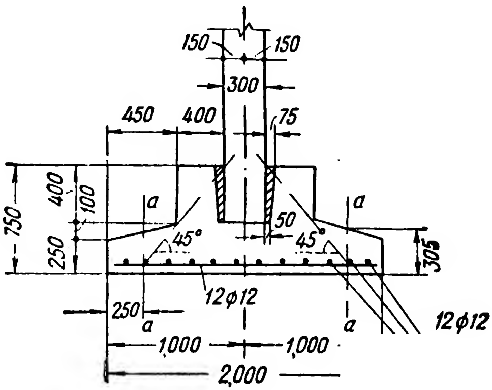


Fig. 142. Illustrating problem 20

According to formula (234), the bending moment at the edge of the column

$$M = \frac{2.25 \times 200\,(200 - 30)^2}{8} = 1,630,000 \text{ kg/cm.}$$

With grade 100 concrete, the service coefficient of St-3 steel bars $m_s = 0.9$. According to formula (235), required bar area

$$F_s = \frac{1,630,000}{0.9 \times 2,100 \times 0.9 \times 70.5} = 13.35 \text{ cm}^2,$$

which is satisfied by 12ø12 in each direction ($F_s = 13.56$ cm^2).

Checking the height of the lower step at plane *a-a* (the step projects 25 cm beyond this plane)

$$Q = 2.25 \times 200 \times 25 = 11,250 \text{ kg;}$$

from formula (231a)

$$h_0 = \frac{11,250}{1 \times 4 \times 200} = 14 \text{ cm.}$$

The actual height of the footing at plane *a-a*

$$h_0 = h - a = 30.5 - 4.5 = 26 \text{ cm} > 14 \text{ cm.}$$

CHAPTER XIII

DESIGN OF SINGLE-STOREY FRAMED BUILDINGS

Sec. 39. GENERAL REMARKS

1. Their Use. Structural Details

Single-storey framed buildings are widely adopted for various branches of industry (steel mills, machine engineering plants, consumers' goods factories, etc.).

The basis in the design of an industrial structure is the blocked layout. This means that the production shops are located under one roof when the technological process allows such an arrangement. Such combinations of layout result in rectangular, parallel blocks of unified heights. Breaks in the height silhouette are avoided because they complicate construction and sometimes even add to the expenditure of materials. Such blocked layouts achieve maximum unification of structural members, simplify connections, and reduce the number of size ranges of standard units. Other advantages in combining shops into a single building are: lowered costs of both construction and subsequent plant maintenance, reduction of plant areas, and shortening of interplant communication and underground and above-ground facilities.

Manufacturing processes in single-storey buildings usually require the movement of material along the bays of the structure. Overhead cranes and monorails, installed for this purpose, are propelled on special rails supported longitudinally by the columns of the building. Monorail cranes may also be suspended from the main spanning members of the roof, in which case the term *suspended transport* is applied.

Another possibility is the implementation of floor-supported cranes which increase the versatility of the floor plan and simplify future modernisation of technological processes.

The structural scheme of the roof of a one-storey industrial building may consist of straight members that act on the principle of beams, or of canopy shells, plications, etc.

The members of a single-storey framed structure with a beam-system roof (Fig. 143,*a*) are: columns resting upon their footings;

main beams (either girders or trusses) supported by the columns; roof panels resting upon the main beams; overhead-crane girders; wall panels; lighting or ventilating monitors and other components. The principal structural unit of the frame will be the transverse bent, composed of columns and main beams. Such a bent may have one or several spans

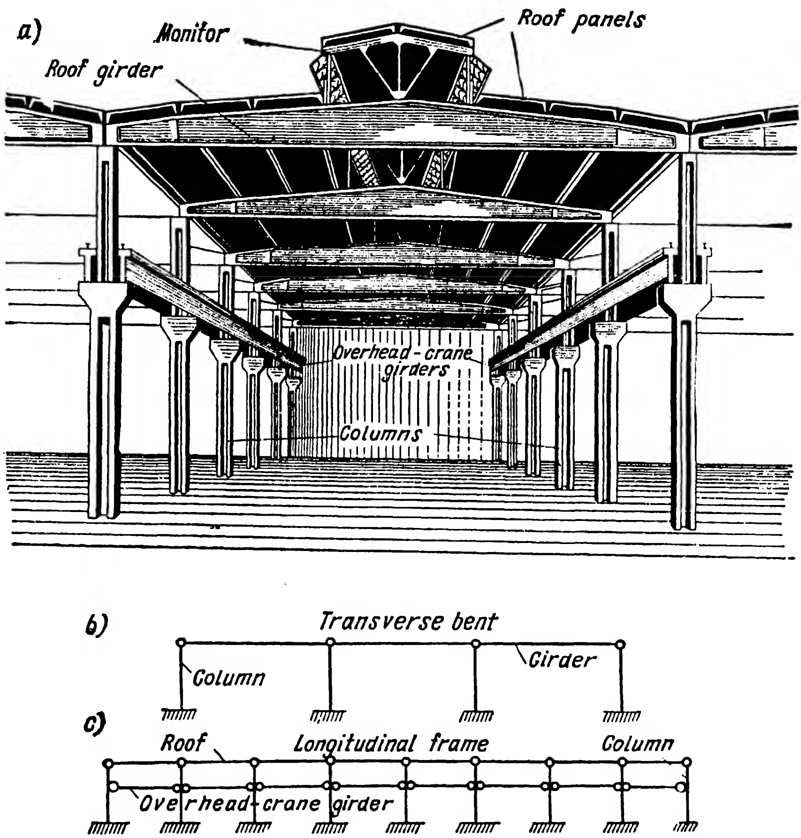


Fig. 143 Construction of a single-storey industrial building
a—general interior view; *b* and *c*—scheme of lateral bent and longitudinal frame

Skeletal rigidity of one-storey framed buildings is assured laterally by the bents (Fig. 143,*b*) and longitudinally by the framework created by the columns, roof deck, and overhead-crane girders (Fig. 143,*c*). Sometimes bracing is also added.

New types of single-storey factories being built both abroad (in the U.S.A. and in Canada) and in this country are distinguished

by floor-supported and suspended types of cranes (instead of overhead cranes), luminescence illumination (instead of monitors), forced ventilation, and in certain cases by the installation of air conditioning systems.

In districts subject to long winters and heavy snowfall, monitors lose their value as a source of natural light, and furthermore are the cause of great heat losses. In such areas it is best to do without monitors, their absence also improving roof drainage and decreasing the number of size ranges of precast reinforced concrete units.

2. Overhead Cranes and Crane Loads

An overhead crane is made up of a truss bridge propelled on four wheels,* and a four-wheeled crane buggy equipped with a hoisting hook and hoisting mechanism (Fig. 144,*a*). Transverse movement of loads in the shop bays is accomplished by propelling the crane buggy along the crane trusses, while for longitudinal transport the crane is run along the crane rails.

Overhead cranes are made with capacities of 5, 10, 15, 20, 30, 50 and more tons, and are sometimes equipped with a second, auxiliary hoisting hook whose capacity is less than that of the main hook.

To assure unhindered passage of the overhead crane, a standard clearance is provided between it and the structure, the dimensions of the crane and this clearance determining the dimensions of the building, including its height. The crane transmits both vertical and horizontal-thrust loads to the building.

The *vertical* loads, which are the sum of the weights of the crane, crane buggy, and the material being handled by the crane, are transmitted to the crane rails via the crane wheels. The heaviest loading scheme borne by the wheels P_{max} occurs when the fully loaded buggy is on the extreme side of the crane trusses (Fig. 144,*b*); in this position the opposite wheels will receive the minimum load P_{min}.

Specified maximum crane loads P^s_{max} for cranes of various capacities are presented in all Crane Standards, while the values of P^s_{min} may be derived by assuming the crane to be a simple beam:

$$P^s_{min} = \frac{Q + Q_{truss} + G}{2} - P^s_{max}, \qquad (244)$$

in which Q—the capacity of the crane,
Q_{truss}—the weight of the trusses,
G—the weight of the crane buggy.

All crane standards give crane weights, including crane buggies.

Horizontal thrust is created when braking either the transverse

* This concerns capacities up to 50 tons.

movement of the buggy on the crane trusses or the longitudinal passage of the crane proper.

The force arising from transverse braking is usually computed by means of the formula

$$Br^{s}_{trans}=\frac{Q+G}{20}; \tag{245}$$

The value Br^{s}_{trans} is borne by one crane rail and is divided equally between the two wheels on that rail.

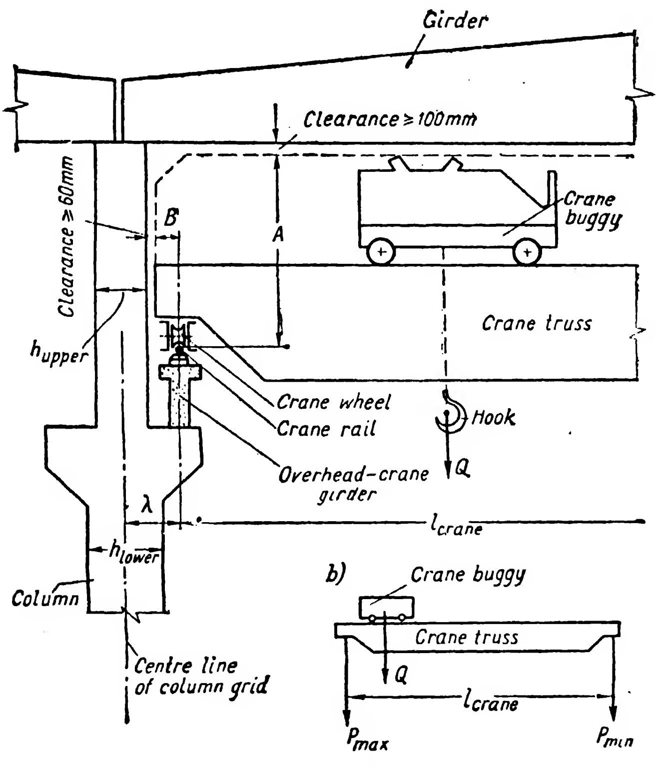


Fig. 144. Schematic cross-section showing the crane and its rail

The force arising from longitudinal braking is determined by the formula

$$Br^{s}_{long}=\frac{P^{s}_{max}}{10}. \tag{246}$$

A load coefficient $n=1.3$ is introduced into the computations of the framing structure when vertical and horizontal crane loads are included therein.

Rapid handling of loads by the crane and impact caused by rail unevenness (especially at rail joints) subject the crane girders to dynamic loads, therefore in the computation of crane girders a dynamic coefficient $k_{dyn} = 1.2$ is introduced into the values of vertical loads and horizontal thrust. Other structural members (columns and foundations) are considered as not effected by this dynamic action.

3. The Layout of the Building

The column grid chosen must satisfy both manufacturing processes to be conducted in the building and comparative technical and economic design-evaluation factors. Experience has proved that the best column spacings are 12×18 and 12×24 m, and—where large machines are to be installed—12×30 and 12×36 m. In all cases the outer line of columns may be spaced at intervals of 6 m, but a 6-metre spacing for interior columns is acceptable only for buildings with one or two spans.

All columns are disposed in relation to the centre lines of the grid layout (Fig. 145): in the case of intermediate columns, their geometric centres will coincide with the grid lines, while the outer columns are placed with their outer surfaces aligning with the grid centres.* Such a disposition of outer columns is known as *edge centering* and makes it possible to retain standard dimensions in marginal roof members and thus standardise structural elements.

The lateral geometrical centres of the columns will coincide with the grid lines, with the exception of the paired columns at compensation joints and those at the ends of the building; these columns are offset 500 mm. Just as in the case of longitudinal centering, the end columns are also edge-centred. Such a centering of walls and columns along the length of the building achieves standardisation of roof panels, equalisation of lengths of overhead-crane girders and tie beams, and unification of beams at the building's ends and at compensation-joint spans.

The distance between the longitudinal grid centre line and the centre line of the crane beam λ (Fig. 144,*a*) depends upon the dimension of the crane B, the dimension of the column above the crane beam h_b, and the required clearance c between the crane and the column. For outer columns

$$\lambda = B + h_b + c,$$

and for intermediate columns

$$\lambda = B + 0.5h_b + c.$$

* This latter is valid when the maximum capacity of cranes is 30 tons.

On the basis of the above, a standard dimension in typified construction $\lambda=750$ mm. With this arrangement, any standard crane span$=l_{crane}$ will fit into all the bays of the building, whether at the sides or in the middle:

$$l_{crane}=l-2\lambda=l-1.5 \text{ m},$$

in which l—the transverse grid span.

In order to reduce vertical size ranges when there is no crane in a one-storey factory, 4, 5, 6 and 7 m heights are adopted from the floor level to the underside of the roof.

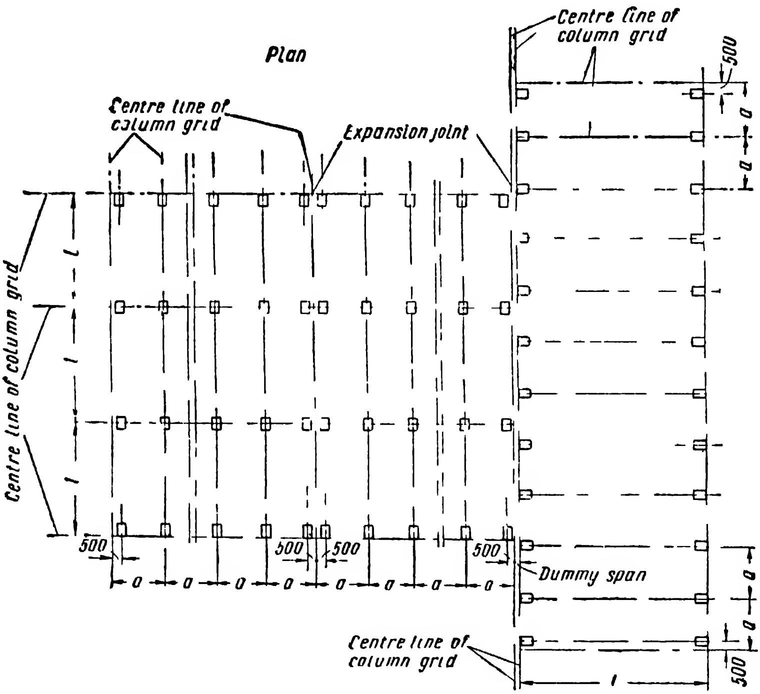


Fig. 145. Column grid for an industrial building

In buildings equipped with cranes, the heights from the floor level to the head of the crane rail are 6, 8, or 10 m for 5-20 ton cranes with spans up to 24 m, and 10, 12, or 14 m for 10-50 ton cranes with spans of 24 to 30 m. 250 mm is the nominal distance from the top of the crane girder to the head of the rail. Above this point, heights are established in multiples of 200 mm and will depend upon the height of the crane and its required clearance.

All single-storey buildings must be divided into transverse and longitudinal compensation blocks by means of compensation joints.

Transverse compensation joints are achieved by the pairing of columns, as already described (edge centres, Fig. 145).

Due to technological regimen in a number of industries (e.g., machine assembly, etc.), it is sometimes required that transverse spans be arranged adjacent to the longitudinal layout. Such a perpendicular grid orientation is achieved by separate rectangular blocks divided by a joint. This joint must satisfy the requirements of both temperature changes and foundation settlement, the latter because the transverse bay is usually higher than its longitudinal neighbour and

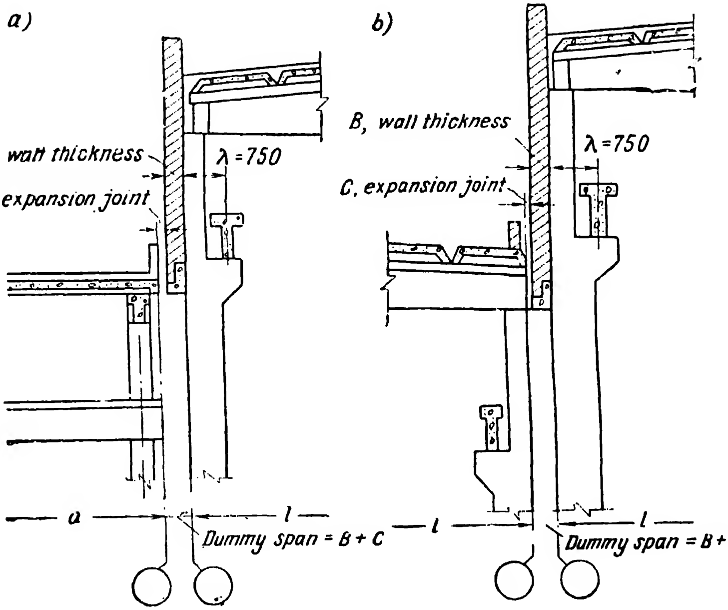


Fig. 146. Details of compensation joints

also carries a greater crane load. With such a layout the members of the transverse bay may also be typified. To do this a *dummy span* (the sum of the wall thickness and the joint width) is inserted between the two perpendicularly orientated blocks (Fig. 146,*a*). The columns of the transverse bay are edge-centred in relation to the grid lines* while the end row of columns of the longitudinal block are placed with a standard offset of 500 mm.

When longitudinal compensation joints occur between spans of equal height, sliding expansion bearings are used at the column supports for one of the girders (or trusses) of the roof. The columns sup-

* This latter is valid when the maximum capacity of cranes is 30 tons.

porting such expansion bearings must be shortened (200 mm) in height.

When a building has two heights, the longitudinal compensation joint is usually arranged along this broken-height line, with columns paired for the purpose. Here also a dummy span, equal to the wall thickness plus the joint width, is fitted in (Fig. 146,*b*). Both of the paired columns are edge-centred to the grid line.*

In all, it must be kept in mind that the basic principle underlying the layouts described above is to retain standard members everywhere, regardless of compensation joints or varying roof heights.

Sec. 40. TRANSVERSE BENTS

1. The Structural Diagram. The Main Beam

In a transverse bent the main beam may be made either with a solid or with a framed web (a girder or a truss, respectively), and with a rigid or a hinged connection to the column. The choice in the girder's profile, its type, and the type of column connection will depend upon the length of the span, the kind of roof, the manner of erection, the conditions of subsequent maintenance, and the intended cost of the building. As illustrated in Fig. 147,*a*, main beams may be made with straight or broken lines, or may be curved (arched), with or without ties.

Bents built with rigid main beam connections are better statically (smaller bending moments in the bent) than those with hinged connections, but are less suited for prefabrication. Furthermore, bending moments will occur both in the column and in the main beam independently of whether the stresses are in the column (from overhead cranes, wind thrust, etc.) or in the main beam (Fig. 147,*b*). Such interlocking of forces in single-storey crane-equipped buildings render standardisation of members extremely difficult.

On the other hand, in transverse bents made with hinged column-beam connections (Fig. 147,*c*), both the column and main girder can be standardised independently of each other since the loads applied to either one will not cause bending moments in the other; moreover, the form of both elements is simplified, as is also their mutual jointing (less steel is required), and if both members be rationally chosen from the point of view of mass production, they will prove more economical in spite of the somewhat greater bending moments stimulated within them in comparison to rigidly connected bents.

The result of all this is that at the present time most framed one-storey buildings are designed with bents restrained at their founda-

* This latter is valid when the maximum capacity of cranes is 30 tons.

tions but with main beams (girders or trusses) built with hinged connections to the columns (Fig. 148,*a*).

As a rule, prestressed girders are employed as main beams for spans up to 18 m, and trusses used if the span is greater. However, girders are satisfactory for 24-metre spans if the dead and snow loads are not too great.

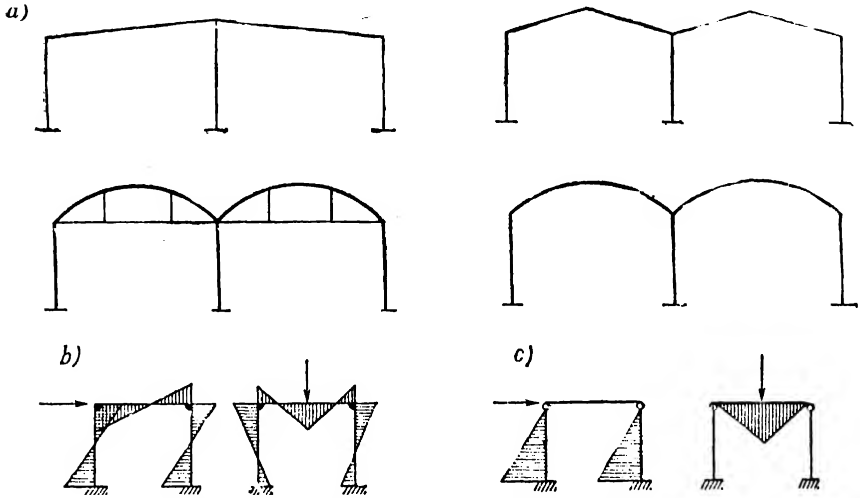


Fig. 147. Investigation of single-storey transverse bents

The main beam is connected to the column by means of anchor bolts protruding from the column (Fig. 148,*b*), for which purpose the main beam is either perforated or supplied with a notched steel plate welded to insertions at the supports before being raised into position. As an added measure, these bearing plates are also welded to similar plates inserted in the column. If the connection is at a compensation joint, one of the beams is supported on a sliding (expansion) bearing and the other on a fixed chair, with both resting on a shortened column.

2. Columns

Columns in buildings equipped with cranes have double crane-beam brackets for the intermediate column and only one bracket for outer columns. Structurally the column may be either a solid rectangular or H-section, or be perforated, that is, two-legged (Fig. 149).

The solid section is used for cranes with maximum capacities of 30 tons and maximum column heights of 15 m. With greater crane capacities the perforated column is more rational. H-columns are more economical than the rectangular (using 30% less concrete and 6% less steel) and are therefore adopted as a standard.

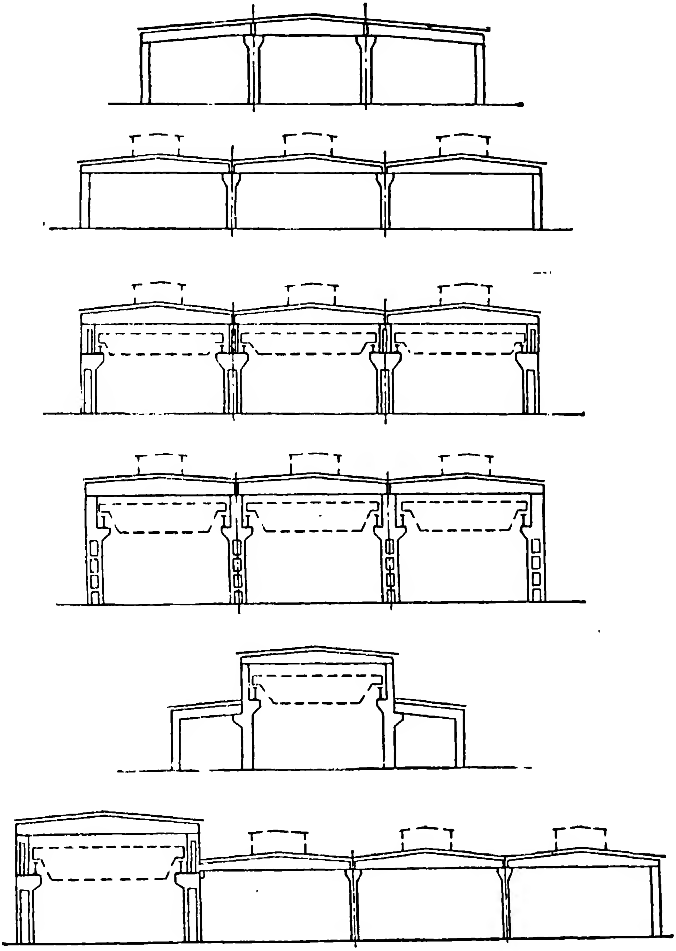


Fig. 148a. Schematic cross-sections of single-storey industrial structures

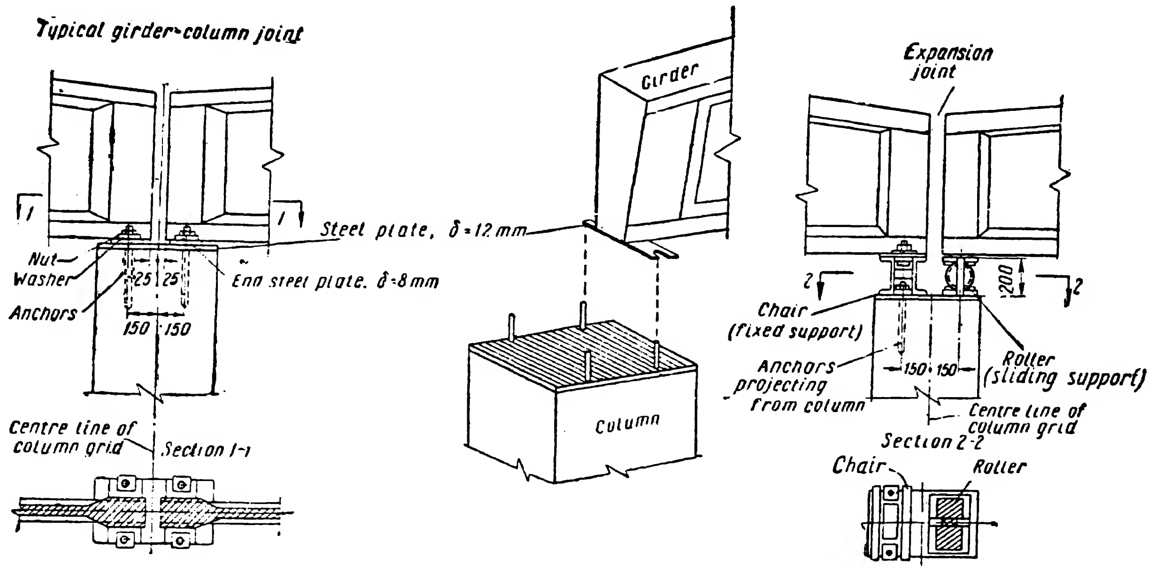


Fig. 148b. Connections of main beams to columns

The upper part of the column—from the crane-girder bracket to the roof—has a length H_{upper}. Here the cross-sectional height h_{upper} will depend upon required bracketless supporting area for the main beam and clearance for the passage of the crane.

The lower part of the column—from the foundation to the crane-girder bracket—has a length H_{lower}. Its cross-sectional height h_{lower}

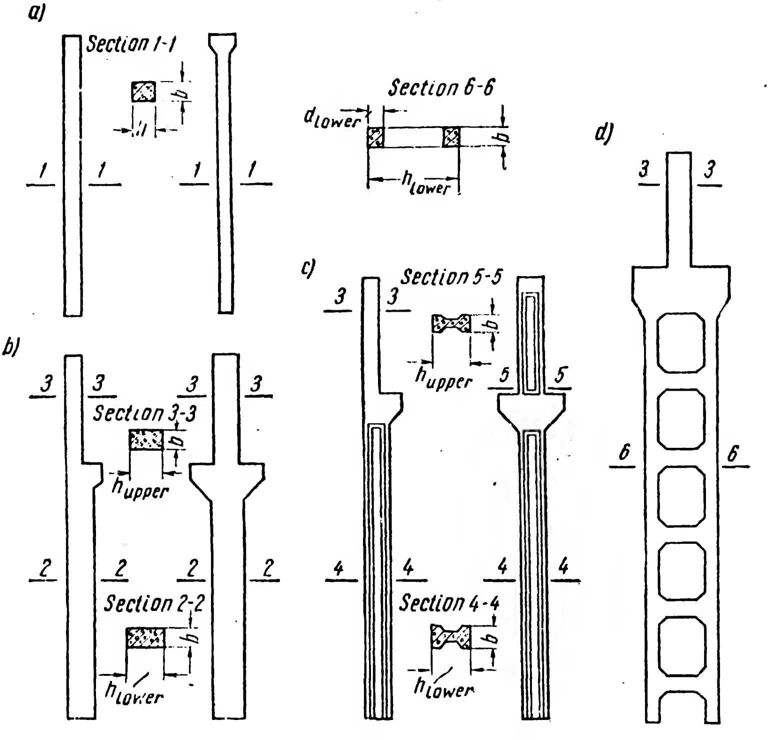


Fig. 149. Types of columns

a—for buildings without cranes; *b*, *c* and *d*—for buildings with cranes: a rectangular column, an H-column and a perforated (two-legged) column, respectively

is governed mostly by requirements of carrying capacity and column rigidity (the latter in its turn requiring that no crane wedging occur when column deformation takes place in the plane of the bent). It has been empirically established that the column will have sufficient rigidity if the cross-sectional height

$$h_{lower} = \text{from } \frac{1}{10} \text{ to } \frac{1}{14} H_{lower}$$

and if the cross-sectional width

$$b = \text{from } \frac{1}{20} \text{ to } \frac{1}{25} H_{lower}.$$

In crane-equipped factory buildings, cross-sectional width of columns $b=40$ cm when the column spacing is 6 m, and $b=50$ cm when the spacing is 12 m. In the part of the column above the crane-girder bracket, $h_{upper}=60$ cm for intermediate columns and 38 or 40 cm for outer rows of columns. For the part beneath the crane-girder bracket, h_{lower} will be 60 or 80 cm, depending upon the column's height and crane capacity.

Perforated columns are made with two rectangular legs below the crane-girder bracket (connected by cross-pieces at intervals of 1.5-2.0 m) and with a solid rectangular cross-section above the crane-girder bracket. For convenience in manufacture, the cross-sectional width is made alike both above and below the crane-girder bracket.

For craneless buildings, column cross-sections are made less than for columns bearing crane loads; but because of the factor of rigidity that enters the computations, $h \geqslant \frac{1}{25} H$ and $b \geqslant \frac{1}{25} H$, in which H is the height of the column from the top of the footing to the bottom of the roof beam. In intermediate columns their tops are made with brackets as a support for the beam, but in the outer line of columns this bracket is not required. Cross-sections of columns for craneless buildings are 30×30, 40×40 and 40×60 cm.

The best computation planes in columns will be found to be directly above the crane girder for the upper part of the column, and the lower and upper planes in the lower length of the column. The required amount of bars F_s and F'_s for each cross-section is found by means of eccentric-load formulae. In determining the coefficient η, column flexibility (the long column) is taken into account and its effective length l_0 established, depending upon the way its ends are connected. Columns of single-storey buildings are fixed at the bottom and hinged at the top, with horizontal expansion connections at the supports; horizontal expansion is made to include all the columns under the roof of an entire compensation block.

The following effective lengths of columns are assumed, based on the above-mentioned method of interbracing:

a) In the plane of the bent: $l_0=H_{lower}$ for the part below the crane girder when the crane load is included, and $l_0=1.25H$ when the crane load is not included; $l_0=2.5H_{upper}$ for the part above the crane-girder bracket.

b) Perpendicular to the above plane: $l_0=H_{lower}$ for the length of the column below the crane-girder bracket, and $l_0=1.25H_{upper}$ for that above the crane-girder bracket.

c) For columns in craneless buildings: $l_0=1.25H$ for the column's plane in either direction.

Concrete grades 200-400 are used for the columns.

When bar areas have been determined for the column's computation planes, its stability outside the plane of the bent must be

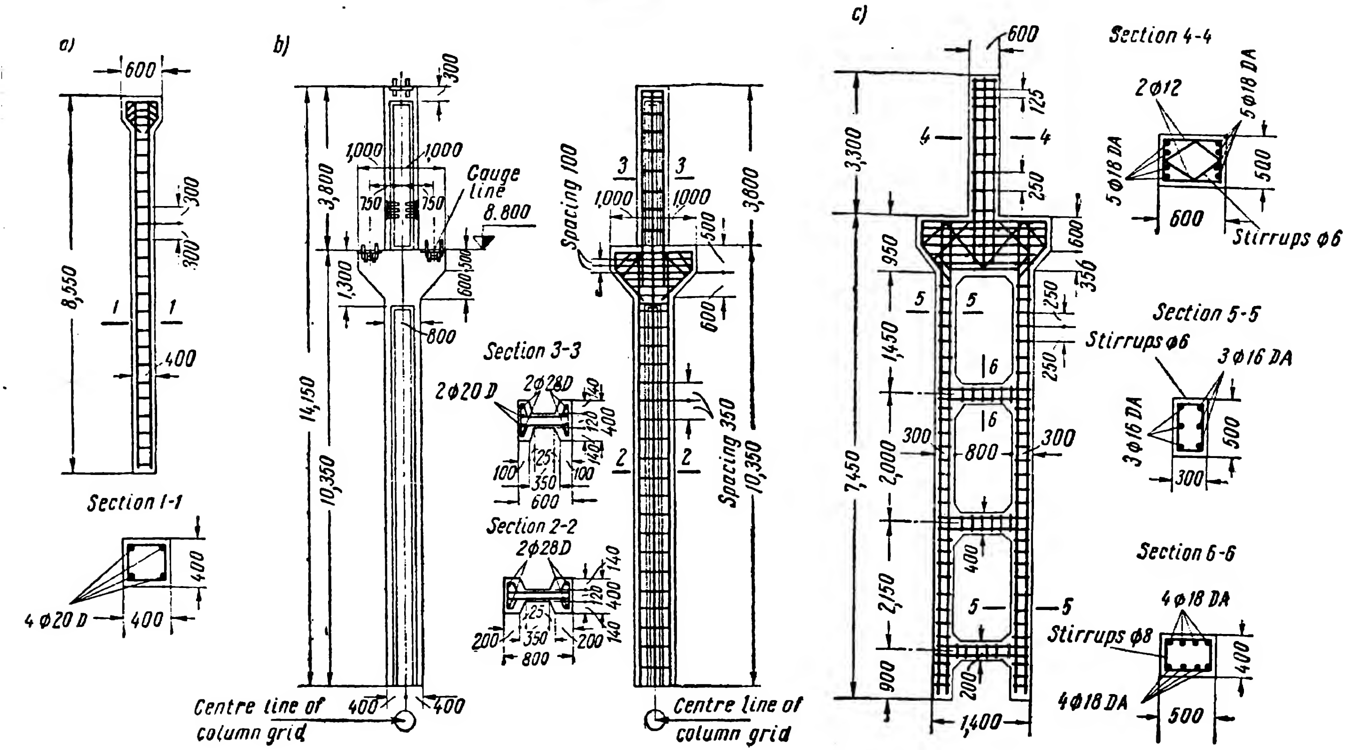


Fig. 150. Reinforcement for intermediate columns

a—a rectangular craneless column; *b* and *c*—crane-supporting columns: H-column and two-legged column, respectively

checked as for an axially loaded member. Furthermore, as already described in Sec. 22, Item 3, its static behaviour during erection operations must be checked.

Intermediate columns are made symmetrical in form and are subject to both plus- and minus-moments, for which reason they must be symmetrically reinforced. Side-row columns may be reinforced unsymmetrically. A 45°-return is the usual angle beneath the brackets that support the crane girder.

As already described in Sec. 26, Item 3, brackets are computed to resist the load of the crane girder Q. Formula (212) is employed for computing bent-up bars.

Examples of reinforcement for intermediate, crane supporting columns are shown in Fig. 150; inserted details (steel plates and bolts) are provided for connecting roof beams and crane girders.

Sec. 41. STRUCTURAL TYPES OF ROOFS

Roofs may be built either of straight bending members—with or without purlins—or canopied in the form of vaulted or plicated shells.

When there are no purlins, the roof deck is laid in the form of giant precast panels, with spans of 6 and 12 m, supported directly on the girders (Fig. 151, *a*).

When purlins are used they bear upon the girders and carry slabs whose spans are either 3 or 1.5 m (Fig. 151, *b*).

1. Roofs without Purlins

Uninsulated roofs of industrial buildings are built of giant 6×3 and 12×3 m prestressed reinforced concrete panels. Additional slabs, 6×1.5 and 12×1.5 m, are available as fillers.

If the roof is to be insulated, giant reinforced foam-concrete slabs are used, 1.5 m in width, which serve the double purpose of decking and insulation. Another method is to place a layer of insulation over the first-mentioned reinforced concrete slabs.

Reinforced concrete roof slabs are made of grade 200-400 concrete and with longitudinal and lateral ribs (Fig. 152). As already stated in Sec. 26, such panels are computed and designed in the same manner as floor slabs. All roof panels, no matter what their width or length, are made with a minimum flange thickness of 25 mm.

Giant reinforced foam-concrete slabs, after pressure-steam curing, have a dry density of 750 kg/m^3 and possess a minimum ultimate compressive resistance of 40 kg/cm^2. Grade 150-200 heavy concrete is used for their ribs, which are reinforced with prefabricated blocks. Their flanges are reinforced with cold-drawn prefabricated mats.

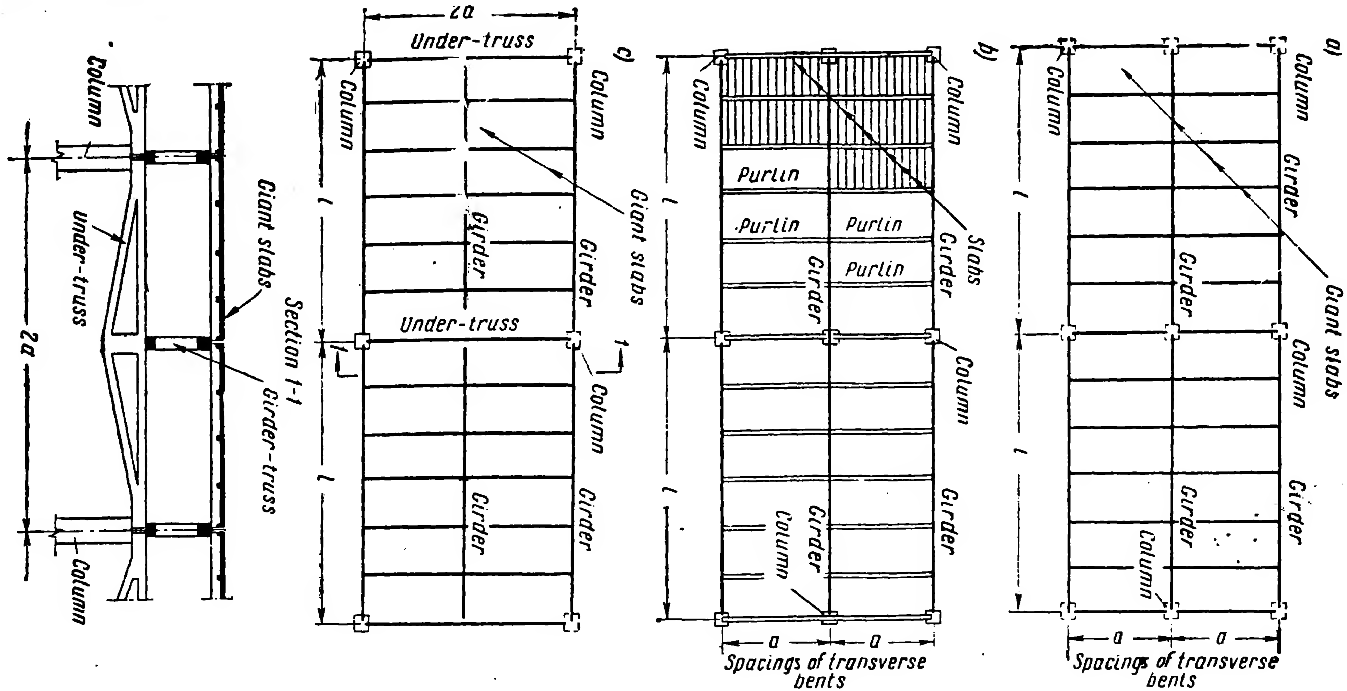


Fig. 151. Structural types of roofs
a—without purlins; *b*—with purlins; *c*—without purlins and with an undertruss

The thickness of flanges (10, 12, 14, or 16 cm) is chosen according to required thermal resistance.

In single-span bearing-wall buildings that have no monitors, the roofs are built of reinforced concrete segment-arch panels with tendons stretched around their perimeters (Fig. 152, *d* and *g*).

Comparative design-evaluation factors for various types of giant roof panels are presented in Table 25, from which it may be seen

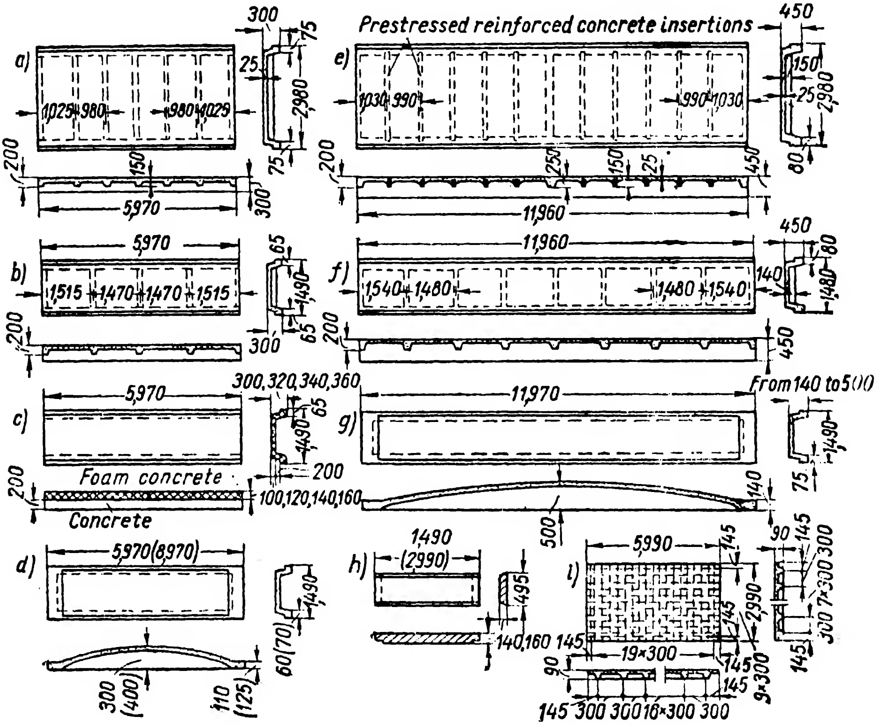


Fig. 152. Roof panels

a—6×3 m of reinforced concrete; *b*—6×1.5 m of either reinforced concrete or lightweight-aggregate (blown-ceramic) reinforced concrete; *c*—6×1.5 m of reinforced foam concrete; *d*—6×1.5 m and 9×1.5 m segment-arch reinforced concrete slabs; *e*—12×3 m of reinforced concrete; *f*—12×1.5 m of reinforced concrete; *g*—12×1.5 segment-arch reinforced concrete slabs; *h*—3×0.5 and 1.5×0.5 m (laid on purlins) of cellular concrete, pressure steam-cured; *i*—3×6 m (laid on purlins) of roll-pressed reinforced concrete

that giant slabs of 3-metre width are the most efficient, since they satisfy the requirements of maximum precast sizes and their 3-metre width also excludes the possibility of a bending moment in the upper chord of the trusses, inasmuch as their loads are transmitted via the truss joints.

Giant roof panels are placed with their end ribs bearing upon the main beams (trusses or girders); steel plates inserted in these latter

Table 25

Comparative Design-Evaluation Factors for Giant Roof Panels

Type of panel and its nominal dimensions, mm	Height of rib, mm	Weight, kg (total)	Weight per m², kg	Transformed thickness of concrete cm*	Grade of concrete	Reinforcement		kg of steel per m² of slab at a design load of 400 kg/m²
						Tendons	Prefabricated blocks and mats	
6,000×3,000 of reinforced concrete	300	2,400	135	5.3	200	—	St-5, cold-drawn wire	6 4
6,000×1,500 of reinforced concrete	300	1,430	160	6 3	200	—	St-5, cold-drawn wire	6 8
6,000×3,000 of prestressed reinforced concrete	300	2,400	135	5 3	300	25Г2С drawn-hardened	25Г2С, cold-drawn wire	5 5
						30ХГ2С	Ditto	5.2
6,000×1,500 of prestressed reinforced concrete	300	1,430	160	6 35	300	High-strength wire	25Г2С, cold-drawn wire	3 3
					200	25Г2С drawn-hardened	Ditto	4 2
					200	30ХГ2С	Ditto	3 8

(Continued)

Type of panel and its nominal dimensions, mm	Height of rib, mm	Weight kg (total)	Weight per m², kg	Transformed thickness of concrete, cm*	Grade of concrete	Reinforcement		kg of steel per m² of slab at a design load of 400 kg/m²
						Tendons	Prefabricated blocks and mats	
6,000×1,500 of concrete with blown ceramic aggregate	300	1,020	115	6.35	200	25Г2С drawn-hardened	25Г2С; cold-drawn wire	4.15
						30ХГ2С	Ditto	3.95
6,000×1,500 of reinforced foam-concrete, thickness of flange—from 100 to 160 mm	300-360	1,300-1,800	146-145	$\frac{10.0\text{-}16.0}{2.7}$	Concrete—150 for ribs; foam concrete—40 for flange	—	25Г2С; St-3; cold-drawn wire	6.5-6.1
12,000×3,000 of reinforced concrete with lateral ribs of prestressed concrete	450	5,900	165	6.6	400	High-strength wire	St-5; cold-drawn wire	4.5
						30ХГ2С	Ditto	4.8

(Continued)

Type of panel and its nominal dimensions, mm	Height of rib, mm	Weight, kg (total)	Weight per m², kg	Transformed thickness of concrete, cm*	Grade of concrete	Reinforcement: Tendons	Reinforcement: Prefabricated blocks and mats	kg of steel per m² of slab at a design load of 400 kg m²
12 000×1,500 of reinforced concrete	450	4,200	237	9 45	400	High strength wire	25Г2С cold drawn wire	4 4
		4,000	225	9 0		30ХГ2С	Ditto	5 4
6,000×1,500 of reinforced concrete, vaulted	110 300	1,200	135	5 5	300	High strength woven strand	Cold drawn wire	3 2
						25Г2С	Ditto	4 15
9,000×1,500 of reinforced concrete, vaulted	125 400	2,100	157	6 2	300	High strength woven strand	Cold drawn wire	4 2
						25Г2С	Ditto	6 1
12,000×1,500 of reinforced concrete, vaulted	140 500	3 100	174	7 05	300	High strength woven strands	Cold drawn wire	4 5
						25Г2С	Ditto	7 5

* In reinforced foam-concrete slabs the numerator gives the thickness of the foam concrete and the denominator indicates the thickness of the heavy concrete

are welded to angles similarly inserted in the ends of the panels. The sequence of laying the panels allows them to be welded to their supports at three corners. Their continuity is achieved by filling their joints with cement grout.

Two structural alternatives are possible in the roof layout when the transverse bents are spaced at intervals of 12 m: 6-metre panels resting on under-girders or under-trusses, or 12-metre panels with neither under-girders nor under-trusses. In the first case the intermediate spanning members supported on the under-trusses (or under-girders) create a 6-metre spacing for the roof decking (Fig. 151, *c*). The same type of girder or truss is used for both the intermediate bearing members and the main beams of the bent. To achieve this, the under-girders (or under-trusses) are correspondingly centred along the longitudinal grid centres.

2. Roofs with Purlins

For insulated roofs that are supported on purlins, use is made of 3×0.5 m or 1.5×0.5 m cell-concrete slabs (pressure steam cured), or insulation-covered rolled reinforced concrete ribbed slabs of 3-metre span and 6-metre width (Fig. 152, *h* and *i*).

Table 26

Comparative Design-Evaluation Factors for Purlin-Supported Slabs

Kind of slab and its nominal dimensions, mm	Height of rib, or, if a flat slab, its thickness, mm	Weight, kg (total)	Weight per m	Concrete grade	Kind of reinforcement*	Index to amount of materials: Transformed thickness, cm	Index to amount of materials: Steel per m², kg
Flat slab of reinforced pressure steam cured concrete (reinforced foam-concrete) 3,000 × 500 ...	140-160	172-196	116-132	$\geqslant 40$	St-3 steel	14-16	5.5-5.2
Ditto, 1,500×500	140-160	86-98	114-132	$\geqslant 40$	Cold-drawn wire	14-16	5.2-5.4
Rolled reinforced concrete slabs, 3,000×6,000	90	1,700	95	200-300	Ditto	3.95	3.05-4.45

* Prefabricated mats.

Slabs of cellular concrete are made without ribs and from 14 to 16 cm in thickness, depending upon required thermal resistance, and are reinforced with prefabricated mats.

The purlin system of roof construction is also best for uninsulated buildings where corrugated asbesto-cement sheet-roofing is used (with such sheets, an underroofing layer is not required).

Table 26 presents comparative design-evaluation factors for various types of purlin-supported slabs.

As may be seen in Fig. 151, *b*, 1.5- or 3.0-metre purlin spacings are employed. T- or channel-shaped purlins are used with 6-metre spacing of transverse bents (Fig. 153). Vertical rigidity ribs, spaced at intervals of 1.5 m, are introduced into prestressed channel-shaped purlins.

Purlins are connected by means of bolts to angles which are welded to steel insertions disposed in the upper part of the girder or truss (Fig. 153, *c*). Such angles not only aid in the proper placing of the purlins, but also resist the inclined thrust of the load.

The design diagram of a purlin is that of a uniformly loaded single-span beam. The normal component of the load q_y (which is equal to $q \cos \varphi$) causes the purlin to bend in its plane, while the inclined component q_x (which is equal to $q \sin \varphi$) will bend it in the plane of the incline. To aid the resistance of the purlins against the action of the inclined component, they are connected by means of steel ties at the centres of their spans, for which purpose a hole is left in the concrete. In the ridge purlin the said forces are mutually neutralised.

When the column spacing is 12 metres, the under-trusses (or under-girders) reduce the spans of the purlins to 6 metres.

3. Monitors

Monitors are usually located in the middle of spans. Their construction consists of lateral monitor trusses and transverse bents (or posts) resting on the main beams of the main transverse bents. But whatever the type of framework, it is made to bear the monitor roof—either of giant panels or small purlin-supported slabs—with the side-slabs resting against the outer posts.

The width of the monitor and the height of its fenestration are chosen according to lighting requirements of the given factory bay, but the most common width is 0.3-0.4*l*. In order to standardise monitor construction, a 6-metre width is adopted for maximum 18-metre bay spans and a 12-metre width when the bay is 24 metres or over (Fig. 154).

Precast reinforced concrete monitor trusses are prefabricated in three parts which are connected in erection: if the overall width is 6 metres there will be one strut-piece and two posts, the latter in the

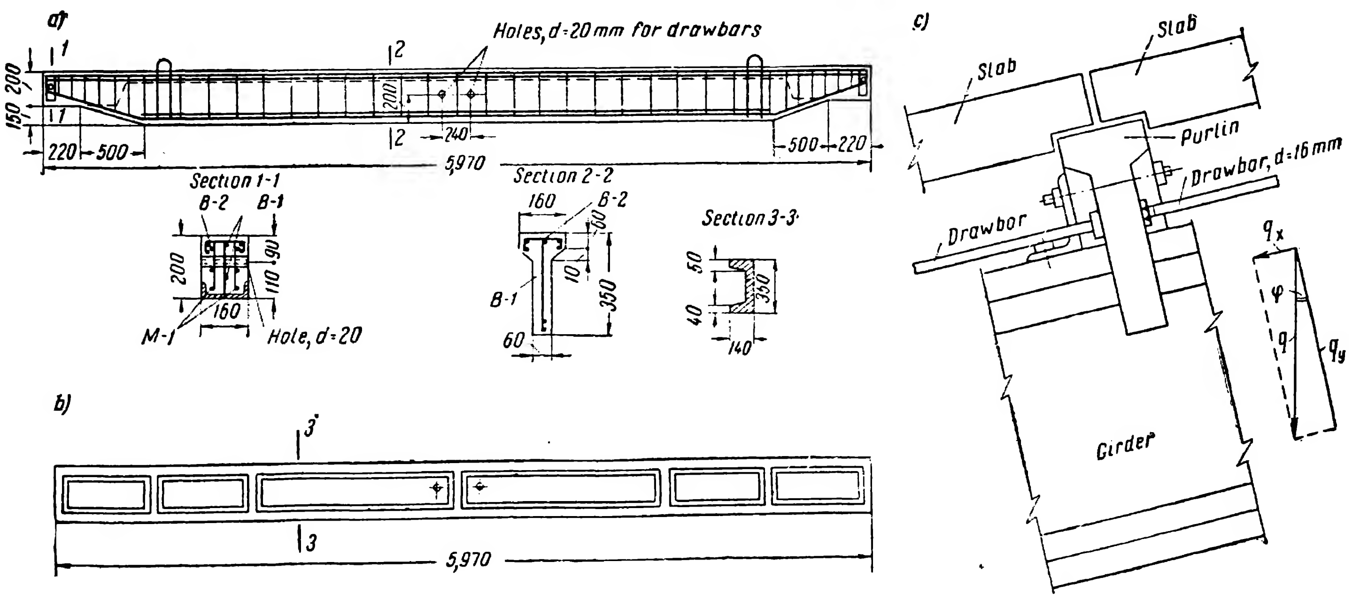


Fig. 153. Reinforced concrete roof purlins

plane of the fenestration; but with a width of 12 m, it will be made up of three strut-pieces. The system of roofing for the monitor is the same as for the main roof of the building (with or without purlins).

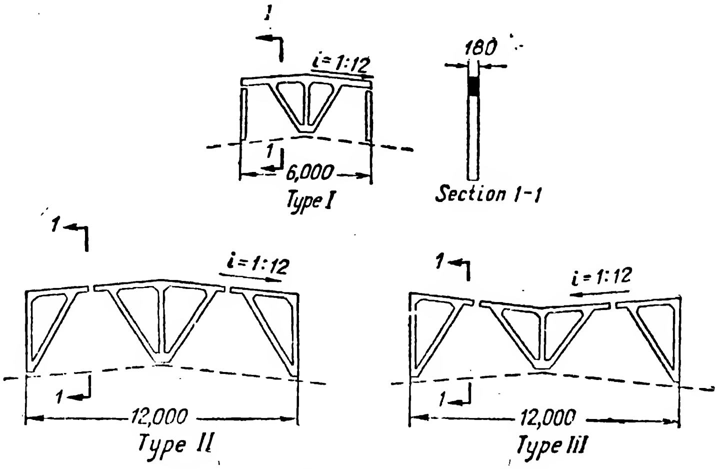


Fig. 154. Types of monitor frames

The posts of monitor trusses are bolted to the main beam of the main bents during erection and subsequently the inserted parts of their connections are welded together.

4. Canopy-Type Roof Construction

Buildings are very effectively roofed by means of thin canopy construction in the form of various types of composite (combined in-situ and precast) vaulted or plicated shells, especially when large unsupported areas are to be covered. In comparison with the beam-type roof, such canopies achieve a saving of at least 25% in steel and concrete. As regards cost, canopies are as yet more expensive than flat construction, but their material economy, as well as the great spans that they can roof, justify their use. If adopted on a mass scale, their cost should decline sharply and make them cheaper than flat construction. The best places to use such canopies is for buildings without overhead cranes.

Fig. 155,a shows a two-way curvature shell thrown over a 40 ×40 area. The slabs which form this shell are 25-40 mm thick and ribbed about their perimeters. Four bordering arches with composite upper chords and precast-prestressed lower chords serve as stiffening diaphragms. There have been two examples of this type of roof shell built in Leningrad. Eight cm is the transformed thickness of the concrete in this shell and 11 kg of steel enter each horizontally projected square metre.

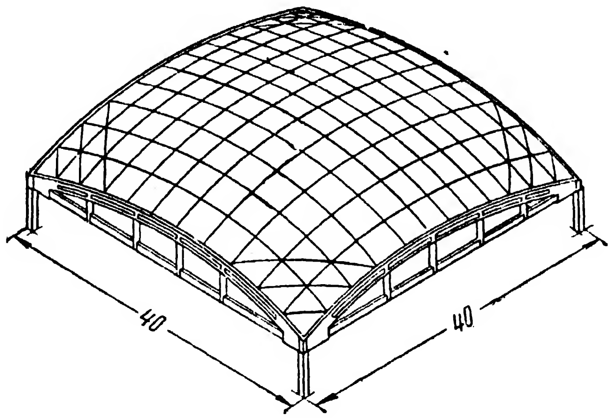


Fig. 155a. A precast two-way curvature roof shell

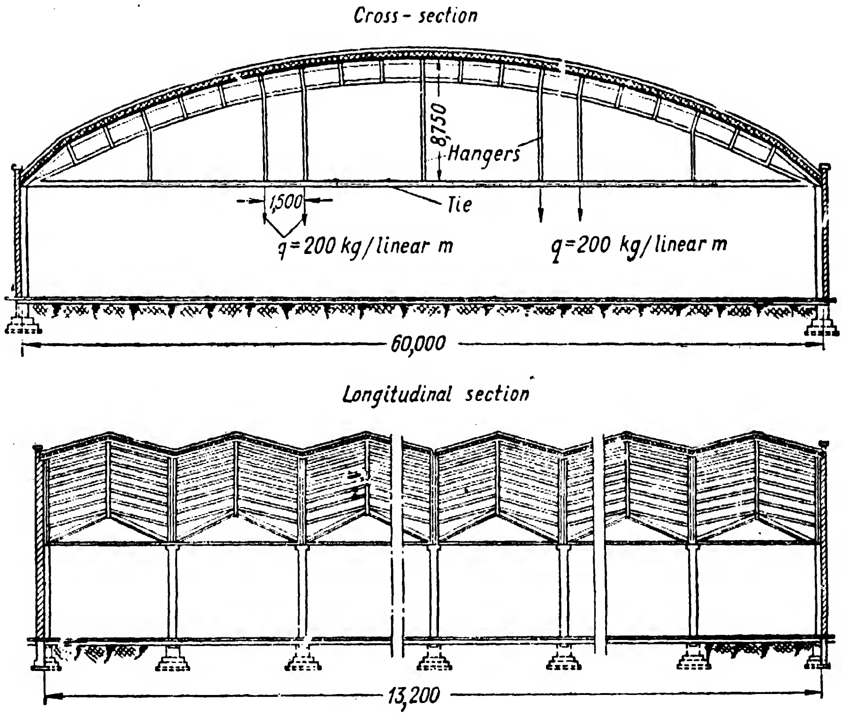


Fig. 155b. A composite roof, without monitors, built in the form of plicated arches

Fig. 155,b is a design for a roof without monitors to be built of composite, plicated multiple arches thrown over a 60-metre span. Each plication is arched in the plane of the span ($l_1=60$ m) and has a breadth $l_2=12$ m. Each plicated arch is assembled from 3×6 m prestressed ribbed slabs, with flanges ranging from 35 to 50 mm in thickness. Vertical mats are placed in the joints between the slabs, which are subsequently grouted with concrete. The tie rods of the arches are composed of stranded tensioned wire. The amount of reinforcement per m^2 of such a roof whose transformed thickness of the concrete is 8 cm, is 9 kg of bars.

Sec. 42. BRACING IN FLAT ROOFS

The vertical and horizontal bracing included in the structural components of single-storey frame buildings have the following functions: 1) to lend rigidity to the roof as a whole, 2) to give stability to the compression flange of the girders of the transverse bents, 3) to resist wind thrust transmitted from the end walls of the building, 4) to bear the braking forces created by the cranes. The entire system of bracing, acting together with the principal members of the frame, assure structure skeletal rigidity.

1. Vertical Bracing

As already stated, longitudinal horizontal thrust (wind acting upon the end of the building, crane braking, etc.) is taken up by the longitudinal framework, with the roof as a whole acting in the capacity of a main beam (Fig. 143,*c*). The roof slabs are interconnected to the columns via the girders (or trusses) which in themselves possess very little rigidity outside their working planes. Hence if there were no bracing, the horizontal forces transmitted to the roof would cause considerable deformation of the girders outside their working plane (Fig. 156, *a*), while if a horizontal force were applied to one of the columns it would deform without obtaining any aid from the other columns (Fig. 156, *b*).

Geometric stability is created in the longitudinal direction of the roof by the vertical bracing arranged along the line of the columns (Fig. 157, *a*). In the spans at the ends of the building or at compensation joints, rigid longitudinal diaphragms are connected to the chords of the trusses and the tops of the columns, while in the intermediate spans of the compensation block longitudinal reinforced concrete spacers are installed at the top level of the columns.

If the trusses are very high at their supports, either a diaphragm in the form of a reinforced concrete truss plays the role of longitudinal bracing, or steel angle cross-bracing is installed. If solid-web girders

are used, no longitudinal bracing is needed; instead, widened ribs are built at the bearings which will transmit to the columns the horizontal thrust received from the roof slabs. In the latter case the steel bearing plates of the girders and the steel insertions of the columns must be connected with a weld capable of taking the moment of these forces $M=Wh$ and the bearing load of the girder N (Fig. 156,*c*).

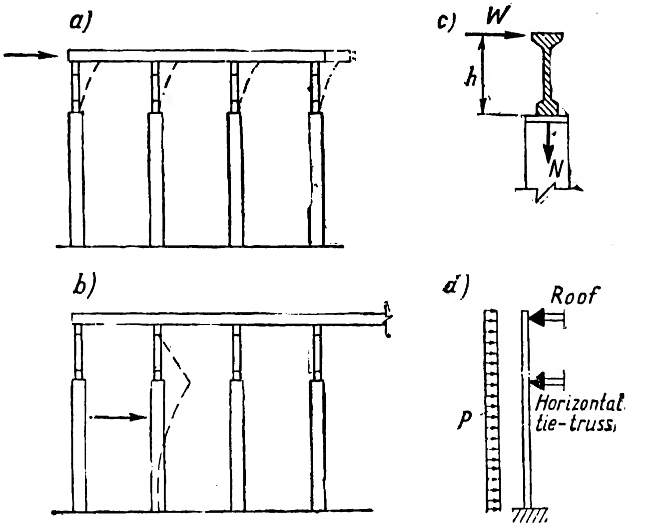


Fig. 156. Deformation of frame members due to horizontal thrust, and design diagrams

The rigidity of the columns is improved along the longitudinal line of the building frame by the installation of vertical bracing between the central columns of the compensation block (Fig. 157, *a*); this cross-bracing is made from steel angles, the gussets of which are welded to the inserted details of the columns.

2. Horizontal Bracing Along the Lower Chords of Trusses

The thrust of the wind upon the end of the building causes the end wall to bend. To reduce the bending force, the roof is made to act as a horizontal support for the wall. But if the building is very high and has very long spans, the horizontal support for the wall should be arranged at the level of the lower truss chord; to do this the lower chord must be braced, otherwise it will not be able to support the wall horizontally. Such a horizontal support is achieved by means of a system of cross-bracing (of steel angles) connecting the lower

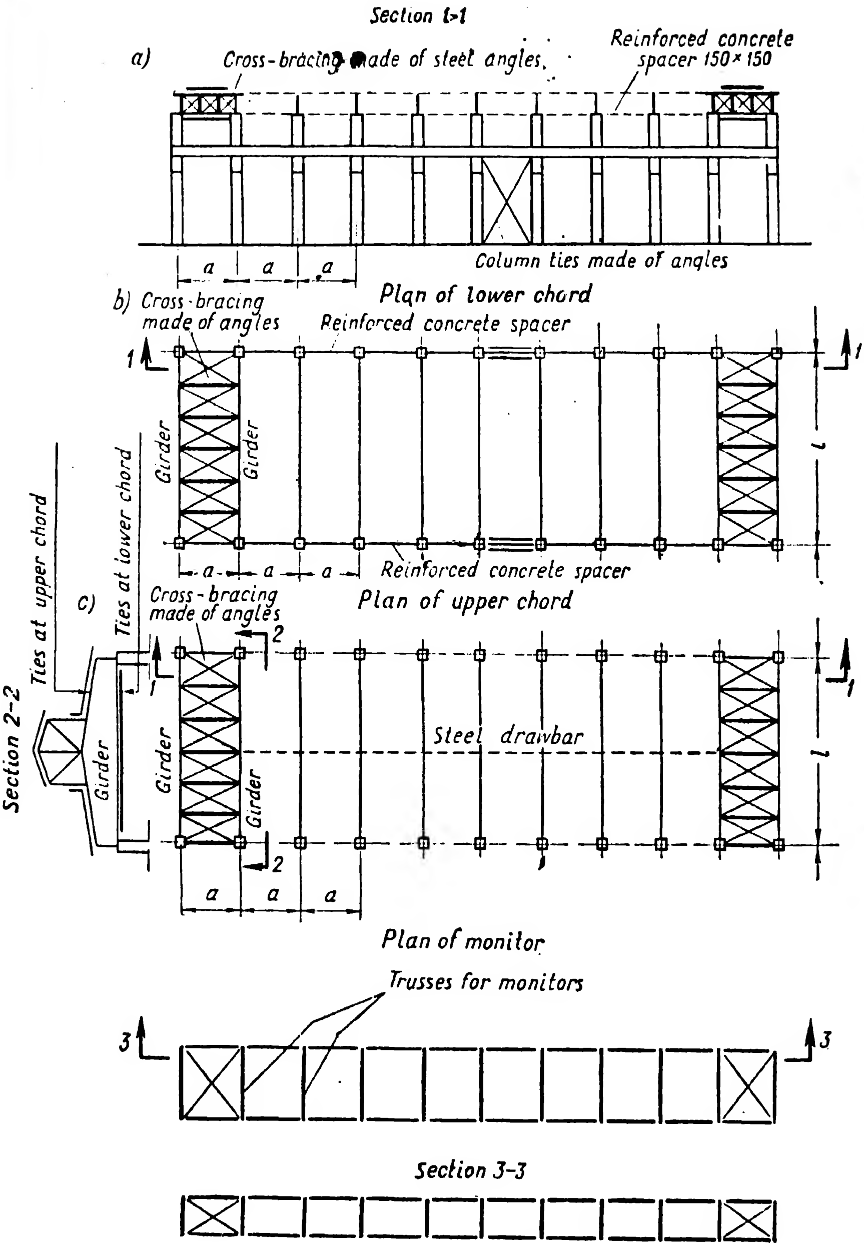


Fig. 157. **Schemes of horizontal roof bracing**

a—vertical bracing; *b*—lower-chord horizontal bracing; *c*—upper-chord horizontal bracing; *d*—monitor bracing

chords of the first two trusses at the ends of the building (Fig. 157, *b*). The thrust received by this bracing is transmitted, via the longitudinal vertical bracing, to all the columns of the compensation block and thus delivered to the foundations.

3. Horizontal Bracing Along the Top Chords

When the structural system of the roof does not include purlins, the stability of the compression zone of the bents' main beam (girder or truss) outside their working planes is assured by giant roof panels, which are welded to the main beams. But a monitor will create a free, unstable length of the beam's compression zone equal to the width of the monitor, and if the compression zone should lack sufficient width it may lose its non-working-plane stability.

This stability of the main beam's compression zone is assured by the installation of horizontal bracing in the end spans of each compensation block. As in the cases already described, the bracing is made of angle irons which are connected, either through steel ties or reinforced concrete spacers, to the ridge (Fig. 157,*c*). This cross-bracing, together with the upper chords (or flanges) of the two end main beams that are thus tied together, will form a horizontal truss that will also sustain the rigidity of the compression zone of the rest of the main beams. In this system the ties will be in tension while spacers will be in compression.

If the monitor does not extend to the end of the compensation block, no horizontal bracing is required in the end spans inasmuch as the reinforced concrete roof slab itself will then function in this capacity. The ties (or the spacers) must be connected to the roof of the end spans.

When the structural system of the roof includes purlins and small slabs, the roof will possess less horizontal-plane rigidity than with the non-purlin system; to overcome this shortcoming and increase the rigidity of the roof, the end spans of each compensation block must always be equipped with horizontal cross-bracing directly beneath the purlins.

4. Monitor Bracing

Steel bracing is used to create skeletal rigidity for the monitor trusses; this bracing will be vertical in the planes of the fenestration and horizontal in the line of the roof (Fig. 157,*d*).

Practical experiments have proved that when the components of a reinforced concrete roof of a frame building are well braced and the

inserted steel details properly welded and all joints grouted, the whole becomes a rigid horizontal diaphragm that binds all the columns at their tops into a single interconnected three-dimensional block.

Sec. 43. OVERHEAD-CRANE GIRDERS

Overhead-crane girders with spans of 6 metres may be cast either of ordinary, or of prestressed, reinforced concrete. But only the prestressed variety is satisfactory for 12-metre girders.

The T-shape is the best section (Fig. 158), the development of whose flange increases its horizontal rigidity against the braking action of the crane and also aids in its erection; subsequently it presents better conditions for servicing the crane and its rails.

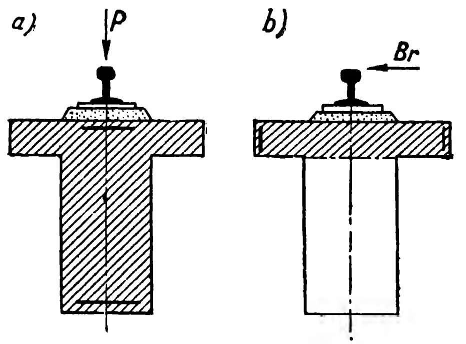


Fig. 158. Design cross-sections of an overhead-crane girder

a—for resisting vertical loads; *b*—for resisting horizontal thrust

If I-shapes are used, they are reinforced with bunched tendons; if the tendons are the endless type, the girder may be composed of two halves.

Producibility of the girder and its erection requirements dictate that its length be not more than one column span and that it be connected together at each column (Fig. 159).

Its strength must be computed on the basis of paired cranes of equal capacity, the dead weight of the girder and its rail also being included in the load.

As may be seen from the design diagram (Fig. 160, *a*), the position of the paired cranes will determine the concentrated loads P_{max}. A load coefficient n and a dynamic coefficient k_{dyn} are to be included in the vertical design loads of the cranes:

$$P_{max} = k_{dyn} n P^{s}_{max}.$$

The crane girder's horizontal design load as caused by transverse braking thrust (with two wheels on one rail)

$$Br_{trans} = \frac{1}{2} k_{dyn} n Br^{s}_{trans}.$$

Crane loads are mobile and may therefore occupy any position along the length of the crane girder; this requires that concentrated loads be assumed in the position along the girder's length that pro-

duces the maximum forces M and Q for the girder's cross-section. We already know that the greatest forces from a moving load are determined by influence lines, at whose apex one of the forces is charted. As may be seen from Fig. 160, *b*, the loads are determined

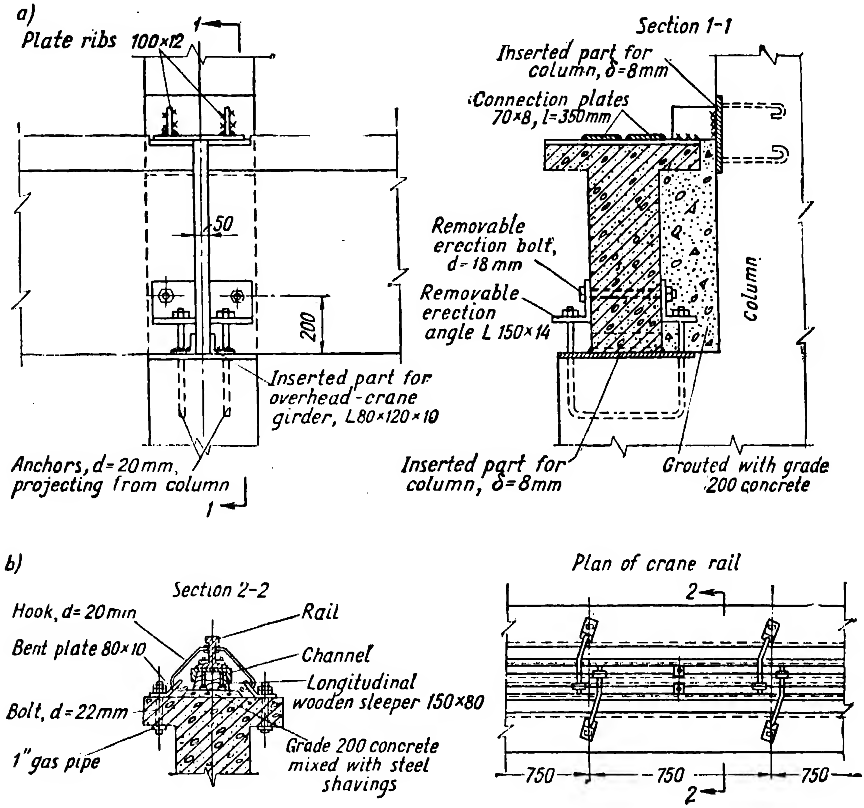


Fig. 159. Connection details for overhead-crane girders

as the sum of the computed forces at corresponding ordinates of the influence lines:

$$M = P_1y_1 + P_2y_2 + \text{etc.} = \Sigma Py.$$

The forces are determined for planes spaced at intervals of 0.1-0.2l, after which a greater-value diagram M and Q is plotted.*

Furthermore, M and Q, corresponding to the dead weight of the girder and the crane rail, must be included in the calculations.

* Tabulated greater values M and Q for the design of overhead-crane girders are presented in Supplement VI.

In order to obtain the most economical bar area and sufficient rigidity, the usual height of the cross-section h will be found to equal $1/6$-$1/8l$ (which may be reduced to $1/10l$ for a prestressed girder), the thickness of the upper flange $h_{fl}=1/7$—$1/8\ h$, and the width of the flange $b_{fl}=1/20l$. Standard heights of overhead-crane girders range from 600 to 1,400 mm with the intervening dimensions varying in multiples of 200 mm.

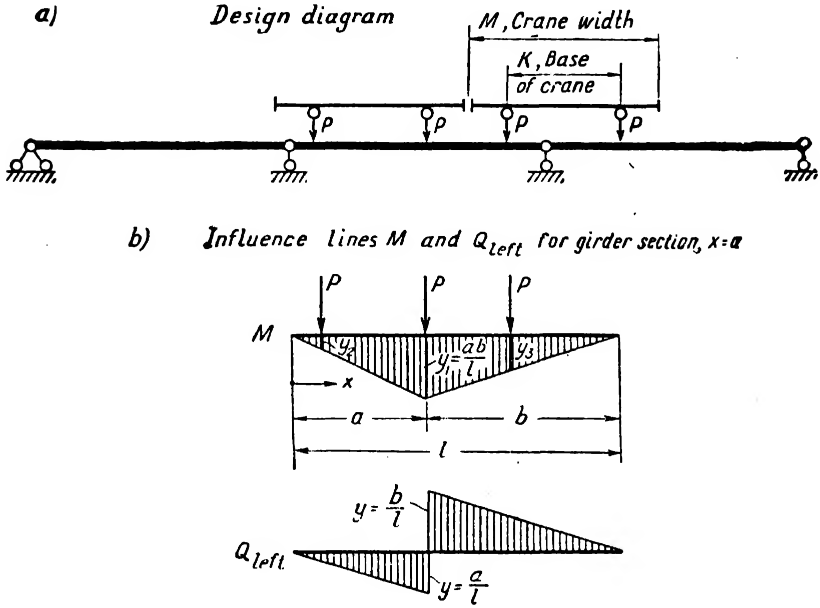


Fig. 160. Investigating an overhead-crane girder

The entire T-section of the crane girder is taken for vertical-load resistance, while the rectangular section of the flange only is assumed to bear horizontal thrust. Although this horizontal thrust is actually applied to the top of the rail, eccentricity is ignored in the calculations and it is assumed that the horizontal force is applied to the horizontal axis of the flange.

In verifying crane girder deformation, the specified loads are used without the dynamic coefficient. Deflection must not exceed $1/600\ l$. In general, deflection from the mobile load may be determined according to influence lines. Deflection in mid-span may also be approximately calculated through the formula

$$f=\frac{Ml^2}{10B},$$

where B—the member's rigidity, determined through formulae already given in Chapter III if ordinary reinforcement is used, and through formulae presented in Chapter VI when a prestressed girder is used;

M—the maximum moment in the larger-moment diagram M.

Not only must the girder be verified for deformation, but also for fissure formation and fissure widening. Fissure widths must not exceed 0.2 mm when ordinary reinforcement is used.

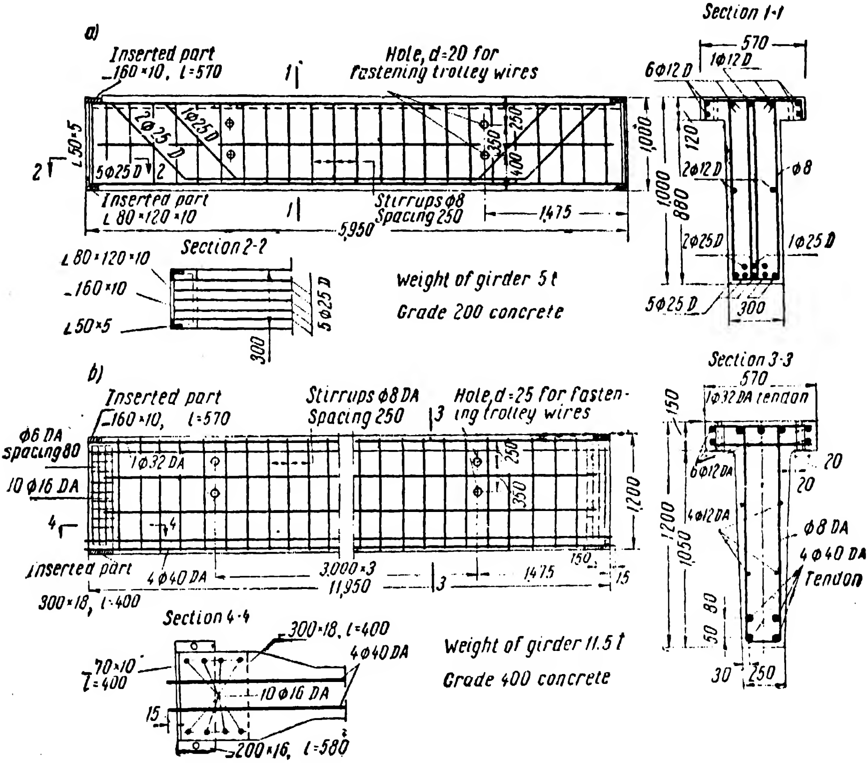


Fig. 161. Reinforcement for overhead-crane girders

a—a 6-metre girder for a crane whose $Q=20$ tons; b—a 12-metre prestressed girder for a crane whose $Q=15$ tons

If the girder is to be prestressed, its fatigue limit must be calculated inasmuch as the girder will be subject to repeated loadings. Such calculations consist in comparing the stresses borne by the concrete and tendons with corresponding fatigue-limit data.

At the present time tied reinforcement blocks are used for the effective reinforcement of crane girders because in prefabricated blocks of reinforcement the stresses become concentrated at the welded

points, cause fatigue of the metal due to an often repeated loading regimen, and eventually lead to a reduction of the member's resistance. Perhaps this difficulty will be overcome in the future through research in this field so as to allow the use of prefabricated reinforcement blocks in overhead-crane girders.

Fig. 161, *a* shows a 6-metre ordinarily reinforced concrete overhead-crane girder. Besides the inserted steel parts for its connection to the columns, it has 25-mm holes for hanging the trolley wires and also is furnished with inserted pieces of gas pipe for the rail connections.

Fig. 161, *b* illustrates a standard 12-metre prestressed crane girder the design of which takes into account the stand-method of its fabrication; for this reason all tendons are extended to the ends where they are weld-anchored to the inserted steel parts. As may be seen in Fig. 159, *a*, the connections between girders, as well as to the columns, are made by welding of inserted parts. Joint spaces (between butting girders and between girder and column) are grouted with concrete.

To insure horizontal stability and also for transmitting transverse horizontal thrust, ribs are welded to the upper inserted plates of the girder and to those of the column.

The rail sleeper is reinforced with a channel on top and fastened to the girder by means of hooked rail clamps (Fig. 159, *b*).

Sec. 44. IN-SITU CONSTRUCTION OF SINGLE-STOREY FRAME BUILDINGS

The lateral frame (transverse bent), just as in precast reinforced concrete, is the principal structural unit of in-situ reinforced concrete single-storey buildings, and supports either a beam-system or shell-type roof.

In order to attain maximum economy in the selection of the type of main beam for the transverse bent, the span will be the greatest factor to be considerd (Fig. 147,*a*). Straight main beams are used for 12-15-metre spans, broken-line contours for 15-18 metre spans, arches without tie rods for 18-24 metre spans, and arches with tie rods for spans of 36 metres and over. Bents with broken-line main beams are best for short vaults, which are the most economical type of roof construction.

The tie-rod girder does not allow horizontal displacement of the tops of the columns, lessens bending moments and shearing stresses in the members of the bent (Fig. 162, *a*) and thus simplifies its design. The same factors reduce the bending moment at the base of columns and also simplify the design of footings. Hence, as already stated, the main beam with a tie rod is the best for spans greater than 18 m. Still, not always is it possible to use tie rods: there are objections of fabrication, such as the need of too high manufacturing equipment, etc.

6 or 12 m is the usual spacing of transverse bents. These create lateral skeletal rigidity, while longitudinal skeletal rigidity is assured through the interconnections of columns and the longitudinal elements of the in-situ roof.

There are two ways of uniting the columns to the footings: fixed (restrained) and hinged connections.

Fixed connections are very simple and economical and are implemented merely by connecting the bars protruding from the footing with those of the column. Therefore the more complicated hinged type of connection is chosen when the fixed type would result in too

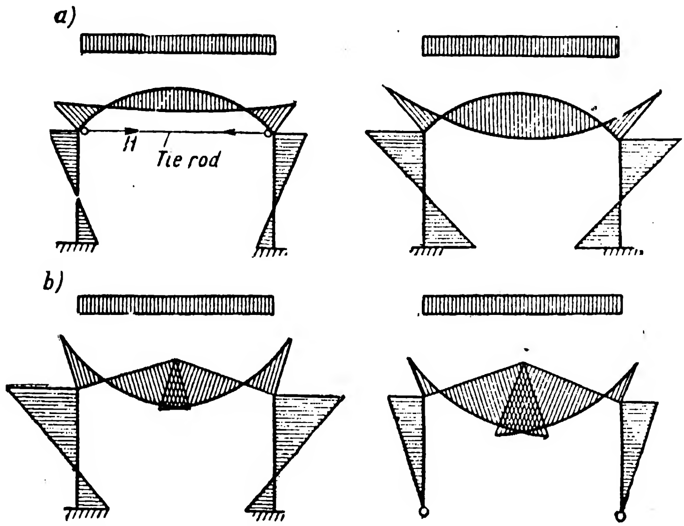


Fig. 162. Bending moments in single-storey transverse bents

great a bending moment at the column's base and too heavy a footing for a given weak bearing soil; for with the hinged type there is no moment at the base of the column and the footing may be smaller.

However, it must be kept in mind that a hinged base results in a greater bending moment in the main beam (Fig. 162, *b*), which must consequently be made somewhat heavier.

Just as in an ordinary bending member, part of the reinforcement of the bent's main beam must be extended to the supports at the negative zone and carried into the columns, which latter are reinforced just as eccentrically compressed elements. From the columns the bars are in their turn carried into the main beam.

Fig. 163,*a* shows a single-span transverse bent reinforced with spliced round bars, while a two-span prestressed bent, built in the German Federal Republic, is illustrated in Fig. 163, *b*. The tendons in the latter example consist of bunched high-strength wire placed

in thin-walled steel piping and subsequently tensioned against the hardened concrete.

Special attention must be directed to the design of connections and other details; the positions of the bars must not only satisfy resistance requirements, but also be convenient for installation.

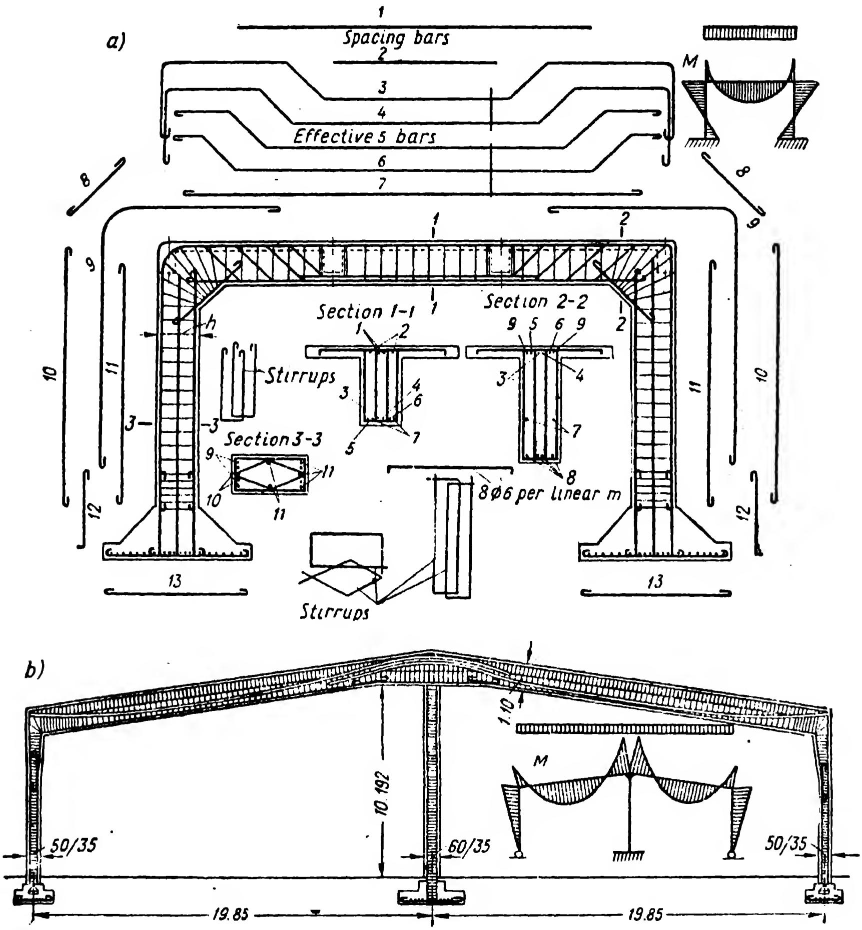


Fig. 163. Reinforcement for in-situ transverse bents
a—spliced block; *b*—bunched tendons

As to mating details: inasmuch as experiments have resulted in the stress diagram shown in Fig. 164, *a* (the greatest tensile stresses occur somewhat away from the corner), the tension bars from the beam into the column, and from the column into the beam, must be

rounded at the corners and further continued for a length corresponding to the moment curve in each case.

Considerable local compression takes place in the compression zone and it is best to make the inside corners in the form of gussets to relieve the stresses somewhat. The compression bars at these points,

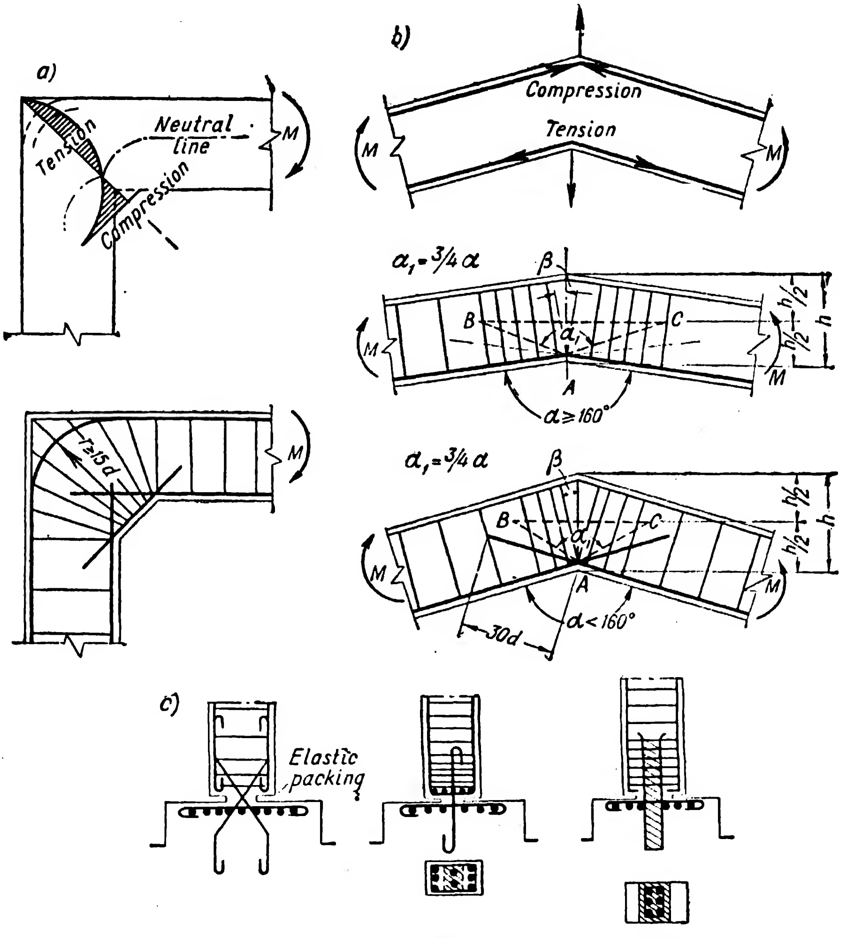


Fig. 164. Details of reinforcement of mating angles and hinged column bases

both of the column and the beam, are also to be continued into each element, while the gusset is independently reinforced. Stirrups in the corners must be spaced as close as possible.

Inasmuch as an addition of gussets complicates the work to a certain extent, they are not adopted if the stresses in the bent are small.

In obtuse connections such as the ridge (Fig. 164, *b*), the forces in the lower (tension) bars create a resultant that bisects the angle and tends to straighten the bars and force out the concrete. The concrete may also be forced out at the top because of the same resultant of forces (only here the stresses are compressive). Special anchors must therefore be provided in the ridge to counteract this tendency. But if the angle is very obtuse ($\alpha > 160°$) it will be enough to connect the stirrups to the bars and let the latter follow the line of the re-entering angle.

If the angle is sharper ($\alpha < 160°$) the lower tension bars are left separate for each mating half so that they cross each other and continue into the concrete for a distance of at least $30d$.

The lateral bars must bear the resultant of forces in the effective reinforcement, which is a condition that gives us the formula

$$m_s R_s f_{st} \cos\beta \geqslant 2 m_s R_s F_s \cos\frac{\alpha}{2}. \tag{247}$$

The number of stirrups that are accommodated along the *BC* side of the *ABC* triangle (Fig. 164, *b*) are entered in formula (247), but in any case the number of lateral bars along the length *BC* must be sufficient to take 35% of the stresses of the longitudinal effective reinforcement, that is

$$\Sigma m_s R_s f_{st} \cos\beta \geqslant 0.7 m_s R_s F_s \cos\frac{\alpha}{2}. \tag{247a}$$

The hinged bearing of the base of the column is achieved by an arrangement known as a *semi-hinge*. Here the cross-section of the column diminishes to 1/2-1/3 of its normal size, with either vertical or crossed bars enclosed in the concrete and lateral mats placed near the mating surfaces of the column and the footing. Longitudinal forces are transmitted to the footing via the reduced butt of the column (Fig. 164, *c*).

Sec. 45. COMPUTATIONS OF A TRANSVERSE BENT

1. The Design Diagram

The transverse bent of a single-storey building (Fig. 165, *a*) is subject to the following loads: 1) vertical loads from snow and the dead weight of the roof; 2) loads from the overhead crane (when the plant is so equipped), which are divided into vertical loads transmitted by the wheels and horizontal thrust caused by braking action; 3) wind thrust, both negative and positive. In special cases seismic and other forces must also be coped with.

The mating of the transverse bent's column with the main beam and with the footings (fixed or hinged base) must be reflected in the design diagram. The column height is considered as the distance

from the top of the footing to the bottom of the main beam. Computations of the transverse bent must reveal the forces in the column and determine its cross-section. If the supports of the main beam are hinged at the column, then it is computed just as an independent single-span girder (or truss).

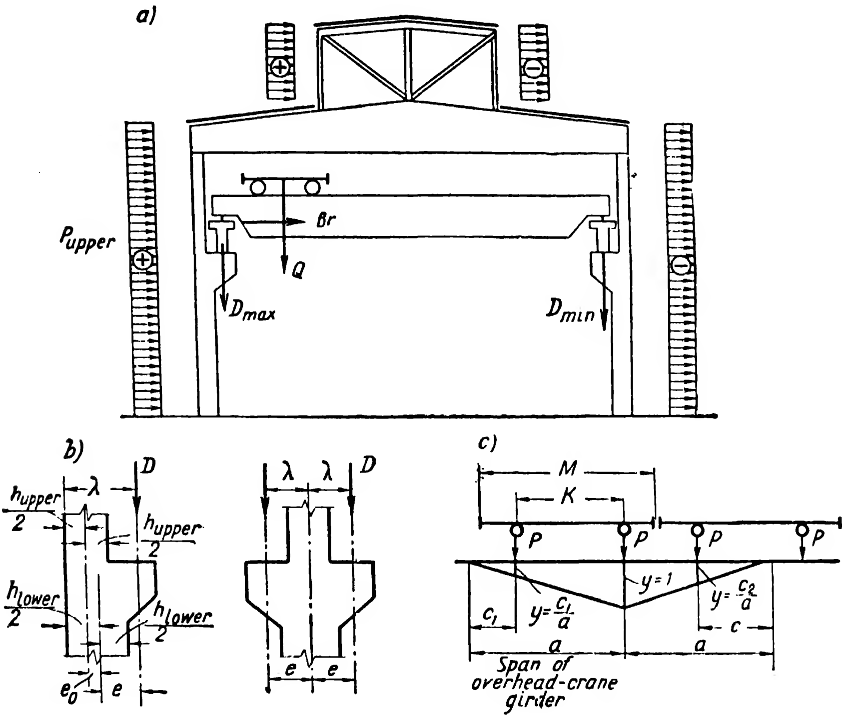


Fig. 165. Design diagrams and structural schemes of transverse bent

2. Design Loads

The load borne by one transverse bent will depend upon the distance between bents.

a) *Vertical loads.* The load N_g delivered to the columns at the supports of the main beams is calculated as being transmitted via the latter from the roof and monitor.

The geographic location of the site and the area of the roof will determine the snow load per m² of roof, which in its turn determines the load N_{snow} on the columns (also delivered via the main beam at its supports).

Both the loads N_g and N_{snow} are considered as applied to the geometric centre line of the upper part of the columns (above the crane girder) and therefore transmit the load to the lower part of the column (below the crane girder) with an eccentricity

$e_0 = \frac{h_{lower} - h_{upper}}{2}$ (Fig. 165, *b*) and create a moment

$$M_g = N_g e_0 \text{ and } M_{snow} e_0.$$

b) *Overhead-crane loads.* It is assumed that two fully loaded cranes in close proximity to each other deliver their loads to the bent via the crane girder. The dynamic character of the cranes' action is ignored in the bents.

The cranes' loads D are computed according to the influence lines of the reactions at the supports of the crane girder, where the greatest ordinate of the influence lines is equal to 1. The greatest load is obtained by assuming one of the loads as located directly at the support; under such circumstances the rest of the loads will be opposite corresponding ordinates of the influence lines, depending upon the distance between wheel centres (Fig. 165, *c*).

Moreover, the dead weight of the crane girder (G_{girder}) must be included.

In this manner

$$D_{max} = P_{max} \Sigma y + G_{girder},$$

while at the opposite column it will correspondingly be

$$D_{min} = P_{min} \Sigma y + G_{girder}.$$

The loads D_{max} and D_{min} are transmitted to the column (below the crane girder) with an eccentricity e (Fig. 165, *b*):

for the outer column $e = \lambda - \frac{h_{lower}}{2}$,

for the intermediate column $e = \lambda$, and the corresponding moments

$$M_{max} = D_{max} e \text{ and } M_{min} = D_{min} e.$$

The horizontal thrust upon the columns of the transverse bent is produced by the braking of the two already mentioned cranes (each of which has two wheels on each crane rail) working in close proximity and disposed in the same relation to the lines of influence as for the vertical loads, namely,

$$Br = 0.5 Br_{trans} \Sigma y.$$

c) *Wind loads.* The geographic location of the site and the intended height of the building will determine the wind pressure q_{wind} per m^2 of vertical surface of the walls and monitors. There will be a positive pressure on the windward side and a negative pressure under the lee. The walls will distribute these loads to the columns uniformly:

$$p_{wind} = q_{wind} a,$$

in which a—column spacing.

The pressure of the wind upon the monitor and upon the wall area situated higher than the columns is transmitted to the latter as a concentrated force W.

3. Computation Procedure

Since a transverse bent is a statically indeterminate system, its rigidity must first be determined before proceeding with computations. The rigidity of the columns $E_c J$ is determined both in its upper cross-section (above the crane beam) and its lower (below the crane beam) just as for a solid concrete section. The rigidity of the main beam is ignored because of its hinged bearing upon the columns. To compute the columns' rigidity, either tentative cross-sections must be assumed as already explained in Sec. 40, or a comparison made with an already designed building. Suffice it to say that for resistance computations of the transverse bent an exact rigidity value is not required, but only comparative values.

The loaded columns of the bent will deform through a displacement of their tops, while the load-free columns will resist this displacement; hence the upper hinged support will become one of elastic displacement.

Inasmuch as all the transverse bents of the entire compensation block are connected to the rigid horizontal diaphragm which forms the roof, they will act together as a three-dimensional system.

But the vertical loads (snow and the dead weight of the roof) and the horizontal thrust of the wind are applied simultaneously to all the bents; under such circumstances they will not act as a three-dimensional whole within the compensation block but must be considered as separate bents. On the other hand, although the greatest bulk of the crane load is applied to one bent, there will be displacement of all the bents, both loaded and load-free, because they are tied together by the roof and the loaded ones are aided by those not loaded. Therefore in computing the transverse bent against crane loads, the united action of the compensation block may be taken into account by a revision of the calculations as explained below.

The best way to compute the transverse bent of a single-storey frame building is by the displacement method which, no matter what the number of bent spans, contains only one unknown Δ_1, viz., horizontal displacement of the tops of the columns.

The fundamental system is derived from the given design diagram by the introduction of a horizontal tie that prevents horizontal displacement of the bent (Fig. 166, *a*). In this fundamental system the columns are rigidly hinged at their tops.

The fundamental system is subject to the action of the unknown which causes a reaction B_Δ and bending moments in the columns (Fig. 166, *b*). Then the fundamental system is loaded in succession with the

loads M, Br, and p_{upper} which result in respective reactions and bending moments in the columns (Fig. 166, *c*, *d*, and *e*). The values of these reactions in solid columns having variable cross-sections are found through the formulae presented in Supplement VII.

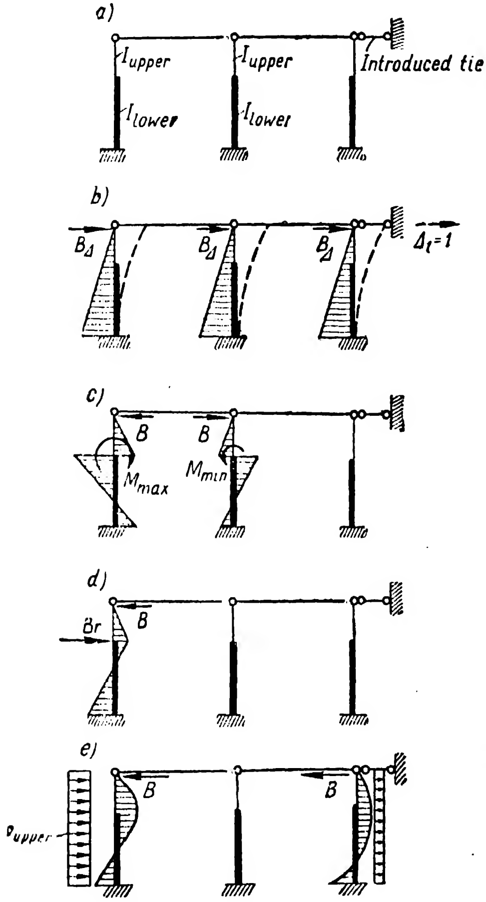


Fig. 166. Stress diagrams of the fundamental system

For each kind of load an equation is formulated in which the reactions in the horizontal tie are equal to zero inasmuch as this tie is imaginary (does not actually exist in the scheme):

$$c_{vol} r_{11} \Delta_1 + R_{1p} = 0,$$

in which r_{11}—the sum of reactions at the top of the column from displacement at the top when $\Delta_1 = 1$;

c_{vol}—the coefficient of united behaviour of the transverse bent. If there is a crane load $c_{vol} = 4$. In all other cases c_{vol}

R_{1p}—the sum of reactions at the top of the column from the loads.

Displacement is found from the equation

$$\Delta_1 = \frac{R_{1p}}{c_{vol} r_{11}}.$$

The bearing reaction at the top of the column, with due regard to the elasticity of the upper hinged bearing, will be the sum of the reactions from the loads and the actual value of the unknown:

$$B_{elastic} = B + \Delta_1 B_\Delta .$$

Moment stresses in the sections of the column will be the result of the load and the reaction at the support $B_{elastic}$, just as in the overhang of a beam.

If the bent has three or more spans, the upper supports of the columns are considered as fixed under the action of crane loads.

The stress diagram is charted on the transverse bent for each type of load, after which the forces M and N are tabulated for the computed sections of the columns and various loading schemes are tried. The dead weights of the roofing, main beam, and columns are always calculated as in their permanent positions, while the rest of the loads are taken in their most disadvantageous combinations, with due regard given to all loading combination coefficients and other special factors.

When the forces M and N have been found, the cross-sections are determined according to eccentric-compression formulae, with the length of columns taken into consideration. Computation planes are usually taken at three places: 1) above the crane-girder bracket; 2) below the crane girder; 3) at the top of the footing. If necessary, corrections are made to the trial dimensions. If in the corrections of cross-sections it is found that rigidity relations of columns have changed more than 3-fold, static computations of the bent must be gone over once more.

Sec. 46. ILLUSTRATIVE PROBLEM 21: A TRANSVERSE BENT

Compute a 2-span transverse bent for an industrial building (Fig. 167), equipped with 20/5-ton overhead cranes.

Given: two 24-metre spans, transverse bent spacing is 6 m, H-columns, roof construction to consist of 8.4-ton reinforced concrete trusses and 2.8-ton reinforced concrete monitor truss, with cladding of giant reinforced concrete

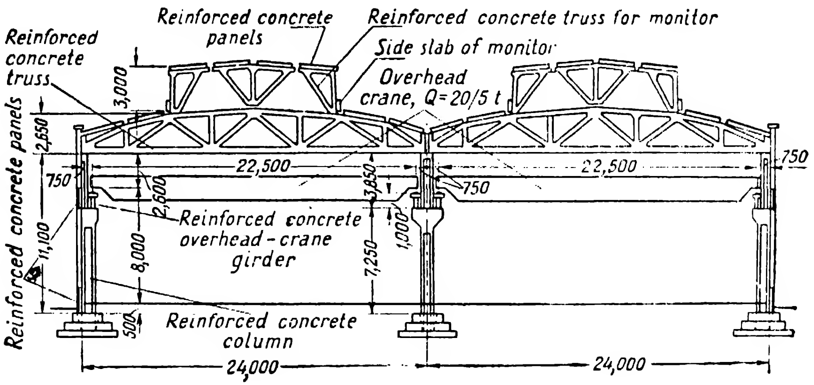


Fig. 167. Cross-section of a building

slabs, 3 metres wide; crane girders of reinforced concrete, dead weight equal to 5 t; finished cladding of composite roofing with an under-insulation of foam concrete $\gamma = 400$ kg/m^3 and 10 cm thick; wind thrust in accordance with geographical region I, snow load in accordance with region III.

a) The *design diagram* is shown in Fig. 168, *a*: height of columns from the crane-girder bracket to the bottom of the main beam is equal to $2.6 + 1.25 =$

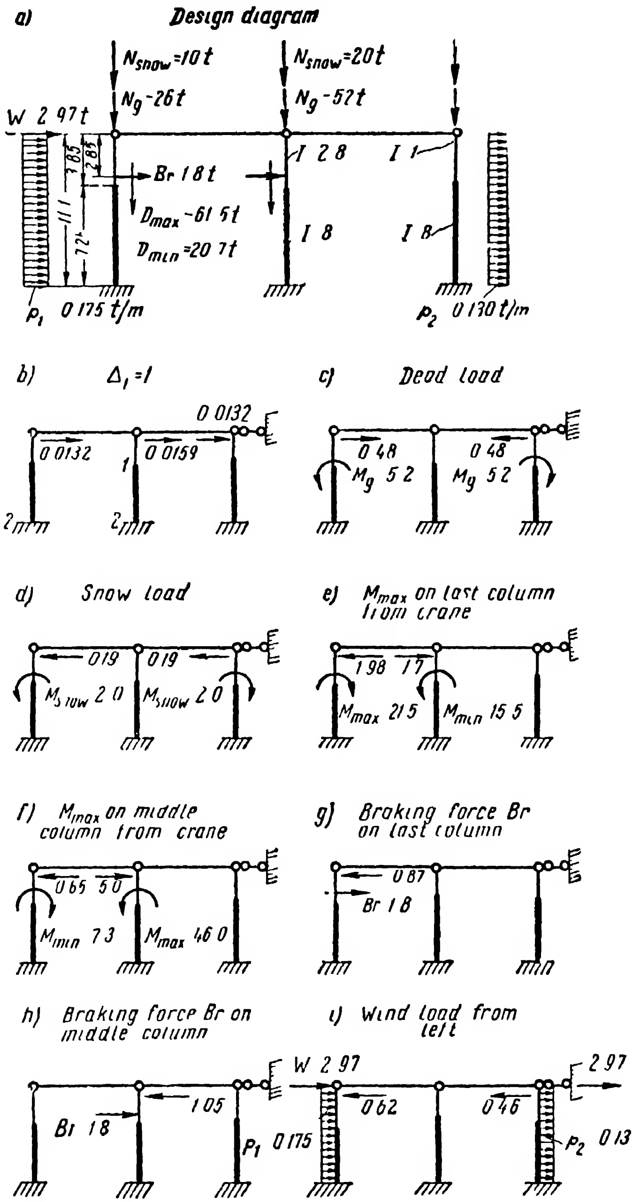


Fig 168 Sketches illustrating problem 21

$=3.85$ m; from the top of footing to the crane-girder bracket it is equal to $8-1.25+0.5=7.25$ m; total height is equal to $3.85+7.25=11.1$ m.

b) *Loads. Specified dead loads per m² of roof.*

Dead weight of giant reinforced concrete slabs . . .	130	kg/m²
Moisture insulation	5	"
Foam-concrete insulation, 0.1×400	40	"
Asphalt underlayer, 2 cm	35	"
Roll-type finished roofing	15	"
	$g^s=225$ kg/m²	

Design dead load $g=225\times1.1=250$ kg/m².

Reaction at the supports of the truss:

from dead weight of roof at a 6-metre transverse-bent span: $0.250\times6\frac{24}{2}=18$ t.

from dead weight of truss . . . . $\frac{8.4}{2}\,1.1=4.6$ t

from dead weight of monitor truss . . . . $\frac{2.8}{2}\,1.1=1.55$ t

from dead weight of fenestration and base slabs $(0.04\times3+0.16)\,6\times1.1=1.85$ t

Total: 26 t

Design dead load from roof on outer-row column $N_g=26$ t.

Design dead load from roof on intermediate column $N_g=2\times26=52$ t.

Specified snow load per m² of roof.

Snow $p=100$ kg/m²; coefficient, depending upon roof outline, $c=1$; $p^s_{snow}=100\times1=100$ kg/m².

Design snow load $p_{snow}=100\times1.4=140$ kg/m².

Ditto on outer-row column

$$N_{snow}=0.14\times6\frac{24}{2}=10\text{ t}.$$

Ditto on intermediate column

$$N_{snow}=2\times10=20\text{ t}.$$

Overhead-crane load. Crane span $l_{crane}=l-2\lambda=24-2\times0.75=22.5$ m.

With a 20-ton crane the specified load carried by one wheel (according to the crane catalogue) $p^s_{max}=22$ t. Dead weight of crane buggy is 8.5 t. Total weight of crane (with crane buggy) is 36 t. Wheel spacing along crane rail $K=4.4$ m. Minimum distance between wheels of two adjacent cranes $B=6.3-4.4=1.9$ m.

Maximum design load of one wheel $P_{max}=22\times1.3=28.6$ t.

Minimum design load of one wheel

$$P_{min}=\left(\frac{20+36}{2}-22\right)1.3=7.8\text{ t}.$$

Design braking force $Br_{trans}=0.5\,\frac{20+8.5}{20}\,1.3=0.93$ t.

Design load on a column from two adjacent cranes and from the dead weight of the crane girder: the influence lines of the reaction at the support

of the crane girder shows a maximum ordinate equal to 1 (Fig. 169, *a*):

$$D_{\max} = \left(1 + \frac{1.6}{6} + \frac{4.1}{6}\right) 28.6 + 5 \times 1.1 = 1.95 \times 28.6 + 5.5 = 61.5 \text{ t};$$

$$D_{\min} = 1.95 \times 7.8 + 5.5 = 20.7 \text{ t};$$

$$Br = 1.95 \times 0.93 = 1.8 \text{ t}.$$

Wind thrust. The wind thrust at a 10-metre height above the ground $Q = 30$ kg/m², and at a 20-metre height above the ground $Q = 40$ kg/m². Aerodynamic coefficients for windward wall surfaces and the surface of the first monitor $k = +0.8$; ditto for the leeward surfaces $k = -0.6$; ditto for the windward side of the second monitor $k = +0.4$, and ditto for its leeward side $k = -0.4$.

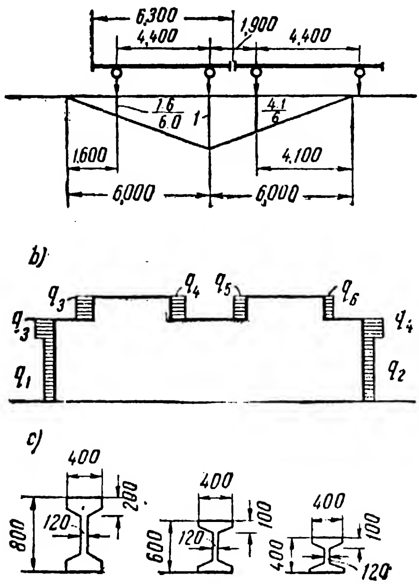


Fig. 169. Loading diagram and column cross-sections

Design wind thrust per m² of vertical surface (Fig. 169, *b*):

$$q_1 = kQn = 0.8 \times 30 \times 1.2 = 29 \text{ kg/m}^2;$$
$$q_2 = -0.6 \times 30 \times 1.2 = -21.6 \text{ kg/m}^2;$$
$$q_3 = 0.8 \times 40 \times 1.2 = 38.4 \text{ kg/m}^2;$$
$$q_4 = -0.6 \times 40 \times 1.2 = -29 \text{ kg/m}^2;$$
$$q_5 = 0.4 \times 40 \times 1.2 = 19.2 \text{ kg/m}^2;$$
$$q_6 = -19.2 \text{ kg/m}^2.$$

The design wind thrust borne by the columns of the transverse bent will be as follows:

Uniformly distributed wind thrust from the walls and fenestration

$$p_1 = 0.029 \times 6 = 0.175 \text{ t/linear m};$$
$$p_2 = -0.0216 \times 6 = -0.13 \text{ t/linear m}.$$

Concentrated wind thrust from that part of the wall situated above the columns and from the monitors

$$W = (0.0384 + 0.029)\, 2.65 \times 6 + (0.0384 + 0.029)\, 3 \times 6 + (0.0192 + 0.0192) 3 \times 6 =$$
$$= 1.07 + 1.21 + 0.68 = 2.97 \text{ t}.$$

c) *Tentative column cross-sections. Rigidity computations.*

The height h_{low} of an H-column below the crane girder will be $\frac{1}{10} H_{lower}$, or $\frac{1}{10} 725 = 7.25 \approx 80$ cm. The width b will be $\frac{1}{20} H_{lower}$, or $\frac{1}{20} 725 = 36.25 \approx$ ≈ 40 cm. The 40×80 cm H-section is assumed as valid below the crane girder for intermediate- as well as outer-row columns.

The cross-sectional height above the crane girder is established structurally so as to be able to support the main beam without additional brackets and also to achieve proper clearance between the crane and face of the column. For the intermediate columns $h_{upper} = 60$ cm, and 40 cm for the outer columns. The cross-sectional width $b = 40$ cm for the entire column height. Web thickness of the column is assumed as 12 cm, while flange thickness is taken as 10 cm above the crane girder and 20 cm below the crane girder (Fig. 169, *c*).

The moments of inertia of the cross-section of the outer column above the crane girder

$$J=\left(\frac{40\times 10^3}{12}+40\times 10\times 15^2\right)2+\frac{12\times 20^3}{12}=19.5\times 10^4\ \text{cm}^4;$$

ditto for the intermediate column above the crane girder

$$J=\left(\frac{40\times 10^3}{12}+40\times 10\times 25^2\right)2+\frac{12\times 40^3}{12}=57.1\times 10^4\ \text{cm}^4;$$

ditto for the section below the crane girder

$$J=\left(\frac{40\times 20^3}{12}+40\times 20\times 30^2\right)2+\frac{12\times 40^3}{12}=155.8\times 10^4\ \text{cm}^4.$$

Relative column rigidity of outer column above the crane girder $\frac{E\times 19.5\times 10^4}{E\times 19.5\times 10^4}=1;$

ditto for the intermediate column above the crane girder $\frac{57.1}{19.5}=2.8;$

ditto for the section below the crane girder $\frac{155.8}{19.5}=8.$

d) *Static computation of the transverse bent.* The transverse bent is calculated by the method of displacement The unknown Δ_1 is the horizontal displacement of the columns' tops, and the fundamental system contains a horizontal tie that hinders this displacement.

It is assumed that the fundamental system is subject to the unit action of the unknown $\Delta_1=1$ (Fig. 168, *b*), from which the reactions at the upper ends of the columns are computed with the aid of the data in Supplement IV when

$$\alpha=\frac{3.85}{11.1}=0.35;$$

for the outer column

$$k=0.35^3\left(\frac{8}{1}-1\right)=0.3;\quad \text{and}\quad B=\frac{3\times 8}{11.1^3\,(1+0.3)}=0.0132;$$

for the intermediate column

$$k=0.35^3\left(\frac{8}{2.8}-1\right)=0.08;\quad \text{and}\quad B_{\bar{x}}=\frac{3\times 8}{11.1^3\,(1+0.08)}=0.0159.$$

The coefficient of the canonical equation $r_{11}=2\times 0.0132+0.0159=0.0423.$
When including the united behaviour of the transverse bents under crane loads, $c_{vol}r_{11}=4\times 0.0423=0.169.$

The *dead loads* (Fig. 168, *c*):

$N_g=26$ t for the outer column whose eccentricity

$$e_0=\frac{0.8-0.4}{2}=0.2\ \text{m}.$$

The outside moment at $M_g=-26\times 0.2=-5.2$ t/m.
The intermediate column is axially loaded when $N_g=52$ t.
The reaction at the upper end of the left column

$$B=\frac{3\times 5.2\,(1-0.35^2)}{2\times 11.1\,(1+0.3)}=0.48\ \text{t},$$

while the reaction at the upper end of the right column $B = -0.48$ t, and the sum of reactions from the load R_{1p} will be $0.48 - 0.48 = 0$. Hence, the above computations of the reactions B are final.

The bending moments in the sections of the left column will be (see Table 27 for the numbers of the sections):

$$M_{10} = 0.48 \times 3.85 = 1.85 \text{ t/m}; \quad M_{12} = 1.85 - 5.2 = -3.35 \text{ t/m};$$
$$M_{21} = 0.48 \times 11.1 - 5.2 = 0.15 \text{ t/m}.$$

Dead weight of columns: the part of the outer column above the crane girder weighs 1.2 t; the intermediate column, also in its upper part, weighs 1.6 t; the lower part (beneath the crane girder) of each column weighs 4.05 t.

The longitudinal design force of the outer column

$$N_{10} = N_{12} = 26 + 1.2 \times 1.1 = 27.3 \text{ t}; \quad N_{21} = 27.3 + 4.05 \times 1.1 = 31.8 \text{ t};$$

ditto for the intermediate column

$$N_{10} = N_{12} = 52 + 1.6 \times 1.1 = 53.75 \text{ t}; \quad N_{21} = 53.75 + 4.05 \times 1.1 = 58.2 \text{ t}.$$

Snow load (Fig. 168, *d*).

The moment due to eccentricity $M_{snow} = -10 \times 0.2 = -2$ t/m.

The bending moments in the sections of the left column are evolved by multiplication of the ratio

$$\frac{M_{snow}}{M_g} = \frac{-2}{-5.2} = 0.385,$$

in which 5.2 is the dead-load moment; hence,

$$M_{10} = 1.85 \times 0.385 = 0.72 \text{ t/m}; \quad M_{12} = -3.35 \times 0.385 = -1.3 \text{ t/m};$$
$$M_{21} = 0.15 \times 0.385 = 0.06 \text{ t/m}.$$

The crane load M_{max} on the outer column (Fig. 168, *e*):

Eccentricity from the force D_{max} on the outer column

$$e = 0.75 - \frac{0.8}{2} = 0.35 \text{ m}.$$

The outside moment at $M_{max} = 61.5 \times 0.35 = 21.5$ t/m.

The reaction $B = -\dfrac{3 \times 21.5\,(1 - 0.35^2)}{2 \times 11.1\,(1 + 0.3)} = -1.98$ t.

The force D_{min} with an eccentricity $e = 0.75$ m, acts upon the intermediate column:

$$M_{min} = -20.7 \times 0.75 = -15.5 \text{ t/m}.$$

The reaction

$$B = \frac{3 \times 15.5\,(1 - 0.35^2)}{2 \times 11.1\,(1 + 0.08)} = 1.7 \text{ t}.$$

The sum of the reactions $R_{1p} = -1.98 + 1.7 = -0.28$ t.

The canonical equation (with due consideration for united action)

$$0.169\,\Delta_1 - 0.28 = 0,$$

therefore $\Delta_1 = 1.65$.

The elastic reaction at the outer column $B_{elastic} = -1.98 + 1.65 \times 0.0132 = -1.96$ t.

Bending moments $M_{10} = -1.96 \times 3.85 = -7.6$ t/m; $M_{12} = -7.6 + 21.5 = 13.9$ t/m; $M_{21} = -1.96 \times 11.1 + 21.5 = -0.3$ t/m.

The elastic reaction at the intermediate column $B_{elastic}=1.7+1.65\times \times 0.0159=1.73$ t.

Bending moments: $M_{10}=1.73\times 3.85=6.65$ t/m; $M_{12}=6.65-15.5= =-8.85$ t/m; $M_{21}=1.73\times 11.1-15.5=3.7$ t/m.

The crane load M_{max} on the intermediate column (Fig. 168, *f*).

The eccentricity of the force D_{max} on the intermediate column $e=0.75$ m. The outside moment at $M_{max}=-61.5\times 0.75=-46$ t/m, and the reaction

$$B=\frac{3.46\,(1-0.35^2)}{2\times 11.1\,(1+0.08)}=5 \text{ t}.$$

The force D_{min}, with an eccentricity $e=0.35$ m, acts upon the outer column:

$$M_{min}=20.7\times 0.35=7.3 \text{ t/m},$$

and the reaction $B=-\dfrac{3\times 7.3\,(1-0.35^2)}{2\times 11.1\,(1+0.3)}=-0.65$ t.

The sum of the reactions $R_{1p}=-0.65+5=4.35$ t. The canonical equation $0.169\,\Delta_1+4.35=0$, therefore $\Delta_1=-25.7$, and the elastic reaction at the outer column $B_{elastic}=-0.65-25.7\times 0.0132=-1$ t.

Bending moments: $M_{10}=-1\times 3.85=-3.85$ t/m; $M_{12}=-3.85+7.3= =3.45$ t/m; $M_{21}=-1\times 11.1+7.3=-3.8$ t/m.

The elastic reaction at the intermediate column

$$B_{elastic}=5-25.7\times 0.0159=4.6 \text{ t},$$

and the bending moments

$$M_{10}=4.6\times 3.85=17.7 \text{ t/m};\quad M_{12}=17.7-46=-28.3 \text{ t/m};$$
$$M_{21}=4.6\times 11.1-46=5.1 \text{ t/m}.$$

The braking thrust Br upon the outer column (Fig. 168, *g*).

The reaction $B=-\dfrac{1.8\,(1-0.35)}{1+0.3}=-0.87$ t, and the sum of reactions $R_{1p}=-0.87$ t, from which the canonical equation takes the form of

$$0.169\Delta_1-0.87=0,$$

therefore $\Delta_1=5.15$ and the elastic reaction at the outer column

$$B_{elastic}=-0.87+5.15\times 0.0132=0.8 \text{ t}.$$

Bending moments: $M_{10}=M_{12}=-0.8\times 3.85+1.8\times 1=-1.3$ t/m;

$$M_{21}=-0.8\times 11.1+1.8\times 8.25=6 \text{ t/m}.$$

The braking thrust Br upon the intermediate column (Fig. 168, *h*).

The reaction $B=-\dfrac{1.8\,(1-0.35)}{1+0.08}=-1.05$ t;

$$R_{1p}=-1.05 \text{ t},$$

hence the canonical equation takes the form of

$$0.169\Delta_1-1.05=0,$$

therefore $\Delta_1=6.2$, and the elastic reaction at the intermediate column

$$B_{elastic}=-1.05+6.2\times 0.0159=-0.95.$$

Bending moments: $M_{10}=M_{12}=-0.95\times 3.85+1.8\times 1=-1.85$ t/m;

$$M_{21}=-0.95\times 11.1+1.8\times 8.25=4.25 \text{ t/m}.$$

Wind thrust at the left (Fig. 168, *i*).

The reaction at the left column from a uniformly distributed load

$$B=-\frac{3\times0.175\times11.1\,(1+0.35\times0.3)}{8\,(1+0.3)}=-0.62\ \text{t}.$$

Ditto at the right column

$$B=-\frac{3\times0.13\times11.1\,(1+0.35\times0.3)}{8\,(1+0.3)}=-0.46\ \text{t}.$$

The reaction at the tie from a concentrated load $W=2.97$ t will be $B=-2.97$ t, and the sum of reactions $R_{1p}=-0.62-0.46-2.97=-4.05$ t; hence the canonical equation

$$0.0423\Delta_1-4.05=0,$$

therefore $\Delta_1=96$, and the elastic reaction at the left column

$$B_{elastic}=-0.62+96\times0.0132=0.64\ \text{t}.$$

Bending moments: $M_{10}=M_{12}=0.64\times3.85+\frac{0.175\times3.85^2}{2}=3.75$ t/m;

$$M_{21}=0.64\times11.1+\frac{0.175\times11.1^2}{2}=18\ \text{t/m}.$$

The elastic reaction at the right column

$$B_{elastic}=-0.46+96\times0.0132=0.8\ \text{t}.$$

Bending moments: $M_{10}=M_{12}=0.8\times3.85+\frac{0.13\times3.85^2}{2}=4.05$ t/m;

$$M_{21}=0.8\times11.1+\frac{0.13\times11.1^2}{2}=17\ \text{t/m}.$$

The elastic reaction at the intermediate column

$$B_{elastic}=96\times0.0159=1.53\ \text{t}.$$

Bending moments: $M_{10}=M_{12}=1.53\times3.85=5.9$ t/m; $M_{21}=1.53\times11.1=17$ t/m.

For clarity, as well as for easy verification, the above static computations must then be composed into tabular form which includes moment diagrams of the various transverse bent loadings and design forces in the columns' cross-sections (Table 27).

e) *Determining bar area for the columns*. Gathering design data: 200-B grade concrete, $R_{be}=100$ kg/cm²; bars of St-5 intermittently deformed steel, $R_s=2{,}400$ kg/cm², $m_s=1$. Design length of columns above the crane girder in the plane of the transverse bent $l_0=2.5\times3.85=9.6$ m, and ditto for the length below the crane girder $l_0=7.25$ m. Area of cross-section F and the radius of gyration $r=\sqrt{\frac{J}{F}}$ for H-section will be:

for the 40×40 section, $F=1{,}100$ cm² and $r=13.3$ cm
„ „ 40×60 section, $F=1{,}350$ „ „ $r=20.4$ „
„ „ 40×80 section, $F=2{,}150$ „ „ $r=26.8$ „

Bar area is chosen through diagramatic data as already explained in Sec. 14.

The Outer Column.

Section 1-0: 40×40 cm H-section,

$$M=9.8\ \text{t/m},\quad N=27.3\ \text{t};$$

$$e_0=\frac{9.8}{27.3}=0.35\ \text{m};\quad \frac{l_0}{r}=\frac{960}{13.3}=72>35;$$

hence, long-column computations are required:

$$\eta=\frac{1}{1-\dfrac{27,300}{1\times4,800\times100\times1,100}\left(\dfrac{960}{13.3}\right)^2}=1.37.$$

The qualified design moment

$$M=\frac{35\times1.37+20-4}{35+20-4}\,9.8=12.3\ \text{t/m}.$$

When $M=12.3$ t/m and $N=27.3$ t, the diagram shows that $\mu=\mu'=0.9\%$; $F_s=F'_s=0.009\times1,100=9.9$ cm², which is satisfied by 3ø20 D ($F_s=9.42$ cm²).

Section 1-2: 40×80 cm H-section, $M=13.8$ t/m,

$N=82.65$ t;

$$\frac{l_0}{r}=\frac{725}{26.8}=27<35;\quad \eta=1.$$

The diagram shows that $\mu=\mu'=0.2\%$; $F_s=F'_s=0.002\times2,150=4.3$ cm², which is satisfied by 2ø 20 D ($F_s=6.28$ cm²).

Section 2-1: 40×80 cm H-section. Two combinations of forces are calculated:

a) $M=21.5$ t/m; $N=97.15$ t.

Through the diagram

$$\mu=\mu'=0.2\%;$$

b) $M=24$ t/m; $N=59.4$ t, and the diagram shows that $\mu=\mu'=0.2\%$; $F_s=F'_s=0.02\times2,150=4.3$ cm², which is satisfied by 2ø20 D ($F_s=6.28$ cm²).

The intermediate column.

Section 1-0: 40×60 cm H-section, $M=22.9$ t/m,

$N=71.75$ t;

$$e_0=\frac{22.9}{71.75}=32;\quad \frac{l_0}{r}=\frac{960}{20.4}=47>35;$$

$$\eta=\frac{1}{1-\dfrac{71,750}{1\times4,800\times100\times1,350}\left(\dfrac{960}{20.4}\right)^2}=1.33.$$

The qualified design moment

$$M=22.9\,\frac{32\times1.33+30-4}{32+30-4}=27.2\ \text{t/m}.$$

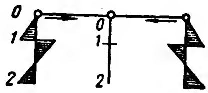
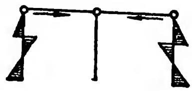
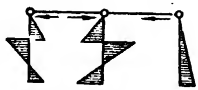
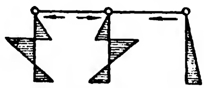
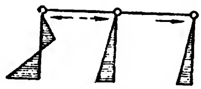
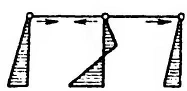
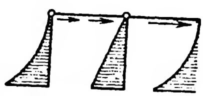
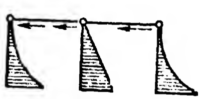
From the diagram it is seen that $\mu=\mu'=1.1\%$; $F_s=F'_s=0.011\times1,350=$ $=14.9$ cm², which is satisfied by 2ø28 D + 1ø20 D ($F_s=15.45$ cm²).

Section 2-1; 40×80 cm H-section, $M=26.35$ t/m, $N=131.55$ t, and the diagram shows that $\mu=\mu'=0.45\%$; $F_s=F'_s=0.0045\times2,150=9.7$ cm², which is satisfied by 2ø28 D ($F_s=12.32$ cm²).

Section 2-1; 40×80 cm H-section, $M=26.35$ t/m, $N=131.55$ t, and the diagram shows that $\mu=\mu'=0.4\%$; $F_s=F'_s=0.004\times2,150=8.6$ cm², which is satisfied by 3ø20 D ($F_s=9.41$ cm²).

Computations are concluded by verifying the stability of the columns as members axially loaded out of the plane of the transverse bent, and also by checking their static behaviour under conditions of transportation and hoisting.

Design Forces in Column

Load combinations	Item No.	Make-up of load	M-diagram	End 1-0 M	End 1-0 N
Main loads	1	Dead loads		1.85	27.3
	2	Snow		0.72	10
	3	Load from overhead crane M_{max} on end column		—7.6	—
	4	Ditto on intermediate column		—3.85	—
	5	Load from overhead crane Br on end column		±1.3	—
	6	Ditto on intermediate column		—	—
	7	Design forces	max. M	2.57	37.3
			min. M	—7.05	27.3
			max. N	2.57	37.3
Additional loads (all loads, with the exception of dead weights are to be taken with a coefficient of 0.9)	8	Wind thrust at left		3.75	—
	9	Wind thrust at right		—4.05	—
	10	Design forces	max. M	5.90	36.3
			min. M	—9.8	27.3

Sections of Bents

column sections				Intermediate column sections					
1-2		2-1		1-0		1-2		2-1	
M	*N*	*M*	*N*	*M*	*N*	*M*	*N*	*M*	*N*
—3.35	27.3	0.15	31.8	—	53.75	—	53.75	—	58.2
—1.3	10	0.06	10	—	20	—	20	—	20
13.9	61.5	—0.3	61.5	6.65	—	—8.85	20.7	3.7	20.7
3.45	20.7	—3.8	20.7	17.7	61.5	—28.3	61.5	5.1	61.5
±1.3	—	±6.0	—	—	—	—	—	—	—
—	—	—	—	±1.85	—	±1.85	—	±4.25	—
11.85	88.8	5.91	103.3	19.55	73.75	0	73.75	9.35	139.7
—4.65	37.3	—9.6	52.5	0	73.75	—30.15	135.25	—0.55	98.9
9.25	98.8	—0.12	103.3	19.55	73.75	0	196.75	0	201.2
3.75	—	18.0	—	5.9	—	5.9	—	17	—
—4.05	—	—17	—	—5.9	—	—5.9	—	—17	—
13.8	82.65	21.5	97.15	22.9	71.75	5.3	71.75	26.35	131.55
—8.20	36.3	—24.0	59.4	—5.3	71.75	—32.4	127.1	—15.8	94.8

CHAPTER XIV

MULTI-STOREY FRAME BUILDINGS

Sec. 47. GENERAL REMARKS

1. In What Fields These Structures Are Used, and Their Details

The number of floors in multi-storey factory buildings usually ranges from three to six, and total heights from 30 to 40 metres. Apartment and civic buildings are carried up eight to twelve storeys, but if it is a very tall structure it may have from 20 and more floors. The number of storeys in any of the above categories is not limited by the structural possibilities of reinforced concrete, but rather by practical requirements and the economics involved in construction and subsequent maintenance.

The depth of storeys in multi-storey industrial structures is limited by natural lighting requirements of manufacturing processes and may be as much as 36 metres, while in apartment houses and civic buildings 14-metre depths are usually the maximum.

Column grids and storey heights are determined by requirements of standardisation of structural members and by unified dimensions of spans and heights (see Sec. 21).

The principal structural elements in multi-storey buildings are the columns and girders, which as a whole constitute the reinforced concrete frame (see Fig. 78). Ribbed panels and flat slabs are both used for floor construction. In flat-slab floors the role of the girder is played by the slab itself, which forms a rigid diaphragm between the columns with the aid of the latter's capitals.

In buildings where the frame is entirely self-sufficient (Fig. 170, *a*) the outer enclosure takes the form of a non-bearing curtain wall with the floor loads transmitted downwards via the exterior and interior columns.

In buildings where the frame is only semi-sufficient, there are no exterior columns and the outer enclosure is built in the form of a bearing wall (Fig. 170, *b*).

In general, multi-storey industrial buildings are built with self-sufficient frames, while apartment houses and civic buildings may have either self-sufficient or semi-sufficient frames.

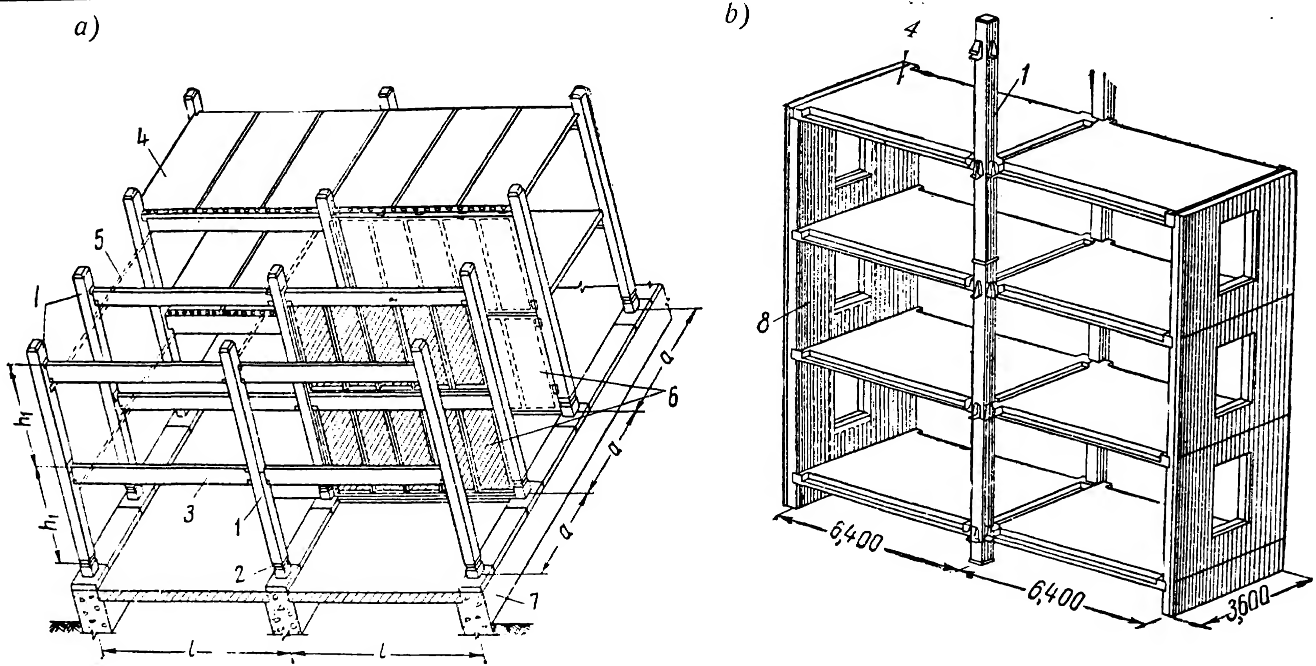


Fig. 170. Multi-storey frame building

a – a building with a self-sufficient frame; *b* – a building with a semi-sufficient frame and girderless floors; *1* – columns; *2* – connection at foot of column; *3* – girders; *4* – slab; *5* – temporary ties; *6* – staircase walls acting as vertical diaphragms; *7* – footings; *8* – exterior bearing wall

2. The Structural Layout of a Multi-Storey Frame Building

The structural layout of a multi-storey frame building may be based on either of two fundamental systems aimed at achieving skeletal rigidity, viz., the rigid-joint frame, or the diaphragm-braced frame.

In the *rigid-joint* frame the vertical loads and the horizontal wind thrust are borne by a reinforced concrete frame whose joints are rigidly fixed.

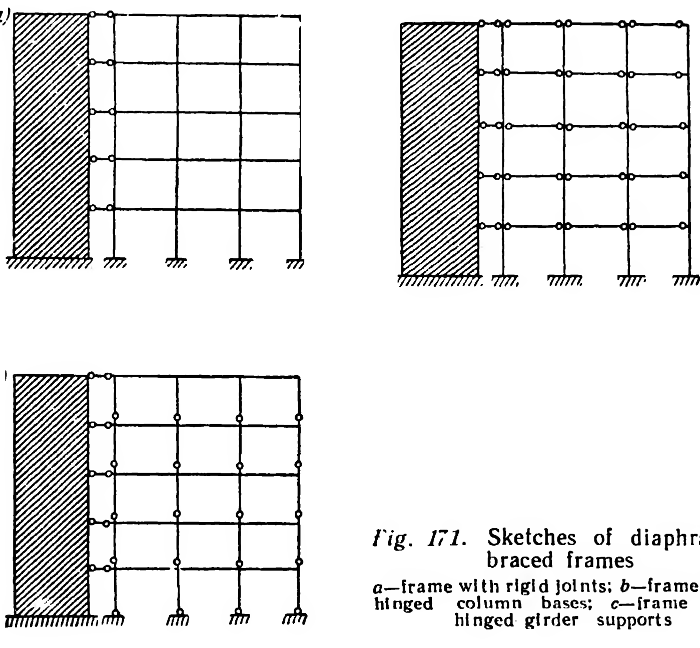


Fig. 171. Sketches of diaphragm-braced frames

a—frame with rigid joints; *b*—frame with hinged column bases; *c*—frame with hinged girder supports

In the *diaphragm-braced* frame the vertical loads are borne by a reinforced concrete frame but the joints may be either hinged or rigidly fixed, while horizontal thrust from the wind or other sources is taken jointly by a vertical bracing diaphragm and the frame (Fig. 171). Vertical bracing diaphragms in multi-storey buildings are created by the walls of staircases, side and other transverse walls, and also by longitudinal walls. Horizontal wind thrust is transmitted in the following sequence: 1) the pressure of the wind is taken by the outer walls which act as beams with spans equal to storey heights and which transmit the thrust to their supports, i.e., the reinforced concrete floors (Fig. 172, *a*); 2) the reinforced concrete floors act as

horizontal diaphragms, that is, as beams in a horizontal plane whose spans are equal to the distance between vertical bracing diaphragms, which latter take the load (Fig. 172, *b*); 3) the vertical bracing diaphragms act as vertical cantilevers (restrained by the foundation) whose height is that of the building (Fig. 172, *c*). All the components of the building—frames, floors, and vertical diaphragms—are interconnected by their welded insertions and the concrete filling of their joints.

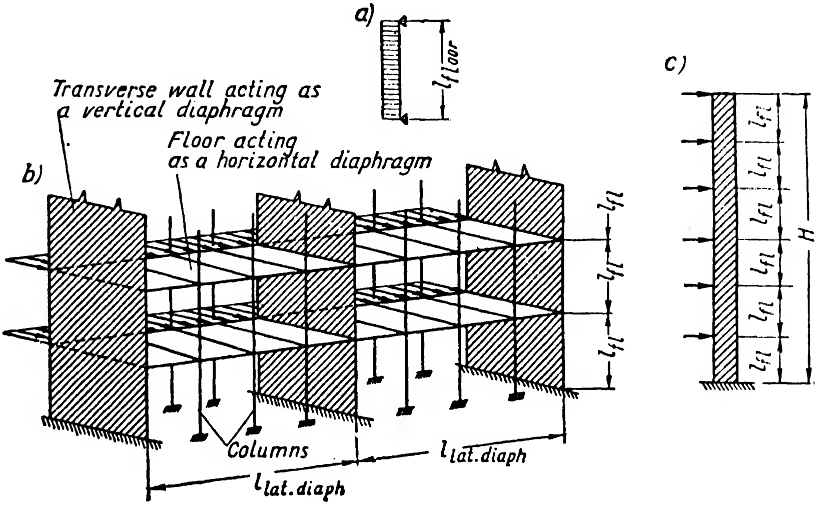


Fig. 172. Sketches of wind thrust transmission via the components of a diaphragm-braced frame in a multi-storey building

In comparison with the rigidity of vertical diaphragms in their resistance to horizontal loads, the frame's resistance (when the building is the usual 30-metre height) to these same horizontal loads is comparatively small. Hence, in the general behaviour of the three-dimensional system the vertical diaphragms bear almost the entire horizontal load and there are almost no bending moments in the frame from that source.

In special cases when the building is very tall or possesses insufficiently rigid vertical diaphragms, the distribution of horizontal thrust between the frame and the diaphragms (in accordance with their relative rigidity) can be established by static computation.

Rigid joints are not a necessity in a diaphragm-braced frame because geometric stability and skeletal rigidity are assured to the greatest extent by the stiffening diaphragms.

Individual hinged joints are possible in the rigid-joint frame, but in such a case there must be no great decrease of the frame's rigidity nor must there be any disturbance of its geometric stability.

In rigid-joint framework it is also possible to have the bases of the columns hinged so as to make up a series of single-storey frames superimposed upon each other (Fig. 171, *b*).

The choice of one of the above structural layouts will depend upon the height of the building (number of storeys), its floor plan,

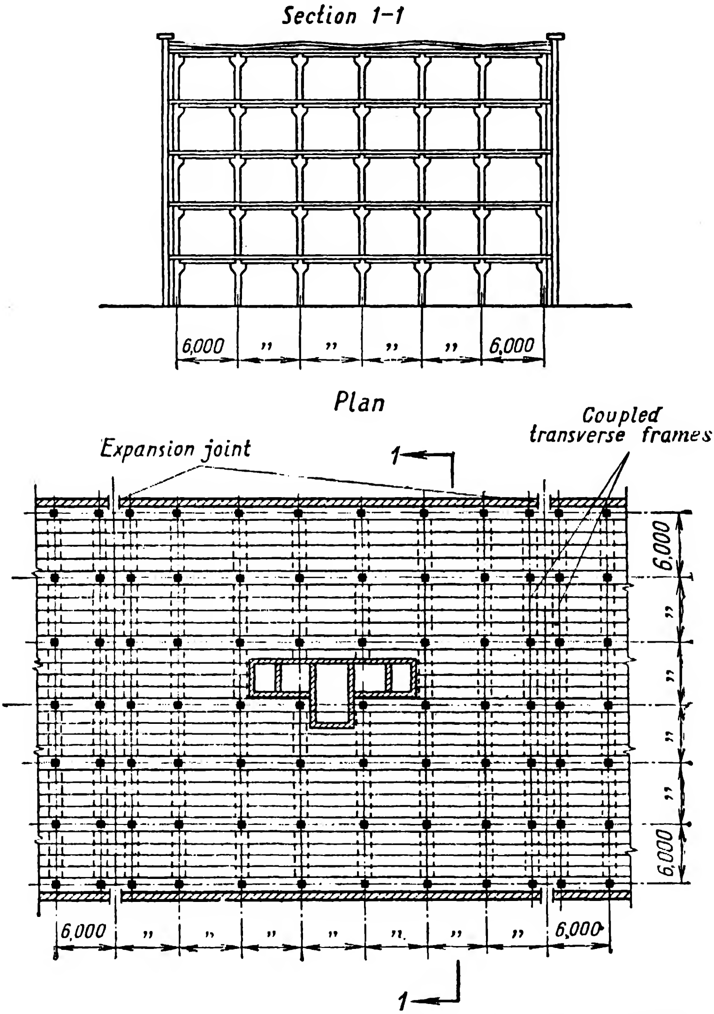


Fig. 173. Plan and cross-section of a multi-storey building

intended manufacturing process, methods of erection and other factors. But in very tall buildings where skeletal rigidity plays a big role in reducing vertical deflection caused by wind pressure, it is

best to use the diaphragm-braced frame. This will simultaneously solve the problem of girder standardisation in each storey, because when the frame is made to resist only vertical loads, the girders can be made all alike. On the other hand, if the frame is designed to also resist horizontal wind thrust, the result will be an ever-increasing bending moment towards the bottom and the necessity of a corresponding increase of reinforcement in the girders, and sometimes even an enlargement of the latter's cross-section.

In industrial structures requiring large unobstructed floor areas because manufacturing processes do not allow partitions that may play the role of bracing diaphragms (as for example in spinning mills, etc.), rigid-joint frames are used. As to the amount of structural material, both framing systems are about alike.

Fig. 173 illustrates a plan and cross-section of a multi-storey factory designed on the principle of a diaphragm-braced frame with ribbed-slab floors. Here the walls of the staircase, situated in the middle of the compensation block of the building, act as bracing diaphragms. The frames run transversely, while the floor panels are placed longitudinally and take the place of spacers between the columns (the floor panels are made with cut-outs to let the columns pass through). Compensation joints are formed by coupled rows of columns which divide the structure into compensation blocks, independent of each other in case of deformation. However, standard bearing members are retained throughout the whole building.

A general view of the frame for an apartment house is shown in Fig. 170, *a*; it has ribbed floors, transversely placed frames, and cut-outs in the floor panels which are laid along the lines of columns. Here, as in the above example, the staircase walls play the role of vertical bracing diaphragms.

A girderless type of frame is shown in Fig. 170, *b*. The giant floor slabs are bordered by ribs (which take the place of girders) supported upon the columns, such supports being considered as hinged bearings. Here again the staircase walls act as vertical bracing diaphragms.

Vertical bracing diaphragms are made in the form of storey-high reinforced concrete panels resting one upon the other and connected with each other by means of welded inserted steel parts. Such a diaphragm is computed to bear horizontal wind thrust and vertical loads and acts as a cantilever beam rigidly fixed by its foundation. Lateral deflection from specified wind loads (thrust)

$$f = \frac{H}{2{,}000},$$

in which H—the height of the building from the top of the foundation to the floor of the uppermost storey.

Sec. 48. HOW TO DIVIDE A MULTI-STOREY FRAME INTO PRECAST COMPONENTS. CONNECTIONS OF SUCH COMPONENTS

The division of a multi-storey frame into precast components is schematically shown in Fig. 83; the girders are divided into straight pieces that fit between the faces of the columns, the columns in their turn being divided into corresponding lengths that are joined at planes somewhat above the level of the floors. Column components can be made two storeys in height, if allowed by weight and length, and connected at alternating floors. But whichever method of frame division is chosen, it must possess producibility and be convenient in erection.

The design of girders and columns, as already presented in Secs. 14 and 26, is valid for multi-storey frames, the only difference being

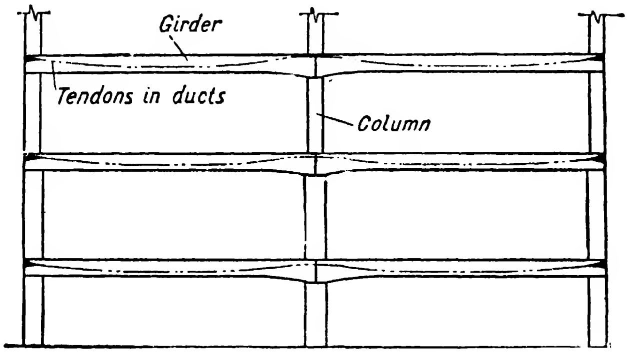


Fig. 174. A multi-storey frame with prestressed girders in which the tendons are tensioned against the hardened concrete

in the construction of joints. Sec. 26 has already explained rigid column-girder connections. The hinged type differs only in its upper longitudinal bars, which are placed as auxiliary reinforcement because the joint does not undergo any bending moment. In hinged girder joints the bars of the supporting brackets are computed through formulae (212) and (213), assuming that $N=\frac{M}{z}=0$.

In jointing the members of the frame, they may be simultaneously tensioned (Fig. 174); the girder tendons are let through ducts in the concrete (left especially in the course of fabrication) and then drawn against the hardened concrete. The position of the reinforcement, both in the length and cross-section of the girder, must correspond to bending moments for a continuous member.

The column joint is subject to the forces N and M, as may be seen in Fig. 85. The shearing force Q is usually small in the column and

can be borne by the friction resistance of the joint. There are two principal types of column joints in multi-storey frames: *hinged*, in which the load is transmitted via the concrete, and *welded*, where load transmission is accomplished by the steel inserted parts together with the concrete.

Columns with *spherical hinged* joints (Fig. 175, *a*) that were tested in the Central Scientific Institute of Industrial Construction have shown that if the workmanship is well done and the member proper-

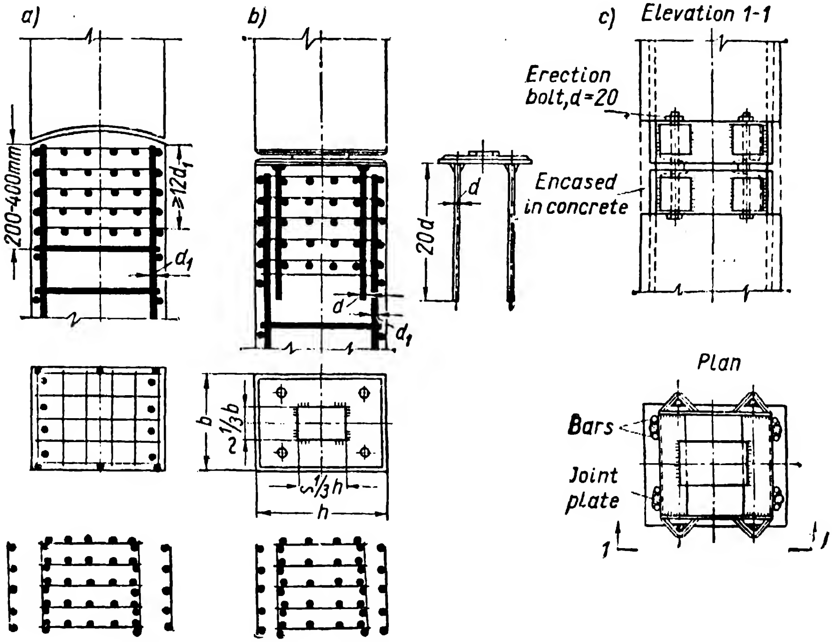


Fig. 175. Column connections in a multi-storey frame

ly reinforced with lateral mats (perpendicular reinforcement) the strength of the joint is not inferior to that of the column itself. Such hinged bearings are especially recommended for heavily loaded columns.

The spherical surface of the lower column is made convex to fit the concave surface of the upper column. The radius of the convex surface is taken as 1.2-1.5 times the dimension of the bigger side of the column, while the radius of the concave end is made 2 or 3 cm greater.

Lateral prefabricated mats are placed in several rows and at intervals of 5-7 cm along a length of the column equal to at least 12 diameters of the column's effective bars. All the mats must be con-

nected, either by welding or splicing, into a unified block; two vertically placed mats are used for this purpose.

The longitudinal bars of the column are passed into the lateral mats. To this end, the first cross bar of the welded reinforcement in the column is placed 200-400 mm from the border of the reinforcing block.

A *welded* joint that will transmit the load via the steel insertions and the concrete (Fig. 175, *b*), is accomplished by contour welding of the steel plate, 10-12 mm thick, at the butt end of the column. Trueing of the columns, so as to centre the transmitted force of the load, is done by welding on 3-4-mm centering plates (with dimensions identical for the upper and lower columns) whose edges are chamfered in accordance with the height of the weld they are to grip. Intermittently deformed anchoring bars of 20*d* length are welded to the edges of the steel plates. The end of the column is reinforced with lateral mats just as in spherical bearings.

Laboratory tests of such a joint have proved its reliability in the transmission of forces. The method of welding the plates along their contours allows the joint to receive a bending moment, although to a limited degree.

If the bending moment is large, the welded joint for transmitting the load through the inserted parts and through the concrete (Fig. 175, *c*) will consist of steel angles placed in the ends of the columns and welded to coupled longitudinal bars (also in the columns). Round connecting bars are inserted in the intervals between each coupled pair and welded to the effective bars of the column. For convenience in erection, short pieces of angle steel are welded to the steel insertions for the reception of bolts.

The joint space is then packed with a stiff mortar mix and finished evenly with the face of the column. The finish is made with fine aggregate and applied upon metal lath. Theoretically such a joint makes the column a continuous unit, but it has a number of shortcomings. It takes a bit too much steel for the inserted parts; furthermore, the responsible workmanship that it requires, and upon which the computed strength of the joint depends, cannot be definitely relied upon.

The resistance of a spherical hinged joint (Fig. 175, *a*) is computed by means of the following empirical formula:

$$N \leqslant mF_{\text{core}}\,(R_{\text{pr}}+2m_s R_s \mu_{\text{perpen. rein}}), \tag{248}$$

n which N—the longitudinal design force in the joint;

$F_{\text{core}}=b_{\text{mat}}h_{\text{mat}}$—the area of the concrete core, limited by the outer dimensions of the lateral mats;

b_{mat} and h_{mat}—dimensions of the mats (from 20 to 25 mm less than the corresponding sides of the column);

R_{pr}—design prism resistance of the column's concrete;

R_s—design resistance of the bars in the lateral mats;

$\mu_{perpen.rein}=\frac{f_{mat}(n_1 h_{mat}+n_2 b_{mat})}{F_{core}s}$

is the coefficient of perpendicular reinforcement of the mats, depending upon the volume ratio of mats and concrete;

f_{mat}—cross-sectional area of the mat's bars;

n_1 and n_2—number of bars in the mat, longitudinally and crosswise;

s— distance between lateral mats along the column length.

The resistance of a welded joint (Fig. 175, *b*) is determined by approximate computations. Under the pressure of the concrete the end steel plates will bend because of their flexibility. Due to this, stresses sharply decrease beyond the contact surfaces of the concrete and steel centering plates. By taking into account the plastic nature of concrete, it may be approximately assumed that the stresses are alike along the entire surface of contact, but are nil beyond its bounds (Fig. 176). With this in mind it is considered that the pressure of the end steel plates is distributed at an angle whose tangent is equal to 1.5, and the total area of contact

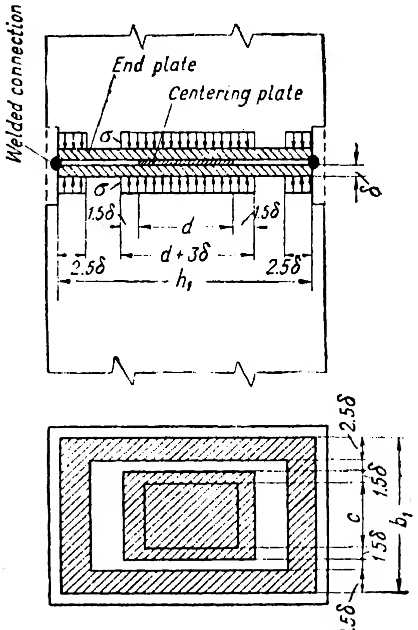


Fig. 176. Normal stresses in the concrete of a joint with centering plates

$$F_{contact}=F_{joint}+F_{face},$$

in which F_{joint}—contact area of the weld around the joint;

F_{face}—face area of contact of the centred plates.

By taking into account the above-mentioned angle of pressure distribution of the end steel plates when their thickness is δ, it is found that

$$F_{joint}=2.5\delta\,[2h_1+2\,(b_1-5\delta)]=5\delta\,(h_1+b_1-5\delta);$$

$$F_{face}+(d+3\delta)\,(c+3\delta),$$

in which b_1 and h_1—dimensions of the end steel plate;

c and d—dimensions of the centred plate.

In order to limit local crushing compression in the concrete, the following condition must be satisfied:

$$\psi = \sqrt[3]{\frac{F}{F_{\text{contact}}}} \leqslant 2,$$

in which F—cross-sectional area of the column;

F_{contact}—contact area.

In practice, the aforesaid condition is always observed if the dimensions of the centred plate $c \approx \frac{1}{3} b$ and $d \approx \frac{1}{3} h$.

If the joint should be only subject to the longitudinal force, then the design stress in the contact $\sigma_{\text{contact}} = \frac{N}{F_{\text{contact}}}$, in which case the stress transmitted to the joint via the weld

$$N_{\text{joint}} = \sigma_{\text{contact}} F_{\text{joint}} = N \frac{F_{\text{joint}}}{F_{\text{contact}}}. \tag{249}$$

The thickness of the welded joint

$$h_{\text{joint}} = \frac{N_{\text{joint}}}{0.7 m_{\text{joint}} R_{\text{weld}} \Sigma l_{\text{joint}}},$$

in which Σl_{joint}—the full length of the welded joint, with poor weld penetration taken into consideration;

R_{weld}—design resistance of the weld.

If the joint is subject to longitudinal forces and a moment, it will be under eccentric compression. If eccentricity is small ($e_0 < 0.15h$), the approximate method of calculation will resolve itself into the transmission of additional forces, caused by the moment, to the lateral welded joints $N_{\text{additional}} = \frac{M}{h}$. The sum of forces transmitted to the lateral weld of the joint

$$N_{\text{lateral}} = \frac{N_{\text{weld}} b_1}{2(b_1 + h_1)} + \frac{M}{h}. \tag{250}$$

and the required thickness of the welded joint

$$h_{\text{joint}} = \frac{N_{\text{lateral}}}{0.7 m_{\text{joint}} R_{\text{weld}} l_{\text{joint}}}, \tag{251}$$

in which l_{joint}—the length of the welded joint, with poor weld penetration taken into consideration.

In most cases the bending moment may be decreased in the column joints of multi-storey buildings by raising the joints close to a mid-ceiling height where, in accordance with the diagram M, the bending moment is almost zero. Such a position of the joints is also very convenient for erection operations.

Sec. 49. DESIGN OF MULTI-STOREY IN-SITU AND COMPOSITE (PRECAST AND IN-SITU) FRAMES

Rigid joints are used in the design of in-situ multi-storey frames. The girders in such frames do not differ from those in ribbed floors, with the exception that at the marginal supports the girders are rigidly connected to the columns (Fig. 177, *a*).

The design of frame joints and the connections of columns to their footings, as presented in Chapter XIII for single-storey in-situ frames, is valid for multi-storey buildings.

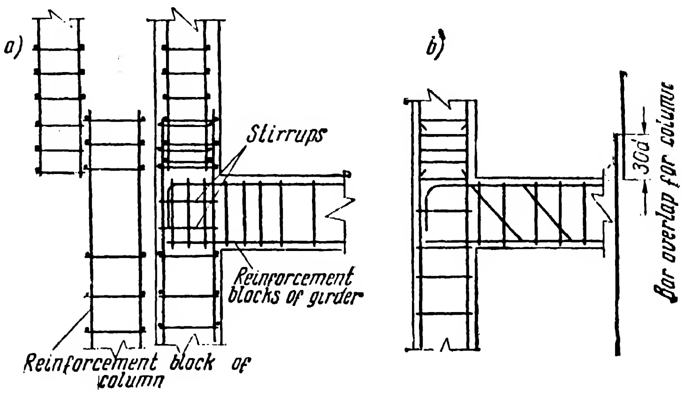


Fig. 177. Joints in a multi-storey in-situ frame

The design of an in-situ frame must anticipate the possibility of concrete splices made because of forced interruptions, such as at the end of the workday or when operations are transferred to another part of the site. In columns, concrete splicing should be done directly above the floor level, where it is convenient because the bars from the lower columns protrude for connection to the upward column (Fig. 177, *b*).

An in-situ reinforced concrete frame is shown in Fig. 178. When spans are very long or floors are very high, the frames should have self-supporting reinforcement blocks. Such blocks of self-supporting bars for a multi-storey powerhouse are shown in Fig. 179. Welded blocks for each girder span are prefabricated in the form of flat trusses and subsequently assembled into a three-dimensional system, tied horizontally at the top and bottom by means of zigzagging, as well as separate, bars.

The blocks of reinforcement for columns are made in the same manner and consist of longitudinal bars, stirrups, and lateral ties placed in the side planes of the block.

Composite (precast and in-situ) reinforced concrete frames are made with rigid joints, just as in-situ frames. An example of a com-

posite reinforced concrete frame with T-girders is given in Fig. 180,*a*. The stirrups of the girders are bared at their tops, as are also the upper bars for the supports. Ribbed panels, with 12-cm spaces left between their butting ends, are placed upon the girders. The rigidity of the joint between the girder and column is assured by the upper bars at the ends of the girders, for which an oval hole is provided in the column.

After the placing of all precast members and providing of reinforcement blocks in the longitudinal interspaces between the panels

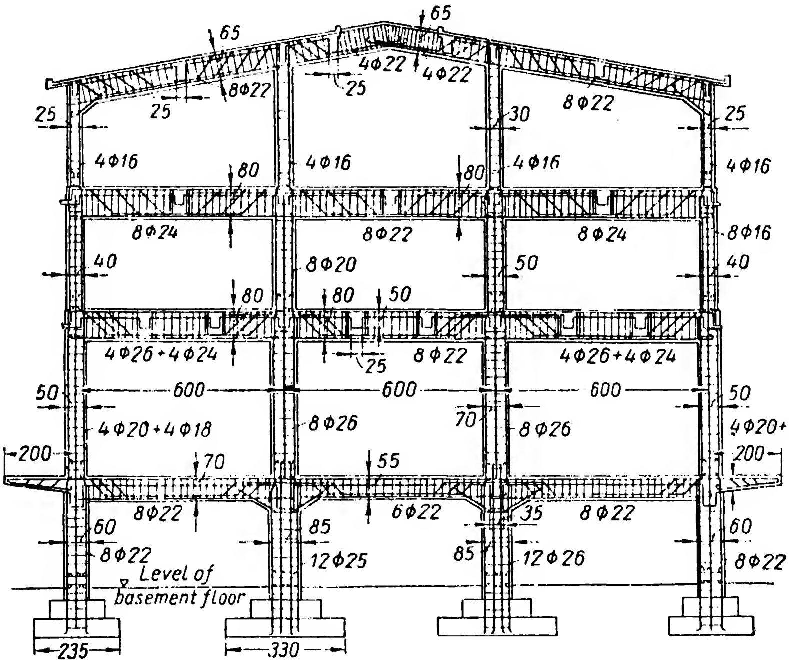


Fig. 178. Reinforcement of a multi-storey in-situ frame

(slabs), the bars at the girders' supports are welded and all spaces (both longitudinal and transverse) filled with 200-grade concrete. This latter operation assures eventual homogeneous action of the frame: individually placed panels will behave as continuous members under their loads and thus possess greater carrying capacity and rigidity than would otherwise have been the case, while the interspan sections of the girders will attain a T-section due to their mutual action with the floor slab, with a final overall height equal to the heights of the girder and slab.

Fig. 180, *b* is a delineation of the mating of girders and columns in a composite reinforced concrete frame, carried out in accordance

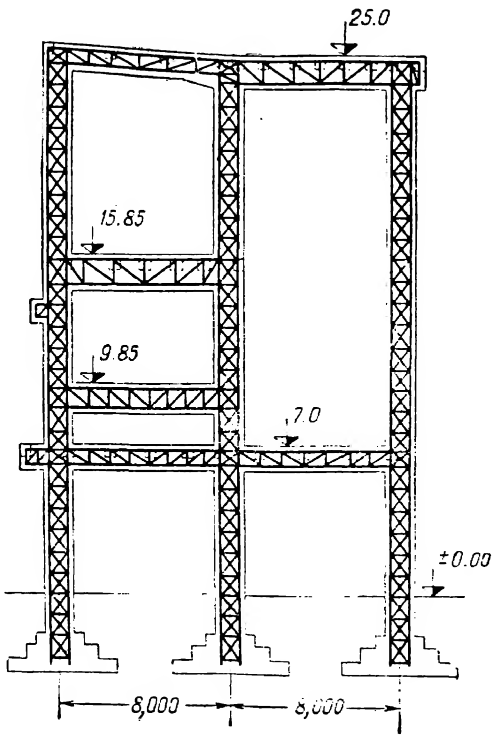


Fig. 179. Sketch of self-supporting reinforcement blocks that make up the multi-storey frame for a powerhouse

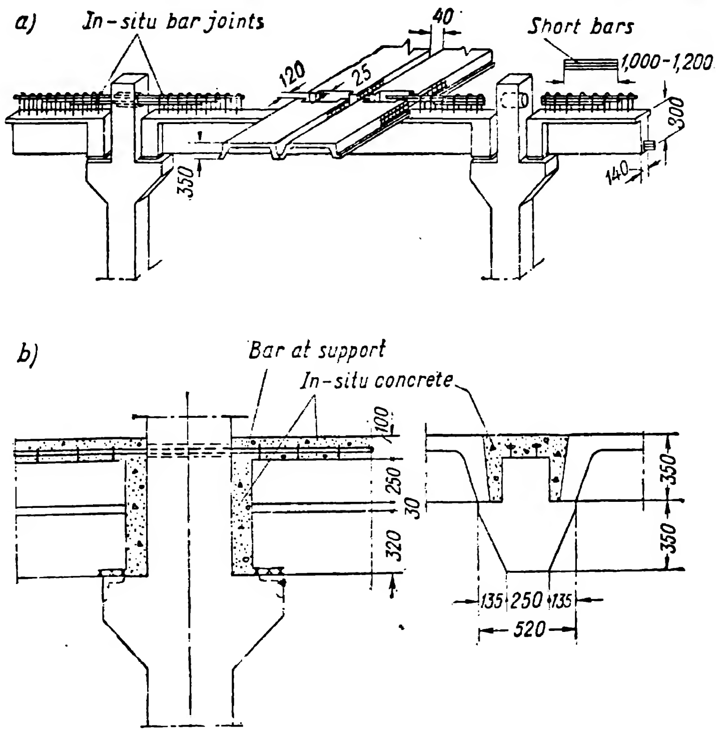


Fig. 180. A frame of composite (precast and in-situ) reinforced concrete

a—before concreting; *b*—after concreting

with unified standards for elements of multi-storey industrial buildings. Here homogeneous action of all members is achieved as already described, except that the girders have projecting shelves for the support of the slabs. The support bars of the girders are connected to the latter's reinforcement blocks after having been let through the holes in the columns.

Sec. 50. GIANT MODULAR BOX-FRAME BUILDINGS

The principal structural elements of a multi-storey giant-modular box-frame building are the precast wall panels, partitions and floors. The most typical examples of this kind of construction are

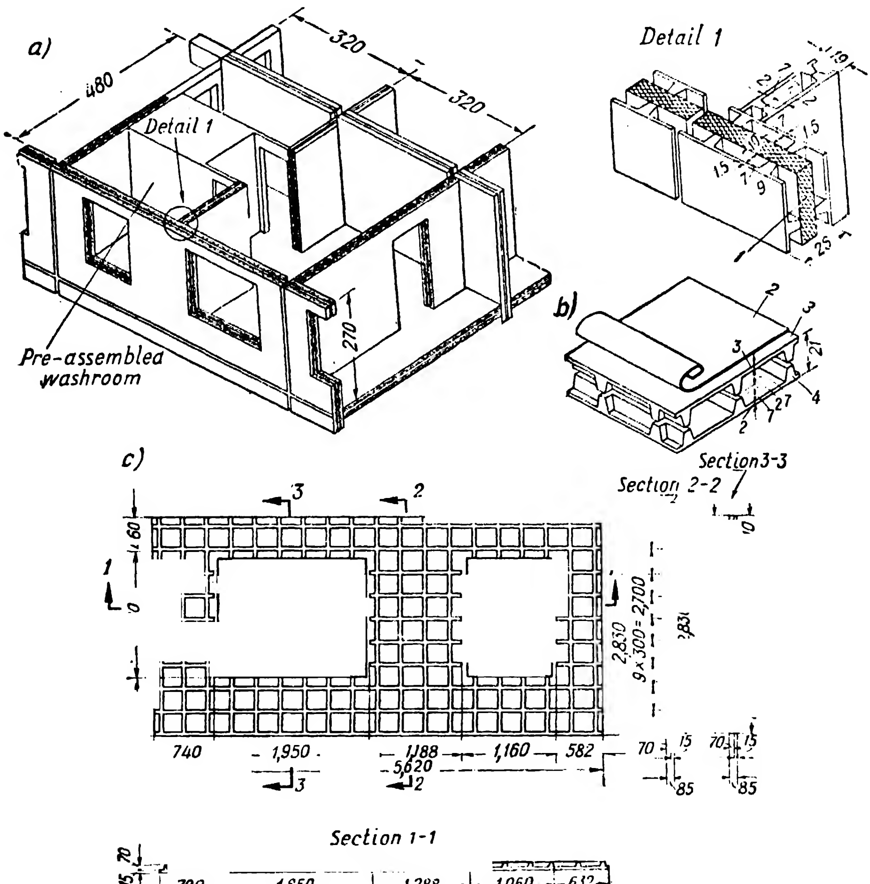


Fig. 181. A building made up of roll-pressed reinforced concrete block panels

found in box-frame apartment houses built without girders or columns and where all the giant panels are at least room-size.

The principle underlying this construction consists in making full use of the carrying capacity of the walls and partitions in the erection of an average four- or five-storey building. The longitudinal and transverse wall and partition panels act as vertical bracing diaphragms and lend the whole structure excellent skeletal rigidity.

One type of box-frame building, invented by N. Y. Kozlov, makes use of pressed, close-ribbed panels. These structures (Fig. 181) have transverse bearing partitions and self-bearing outer walls. All the structural elements—partitions, walls, and floors—are made of doubled close-ribbed slabs with the ribs placed inward. The panels for the outer walls have thermal insulation placed between the doubled slabs, while the inner spaces of the panels that separate the various apartments have a layer of sound insulation (the panels acting as partitions within the apartments proper are not insulated, but an air space is left between the doubled slabs). Floor panels are sound-insulated around their perimeters between the doubled slabs. Doubling of the panels is done at the factory with the aid of welded steel ties.

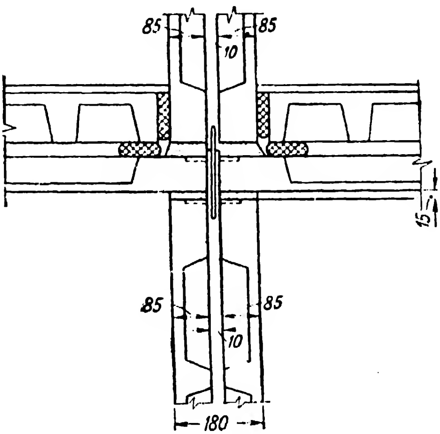


Fig. 182. Detail showing how the bearing partition supports the floor

The doubled-slab construction of the panels results in a division of structural functions in the floor: the lower slab rests upon the bearing partition as a bending member supporting its own weight, while the upper slab transmits its dead and live loads through its bordering ribs (thus also acting as a bending member) by resting upon corresponding bordering ribs of the lower slab, with the insulating strip between the two. A detail of the bearing partition and divided floor slab is shown in Fig. 182.

The ribs of all pressed types of slabs run in both directions, forming square coffers 300 × 300 mm on rib centres, and 70 mm deep (i.e., the ribs project 70 mm). The shell's thickness ranges from 15 to 30 mm, depending upon intended loads. The ribs are reinforced with flat welded reinforcement blocks. There is no reinforcement in the shell between the ribs.

Another type of this construction, proposed by the Soviet engineer V. P. Lagutenko, differs from the other in that the transverse

reinforced concrete bearing partitions act as bending members instead of being subject to direct compression (Fig. 183, *a*). These partitions are 4 cm thick, are bordered by ribs and may be pierced by doors if necessary.

The floor panels of this alternate type also have divided structural functions: the reinforced concrete room-size ribbed floor slab rests upon the lower horizontal rib of the bearing partition of the given storey, while the ceiling panels are carried by the upper horizontal ribs of the bearing partitions of the storey below. The ceiling panels are made of plasterboard (Fig. 183,*b*).

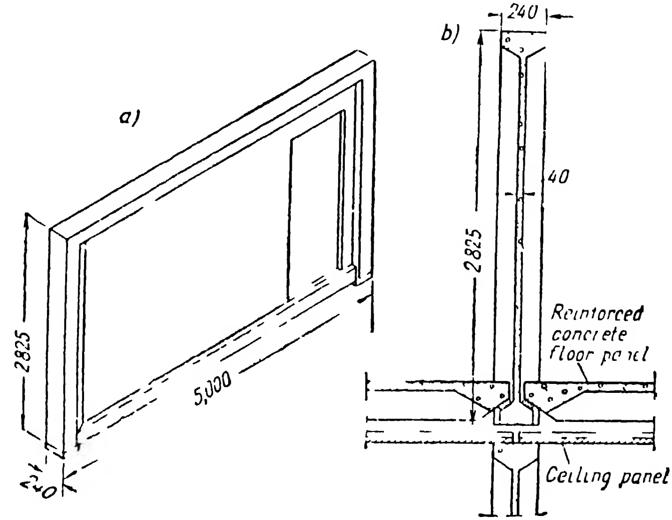


Fig. 183 Details of box-frame proposed by the Soviet engineer V. P. Lagutenko

a—partition panel *b* vertical cross section

Each bearing partition rests only with its ends upon the partition below, thus the load is transmitted only through the vertical ribs. The ribbed floor panel acts as a beam with a span equal to the distance between bearing partitions. The outer walls are formed by insulated reinforced concrete curtain-wall panels connected to the transverse bearing partitions by means of welded steel insertions.

Both the floor and wall panels are reinforced with welded mats and blocks (Fig. 184), with additional bars placed around door openings. All units are manufactured in vertical moulds.

Giant-modular box construction of thin-walled reinforced concrete members for the erection of apartment houses is the most economical in material, labour, and cost, and offers the highest standard of prefabricated design.

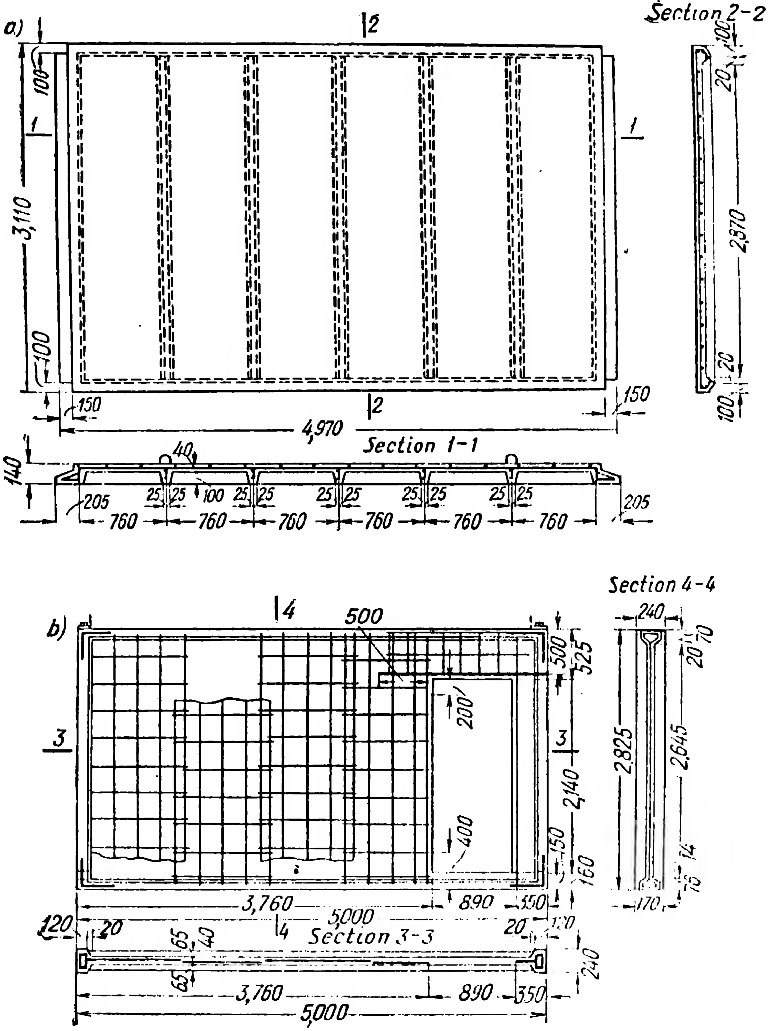


Fig. 184. Panel reinforcement

a – a floor panel; *b* – a bearing partition

1. Vertical Loads

The forces acting upon the frame of a multi-storey building include vertical dead and live loads, horizontal wind thrust, and sometimes other loads. Furthermore, buildings constructed in regions of seismic activity are required to cope with horizontal seismic forces of inertia.

Multi-storey multi-span frame structures are usually built with a simple geometrical layout, in which the floors are equally loaded and the spans are either all alike (Fig. 185,*a*) or with a shorter axially centred span (Fig. 185,*b*).

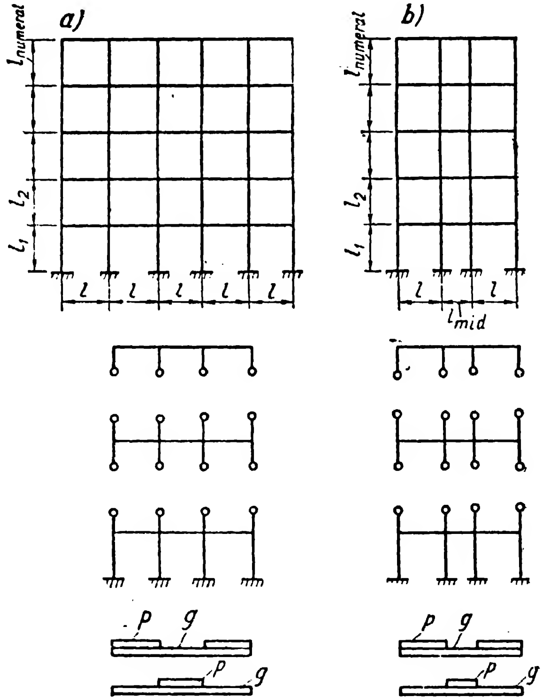


Fig. 185. Design diagrams for multi-storey frames

With such a simple geometrical layout all the joints situated along a given vertical line will have approximately the same angle of rotation, resulting in equal joint moments which decrease to zero in mid-height between floors (Fig. 186). This makes it possible to assume the design diagram of the multi-storey frame as a divided number of single-storey frames whose columns are half the floor heights and hinged at their ends.

Three such single-storey frames are then assumed as being subject to vertical load calculations: the frame of the upper storey, the frame of the intermediate storeys, and the frame of the ground floor. If the entire frame has more than three spans, it can nevertheless be assumed as triple-spanned for practical computations. Fig. 185 gives the design diagram for such computations.

Vertical-load calculations of the frame, in which account is taken of moment redistribution due to plastic deformation, are best carried out by using an equalised-moment diagram evolved from the loading schemes shown in Fig. 185; that is, with live loads placed in alternate spans. This will give maximum moments both in the girders and the columns. The moments thus derived at the ends of girders, with a possible load relationship of $\frac{p}{g} \leqslant 5$, will be about 30% less than the moments in an elastic frame.

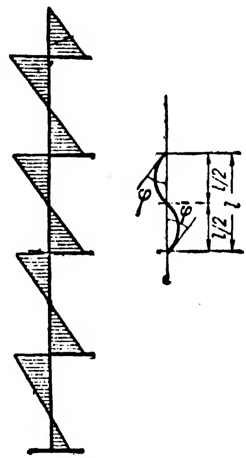


Fig. 186. Vertical-load moment diagram for a line of multi-storey columns

Thus in this method the computation of a multi-storey multi-span frame is reduced to determining bending moments in single-storey symmetrical triple-span frames, loaded symmetrically. Two unknowns are met with in such a frame when using the displacement method: the angle of rotation of the outer joint and the angle of rotation of the intermediate joint.

The moments at the supports, when all columns of a given storey have equal cross-sections, can be found by table*; the moment at the support of the girder

$$M = (\alpha g + \beta p)\, l^2,$$

in which α, β—tabular coefficients, depending upon the loading scheme for the dead and live loads, and upon the ratio of the sum of the columns' linear rigidity and the linear rigidity of the girder,

g and p—dead and live loads per linear metre of girder,

l—the span of the girder.

The bending moment in the columns for each loading scheme is determined, according to the difference in the girders' support moment and in proportion to the linear rigidity of the columns, as follows.

For the frame of the upper storey:

$$M_{1\,\text{lower}} = M_{12} \text{ for the outer columns;}$$

$$M_{2\,\text{lower}} = M_{21} - M_{23} \text{ for the intermediate columns.}$$

* See Supplement VIII at the end of book.

For the frame of the intermediate storey:

$$M_{1\,\text{lower}}=M_{12}\,\frac{i_{\text{lower}}}{i_{\text{lower}}+i_{\text{upper}}}\ \text{and}$$

$$M_{2\,\text{lower}}=(M_{21}-M_{23})\,\frac{i_{\text{lower}}}{i_{\text{lower}}+i_{\text{upper}}}\ \text{for the outer columns;}$$

$$M_{2\,\text{upper}}=(M_{21}-M_{23})\,\frac{i_{\text{upper}}}{i_{\text{lower}}+i_{\text{upper}}}\ \text{for the intermediate columns.}$$

For the frame of the ground floor:

$$M_{1\,\text{lower}}=M_{12}\,\frac{i_{\text{lower}}}{i_{\text{lower}}+1.5i_{\text{upper}}}\ \text{and}\ M_{1\,\text{upper}}=M_{12}\,\frac{1.5i_{\text{upper}}}{i_{\text{lower}}+1.5i_{\text{upper}}}$$

for the outer columns;

$$M_{2\,\text{lower}}=(M_{21}-M_{23})\,\frac{i_{\text{lower}}}{i_{\text{lower}}+1.5i_{\text{upper}}}\ \text{and}\ M_{2\,\text{upper}}=(M_{21}-M_{23})\times$$

$$\times\frac{1.5i_{\text{upper}}}{i_{\text{lower}}+1.5i_{\text{upper}}}\ \text{for the intermediate columns.}$$

The bending moment of the lower restrained end of the columns at the ground floor will be equal to $\frac{1}{2}$ the bending moment at the opposite end.

In the above formulae the expressions

$i_{\text{girder}}=\frac{I_{\text{girder}}}{l}$ is the linear rigidity of the girder;

$i_{\text{lower}}=\frac{I_{\text{lower}}}{l_{\text{lower}}}$ is the linear rigidity of the column below the joint;

$i_{\text{upper}}=\frac{I_{\text{upper}}}{l_{\text{upper}}}$ ditto for the column above the joint;

l_{lower} and l_{upper} are the full lengths of corresponding lower and upper columns (i.e., the storey height).

The bending moments and shearing forces in the interspan sections of the girders are determined by regular methods as when computing a loaded single-span beam, and by the moments at the supports.

After the completion of calculations a larger-moment diagram is plotted for the frame, as already explained in Sec. 26 for floor girders.

Girder and column cross-sections and their rigidity are tentatively determined through approximate trial computations for the frame, or are chosen by comparison with an already designed, or standard type, structure.

In the tentative calculations, the cross-sectional height of the girder is derived through the formula

$$_0=2\sqrt{\frac{M}{R_{be}b}}$$

in which

$M\approx 0\ 6\text{-}0.7M_0$,

M_0—the beam moment of the girder.

The cross-sections of the columns are found by the following approximate formula:

$$F_c=1.2\text{-}1.5\frac{N}{R_{pr}}.$$

The moment of inertia of the girders' and columns' sections are determined tentatively as for a solid concrete cross-section. For an in-situ floor the girders' moments of inertia are based upon a T-section whose width is equal to the full span of the frame's columns.

When the girder is freely supported (as in a diaphragm-braced frame), the moment of the vertical force Q in relation to the axis of the column (Fig 187, *a*)

$$M=Qe,$$

in which

$$e=\frac{2}{3}l_{bracket}+h_{col}$$

$l_{bracket}$—the overhang of the bracket,

h_{column}—the cross-sectional height of the column.

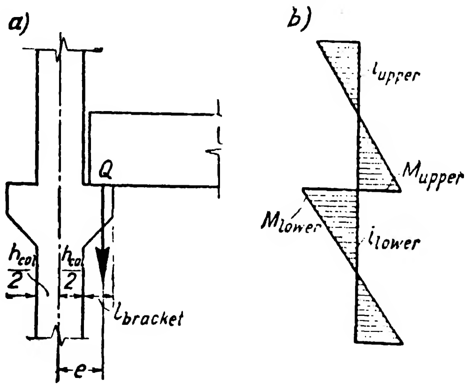


Fig. 187. Investigating a column of a multi-storey frame

a—structural scheme for a free end girder support; *b*—moment diagram of a multi-storey column

In the intermediate columns a moment M at the joint will be created only when the girder has a one-sided live load (and when the spans are equal); hence, here the live load only is used in order to determine the moment, while in the outer columns the moment is computed on the basis of both dead and live loads.

The bending moments in the sections of the column at the joints above and below the given girder (Fig. 187, *b*) are computed by distributing the moment at the joint in proportion to the linear rigidity of the column:

$$M_{lower}=M\frac{i_{lower}}{i_{lower}+i_{upper}};\qquad (252)$$

$$M_{upper}=M\frac{\text{upper}}{i_{lower}+i_{upper}}.\qquad (253)$$

When the columns of adjacent storeys are similar in linear rigidity, the moments at the joints are equally divided between the columns, that is,

$$M_{\text{upper}} = M_{\text{lower}} = 0.5M.$$

When the floor construction in a multi-storey frame building is the flat-slab type, column bending moments can also be computed through formulae (252) and (253), in which case M in the formulae will be the difference in the girders' support moments when their live loads are considered as in alternate spans.

2. Wind Thrust

The horizontal forces P are assumed as being applied to the joints of the frame (Fig. 188), and the transverse force within the columns of each tier is determined by the following formulae:

$$Q_1 = P_1 + P_2 + \ldots\ldots\ldots\ldots + P_n;$$
$$Q_2 = P_2 + P_3 + \ldots\ldots\ldots\ldots + P_n, \text{ etc.}$$

The transverse force at each tier is approximately distributed between the columns as dictated by the ratio of linear rigidity of the

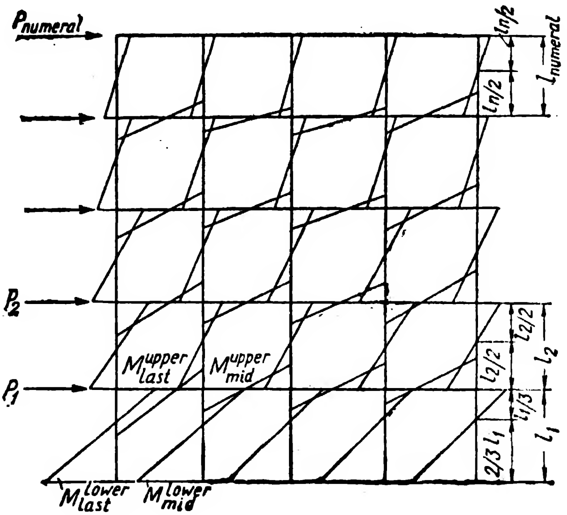


Fig. 188. Moment diagrams evolved from horizontal thrust in a multi-storey frame

girders and columns. The values thus obtained are used in determining the bending moments by assuming that the zero point of the moment-diagram in the columns is situated in the columns' mid-

height (but in the ground storey the point is considered as 2/3 l from the lower end of the column). The sum of moments in a joint from the columns situated above and below is distributed between girders in proportion to their linear rigidity.

If the columns of one storey possess equal cross-sections, the distribution of the transverse force between them at each tier will be as follows:

for the intermediate columns of the tier

$$Q_{\text{intermediate}}=\frac{Q}{m-2(1-\beta)};$$

for the outer columns of the tier

$$Q_{\text{outer}}=\beta Q_{\text{intermediate}};$$

in which m—the number of columns in one tier,
β—a coefficient taken from Table 28.

Table 28

Values for the Coefficient β in the Computation of Horizontal Thrust in the Frame

Ground floor	All other storeys when $\frac{i_{\text{girder}}}{i_{\text{lower}}}$* is:					
	4	3	2	1	0.5	0.25
0.9	0.79	0.75	0.7	0.62	0.56	0.54

* i_{girder}—linear rigidity of the girder.
i_{lower}—linear rigidity of the column below the joint.

At the ground floor the bending moments at the tops of columns will be:

$$M_{\text{outer}}^{\text{upper}}=Q_{\text{outer}}\frac{l_1}{3}; \quad M_{\text{intermediate}}^{\text{upper}}=Q_{\text{intermediate}}\frac{l_1}{3};$$

ditto for the bottoms of columns:

$$M_{\text{outer}}^{\text{lower}}=Q_{\text{outer}}\frac{2}{3}l_1; \quad M_{\text{intermediate}}^{\text{lower}}=\frac{2}{3}l_1.$$

For all the rest of the storeys the bending moments at the tops and bottoms of the columns will be:

$$M=Q\frac{l}{2}.$$

CHAPTER XV

SPECIAL KINDS OF STRUCTURES

Sec. 52. RESERVOIRS

1. Their Layout

Reinforced concrete reservoirs are used for the storage of large volumes of potable, sanitation, and industrial water, petroleum products, industrial acids, and other liquids, and also for sewage oxidation and settling tanks.

Reinforced concrete reservoirs must not only be strong enough to safely hold their contents, but possess impermeability. For this reason special attention is given to the concrete's density; moreover the interior surfaces of reservoirs are either gunite-treated or cemented, or a layer of soluble glass applied. If an aggressive chemical liquid (that eats the concrete away) is to be stored, the interior surface is covered either with ceramic or glass tiles layed in acid-proof mortar.

Prestressing of the concrete is the most effective way of assuring crack resistance and impermeability. The concrete used must be at least grade 200. Reservoirs built in this way will prove durable and economical.

Both round (cylindrical) and rectangular forms are used for reservoirs, but the round is the more advantageous since with a given volume it has less surface area than the rectangular type. Furthermore, the round wall can be made thinner because it is subject only to axial tension, whereas rectangular walls undergo both tension and bending stresses. Hence round reservoirs are more economical and enjoy greater popularity. Although rectangular walls are used with very large reservoirs (more than 2,000 m^3), still even in such cases the round form will prove more rational if prestressed construction is adopted.

The height of a round reservoir is usually 0.3 of its diameter but not more than 4 or 5 m (which is also true of the rectangular type).

The thin-shell dome, from 7 to 8 cm thick, is the usual type of reservoir roof (Fig. 189, *a*), the least amount of reinforced concrete

being expended in its construction. However, with a reservoir volume over 600 m^3 and a diameter greater than 15 m, dome formwork becomes very complex and horizontal flat slabs are used instead (Fig. 189, *b*).

If precast construction is chosen for the roof, the best type will be horizontal ribbed slabs, no matter what the volume or form of the reservoir may be (Fig. 189, *c*).

With a dome (cupola) roof, all loads are transmitted to bearing soil through circumferential expansion of the cylindrical walls,

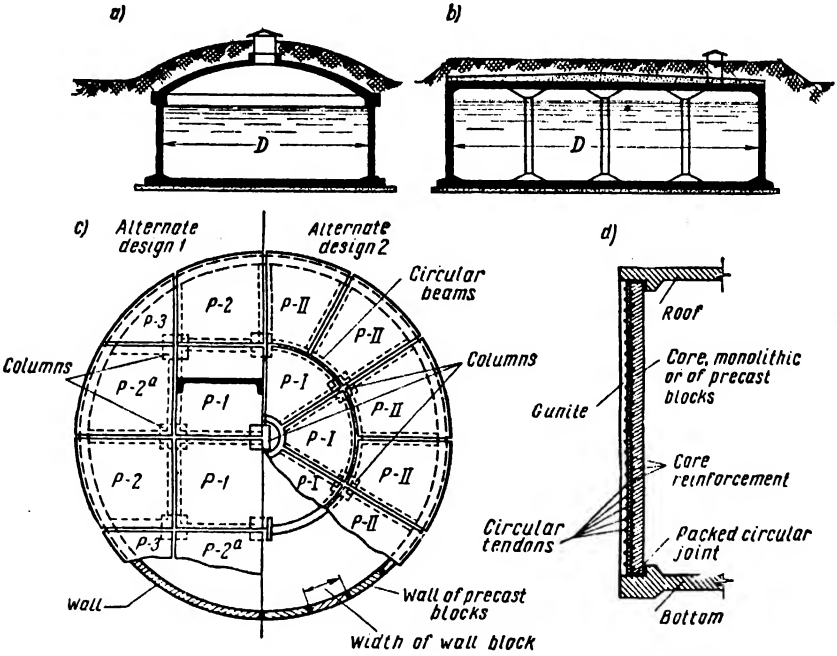


Fig. 189. Cylindrical reinforced concrete reservoirs

a—roofed with a dome; *b*—roofed with a horizontal flat slab; *c*—alternate design of a precast roof; *d*—cross-section of a prestressed reinforced concrete wall

with no stresses whatsoever occurring in the reservoir bottom (assuming that there is no subsurface hydraulic pressure). But when the roof is horizontal and rests on the walls and intermediate columns, the bottom is subject to bending action because of the reaction of the bearing soil, in which case the thickness of the bottom slab and its reinforcement must be determined by computation.

For prestressed reservoirs a sectional design diagram is used: the walls are separated from the bottom and the roof by circumferential joints (Fig. 189, *d*), a method which does not evolve large prestressing bending moments in the walls.

The walls of prestressed reservoirs consist of a reinforced concrete core around which high-strength wire tendons are tightly drawn in an endless spiral by a special machine. This post-tensioning operation causes circumferential compression of the core. Annular tendons may also consist of separate bars connected to steel pegs and then tightly drawn with the aid of a manual long-handled spanner. The core, tendons and interior surface then receive a protective covering applied by the gunite method.

The circumferential joints separating the wall from both the bottom and the roof must fulfil a double function: they must lend adjustability to the wall when it is compressed by the tendons and at the same time must allow no breach of the reservoir's impermeability. To do this, if the reservoir is for the storage of water the joints are tightly calked with cold bitumen mastic; but if petroleum products are to be stored, an oil-resisting rubber ribbon is used for the calking. Impermeability of the lower circumferential joint may also be attained by the insertion of special steel seals.

The concrete core may be assembled of precast curved slabs and the joints filled with expansion cement. Subsequent drawing of the tendons will tighten the joints and assure impermeability.

2. Computation and Design of the Walls of Circular Reservoirs

Under the action of hydrostatic pressure the cylindrical walls of the reservoir are subject to circumferential axial tension. For reservoirs with a maximum volume of 100 m^3 the tensile stresses are approximately determined without consideration of wall connection to the bottom and the roof.

Let us investigate the wall of a small open cylindrical reservoir (Fig. 190). In order to determine the forces involved and compute the reinforcement, the wall is figuratively divided into rings, each a metre high, beginning from the bottom, in which case the assumed uppermost ring may be somewhat higher (i. e. wider) or lower than the others. It is known that the magnitude of hydrostatic pressure is equal to the weight of a liquid column situated above a given level. From this it is found that the design hydrostatic pressure for each of the rings will be as follows:

$$p_0 = n\gamma l;$$
$$p_1 = n\gamma (l - 1);$$
$$p_2 = n\gamma (l - 2), \text{ etc.},$$

in which l—the height of the assumed ring in metres;
γ—specific gravity of liquids and which will be equal to 1 t/m^3 for water,
n—a load coefficient of 1.1.

It has been established by Strength of Materials that if a radial pressure p be directed against a ring whose radius is r, the tension created in the cross-section of the ring

$$T^0 = pr, \tag{254}$$

whence the circumferential design force in the cross-sections of the vertically situated rings

$$T_0^0 = p_0 r; \quad T_1^0 = p_1 r; \quad T_2^0 = p_2 r; \text{ etc.}$$

The tensile forces T^0 are considered as being axially applied to the vertical cross-section of each ring, and the area of tension rein-

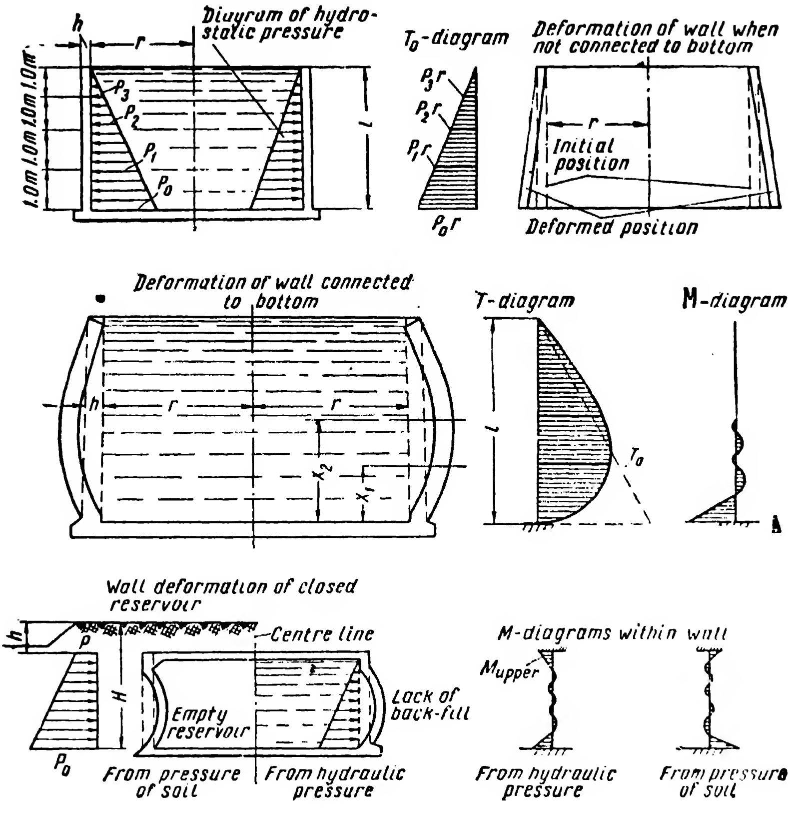


Fig. 190. Behaviour of the loaded walls in cylindrical reservoirs

forcement is determined for the height of each ring. But in a simplified method, which gives a small positive error (towards an additional margin of safety) the reinforcement is determined according to the greatest force T^0 at the end of each ring, i.e., a stepped hydrostatic diagram is used instead of a triangular one (see Sec. 15).

In computations of a large reservoir, the mating of its bottom and wall is taken into account. The bottom is considered as a tie that hinders free radial tensile deformation of the wall but deforms the latter in height. The result is that at the connection of the wall and reservoir's bottom a bending moment M is produced that bends the wall vertically and causes a thrust H that lessens the magnitude of the circumferential force T°.

The magnitudes of the contour forces M and H and the circumferential force T can be derived, with due allowances for the connection at the bottom, through the following formulae:

$$M=\frac{p_0}{2m^2}\left(1-\frac{1}{ml}\right); \tag{255}$$

$$H=\frac{p_0}{m}\left(1-\frac{1}{2ml}\right); \tag{256}$$

$$T=T^{\circ}-p_0 r\left[\eta_1+\eta_2\left(1-\frac{1}{ml}\right)\right], \tag{257}$$

in which $m=\frac{1.3}{\sqrt{rh}}$ is the rigidity of the cylindrical reservoir wall;

p_0—the magnitude of hydrostatic pressure at the foot of the wall;

T°—circumferential force in the wall in accordance with formula (254), i.e., ignoring its connection at the bottom;

η_1 and η_2—coefficients given in Table 29 and depending upon $\varphi=mx$;

x—the distance from the bottom to the cross-section under consideration;

Table 29

Coefficients for Computing Cylindrical Reservoir Walls with Due Consideration for Their Mating with the Bottom

φ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
η_1	1	0.9	0.8	0.71	0.62	0.53	0.45	0.38	0.31	0.25	0.2
η_2	0	0.09	0.16	0.22	0.26	0.29	0.31	0.32	0.32	0.32	0.31
φ	1.1	1.2	1.3	1.4	1.5	1.6	2	2.5	3	4	5
η_1	0.15	0.11	0.07	0.04	0.02	—0.01	—0.06	—0.07	—0.05	—0.01	0
η_2	0.3	0.28	0.26	0.24	0.22	0.2	0.12	0.05	0.01	—0.01	—0.01

An underground reservoir is subject to side pressure from the backfill. The lower ordinate of the pressure diagram

$$p_0=n\gamma kH,$$

and the upper ordinate

$$p = n\gamma kh,$$

in which H and h—depth of the overlayer, measured to the top and bottom of the reservoir, respectively;

k—coefficient of side pressure, depending upon φ (angle of repose of the earth), i. e.,

$$k = \tan^2\left(45 - \frac{\varphi}{2}\right); \tag{258}$$

γ—bulk weight of the earth;

n—load coefficient, but not less than 1.2.

Prior to the filling of the reservoir with liquid, the pressure of the ground will cause an inverted-sign contour force upon the walls, which, when $p_0 \geqslant p$, can be determined by the following formulae:

$$M = \frac{p_0}{2m^2}\left(1 - \frac{1 - \frac{p}{p_1}}{ml}\right); \tag{259}$$

$$H = \frac{p_0}{m}\left(1 - \frac{1 - \frac{p}{p_1}}{2ml}\right). \tag{260}$$

The possibility of a plus- or a minus-bending moment in the joint of the reservoir wall and its bottom, occasioned by various load forces, either when the reservoir is full or empty, must be provided for when determining the area of vertical reinforcement. The relieving effect of the earth backfill, when the reservoir has been completely filled with liquid, is usually ignored in the calculations.

The walls of small reservoirs are made of equal thickness throughout their height. Practical considerations of construction do not allow this thickness to be less than 8-10 cm.

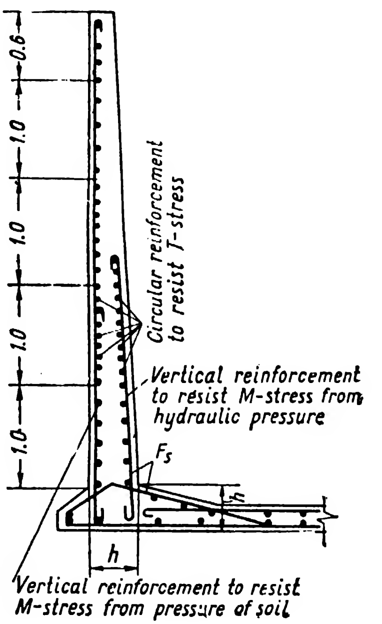


Fig. 191. Wall reinforcement in a cylindrical reservoir

In a large reservoir the wall may be either of equal thickness throughout, or trapezoidal in cross-section (of course, thicker at the foot of the wall).

Reinforcement of the wall will consist of annular horizontal bars arranged in one or two rows, and vertical bars (Fig. 191), the annular bars being lap-connected according to standard requirements. The vertical bars serve the auxiliary function of installation steel

and at the same time improve resistance to vertical bending moments. The connection of the foot of the wall and the bottom of the reservoir is strengthened with tapered ribs and additional bars to take tensile and bending stresses.

In prestressed reservoirs the annular tendons bear circumferential tensile forces T, while the vertical steel of the core is computed to resist the vertical bending moment M. Instructions in computing prestressed walls have already been given in Chapter VI. Annular reinforcement placed in the core is considered as auxiliary steel.

The force of friction $H_{friction}$ incidental to the connection of the bottom of the reservoir and the foot of the wall along the entire circumference is included when calculating circumferential tension T in the wall. Friction decreases the magnitude of circumferential forces but also produces a vertical bending moment:

$$M = \frac{H_{friction}}{m} \eta_2; \tag{261}$$

$$T = T^0 - 2mrH_{friction}\eta_1. \tag{262}$$

From formula (261) we evolve the maximum value of the moment produced at a distance of

$$x = \frac{\pi}{4m} = 0.6\sqrt{rh}$$

from the foot of the wall.

The magnitude of friction around the whole contour

$$H_{friction} = Nf, \tag{263}$$

in which N—design pressure from the dead weight of the wall and roof per linear metre along the circumferential length at the foot of the wall;

$f = 0.5$—the coefficient of friction between the foot of the wall and the bottom of the reservoir.

3. Rectangular Reservoirs

The walls of rectangular reservoirs are subject to bending moments in one or two directions and must therefore be made thicker than in cylindrical reservoirs.

If the walls of a closed rectangular reservoir are very long they will undergo only vertical bending stresses (Fig. 192, *a*) and must be computed just as a beam partly restrained at the supports.

In an open rectangular reservoir with transverse partitions (Fig. 192, *b*) the walls will act as slabs fixed on three sides and undergoing 2-way tension and bending stresses from the triangular hydrostatic load. If the transverse partitions are raised above the bottom so as to allow an even level of the reservoir's liquid contents,

they will act as vertical supports for the walls but will themselves be subject to tension. The bottom of the reservoir serves as the wall's horizontal spacer. In a small reservoir (Fig. 192, *c*) the walls act as slabs supported on all four sides.

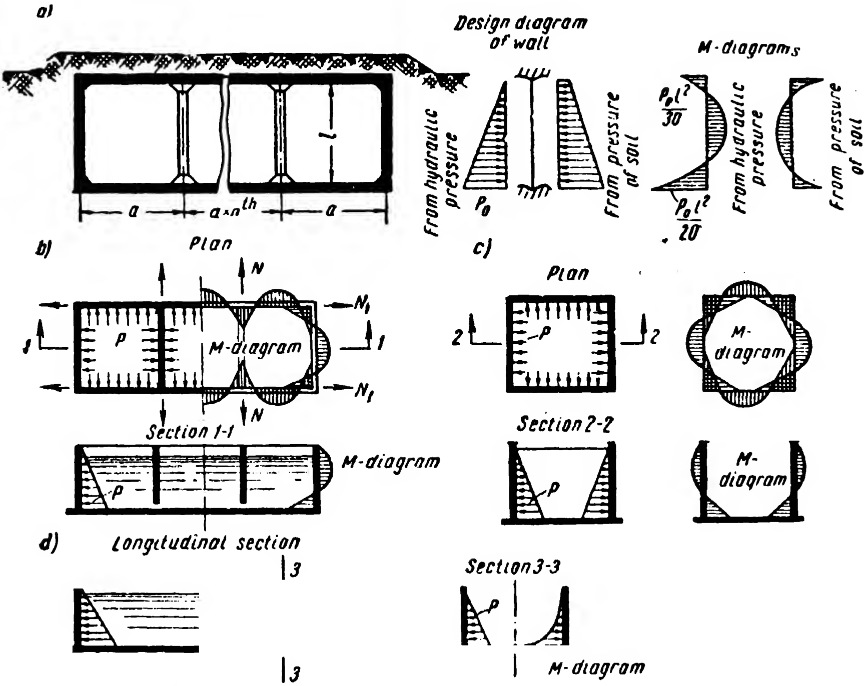


Fig. 192. Wall behaviour of rectangular reservoirs when subject to various loads

All walls must be doubly reinforced, the bars being bent around the corners and carried from one wall into another. In very long open rectangular reservoirs (Fig. 192, *d*), the walls will behave just as cantilevered slabs and must be reinforced with effective vertical bars on their inner sides.

Sec. 53. HOPPERS

Hopper is a term applied to a structure used for storing bulk material, loaded at the top and discharged at the bottom. Its height is small as compared to its cross-sectional dimensions (Fig. 193, *a*). The dimensions are commonly limited to $\frac{H}{l} \leqslant 1.5$. Gravity discharge is accomplished by arranging the bottom in the form of a funnel with walls sloping 5% more than the angle of repose of the material being

handled. Most hoppers are rectangular in plan, but the round type is also possible. Individual rectangular hoppers can be banked together into batteries (Fig. 193, *b*). Very large hoppers are sometimes built in the form of long cylindrical or plicated units.

Until recently in-situ reinforced concrete was the usual material for hoppers, but today the precast type is also being adopted, constructed of separate slabs assembled on the site by welding their inserted steel parts and grouting their joints. Giant ribbed panels

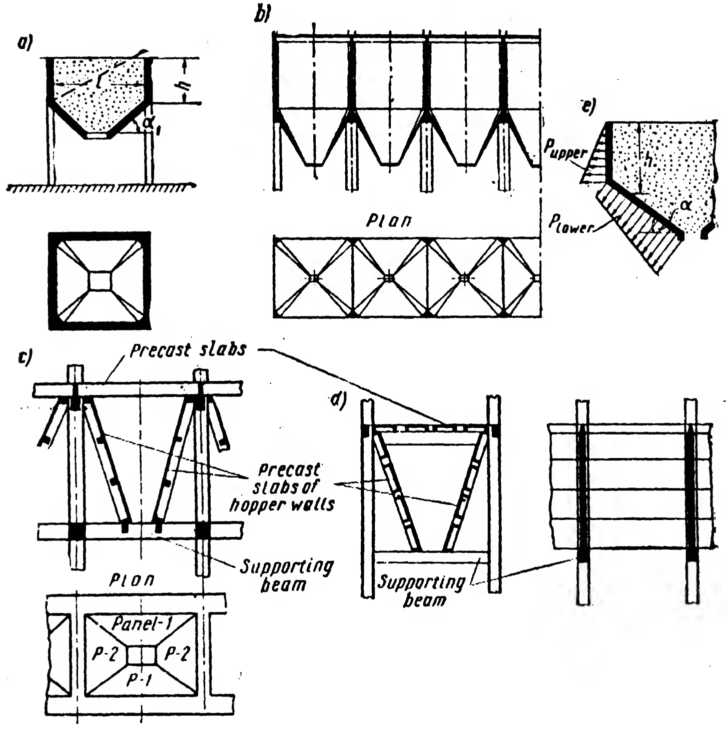


Fig. 193. Reinforced concrete hoppers

a and *b*—in-situ hoppers; *c* and *d*—the precast type; *e*—pressure of bulk material upon the hopper walls

are likewise used (with the ribs on the outside), mounted on specially designed girders (Fig. 193, *c*). In precast hoppers the type with the beam-supported slabs is the simpler form; the funnel's sides are made of trapezoidal slabs placed at the desired angle, and if the hopper has two sloping sides and two vertical ones (Fig. 193, *d*), it is built of rectangular slabs reaching from column to column. The ribs of the slabs are placed on the outside to form a smooth inner surface. Each slab can be made as large as the erection crane can handle, i.e., 5 tons and more.

Bins, just as hoppers, are also used for storing bulk material, but their height is much greater than their width: $\frac{H}{D} > 1.5$ (Fig. 194). Just as hoppers, they are loaded from the top and discharged from below. Bins are usually cylindrical in plan with diameters of either 6 or 12 m, but it is possible to make them rectangular in plan. A battery of individual bins is made into a single structure. The structural elements of a bin include its walls, bottom, roof and upper gallery.

Reinforced concrete bins are customarily used as grain elevators, cement and coal bunkers, and also serve the metallurgical industry.

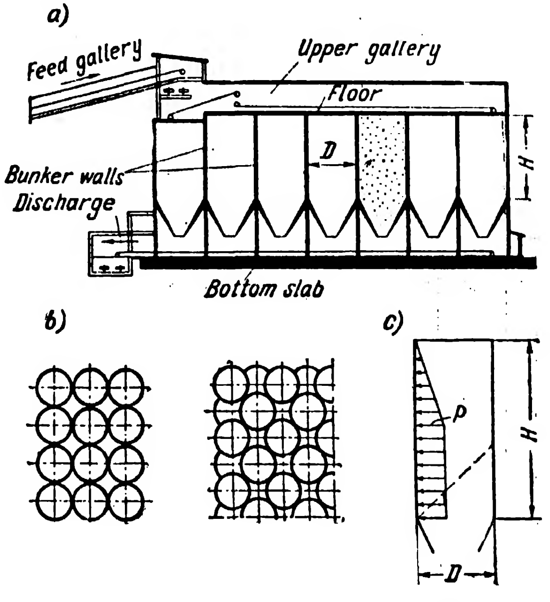


Fig. 194. Reinforced concrete bins
a – longitudinal section of battery of bins; *b* — layouts of bins in a battery: straight and diagonal; *c* – thrust of bulk material on a bin wall

In the past, bins have been built of in-situ reinforced concrete with the use of sliding formwork, but at present precast methods have been inaugurated by mounting prefabricated reinforced concrete rings (either circumferential or segmental units) one upon the other. Another possibility is prestressed construction with high-strength wire tendons drawn against the hardened concrete, as already described for prestressed reservoirs.

Bin walls are subject to axial tension from the radial thrust of the bulk material. Their computation must include carrying capacity, fissure anticipation, and fissure widening. Wall thicknesses in in-situ bins range from 15 to 20 cm, while their precast counterparts are about 35% thinner.

One of the forces to be reckoned with in bin computation is friction, both within the mass of bulk material and upon the bin walls. Wall friction lessens the pressure of the upper strata of the bulk contents upon the lower and decreases the thrust against the bin walls.

CROSS-SECTIONAL AREAS AND WEIGHTS OF BARS

Dia in mm or No. of intermittently deformed bars	Cross-sectional areas in cm² for the following number of bars										Weight in kg/linear m	Dia. in mm or No. of intermittently deformed bars
	1	2	3	4	5	6	7	8	9	10		
2.5	0.049	0.10	0.15	0.20	0.25	0.29	0.34	0.39	0.44	0.49	0.038	2.5
3	0.071	0.14	0.21	0.28	0.35	0.42	0.49	0.57	0.64	0.71	0.055	3
3.5	0.096	0.19	0.29	0.38	0.48	0.58	0.67	0.77	0.86	0.96	0.075	3.5
4	0.126	0.25	0.38	0.50	0.63	0.76	0.88	1.01	1.13	1.26	0.098	4
4.5	0.159	0.32	0.48	0.64	0.80	0.95	1.11	1.27	1.43	1.59	0.125	4.5
5	0.196	0.39	0.59	0.79	0.98	1.18	1.37	1.57	1.77	1.96	0.154	5
5.5	0.238	0.48	0.71	0.95	1.19	1.43	1.66	1.90	2.14	2.38	0.188	5.5
6	0.283	0.57	0.85	1.13	1.42	1.70	1.98	2.26	2.55	2.83	0.222	6
7	0.385	0.77	1.15	1.54	1.92	2.31	2.69	3.08	3.46	3.85	0.302	7
8	0.503	1.01	1.51	2.01	2.51	3.02	3.52	4.02	4.53	5.03	0.395	8
9	0.636	1.27	1.91	2.54	3.18	3.82	4.45	5.09	5.72	6.36	0.499	9
10	0.785	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85	0.617	10
12	1.131	2.26	3.39	4.52	5.65	6.79	7.92	9.05	10.18	11.31	0.888	12
14	1.539	3.08	4.62	6.16	7.69	9.23	10.77	12.31	13.85	15.39	1.208	14
16	2.011	4.02	6.03	8.04	10.05	12.06	14.07	16.08	18.10	20.11	1.578	16
18	2.545	5.09	7.63	10.18	12.72	15.27	17.81	20.36	22.90	25.45	1.998	18
20	3.142	6.28	9.41	12.56	15.71	18.85	21.99	25.14	28.28	31.42	2.466	20
22	3.801	7.60	11.40	15.20	19.00	22.81	26.61	30.41	34.21	38.01	2.984	22
24	4.524	9.04	13.56	18.10	22.62	27.14	31.67	36.19	40.71	45.24	3.551	24
25	4.909	9.82	14.73	19.63	24.54	29.45	34.36	39.27	44.18	49.09	3.853	25
26	5.309	10.62	15.93	21.24	26.55	31.86	37.17	42.47	47.78	53.09	4.168	26
27	5.726	11.45	17.81	22.90	28.63	34.35	40.08	45.80	51.53	57.26	4.495	27
28	6.158	12.32	18.47	24.63	30.79	36.95	43.10	49.26	55.42	61.58	4.834	28
30	7.069	14.14	21.21	28.28	35.34	42.41	49.48	56.55	63.62	70.69	5.549	30
32	8.042	16.08	24.13	32.17	40.21	48.25	56.30	64.34	72.38	80.42	6.313	32
36	10.18	20.36	30.54	40.72	50.90	61.08	71.26	81.44	91.62	101.80	7.99	36
40	12.56	25.12	37.68	50.24	62.80	75.36	87.92	100.48	113.04	125.60	9.87	40

DIAMETERS AND KINDS OF BARS AND THEIR SYMBOLS (DOTS INDICATE THE DIAMETERS USED IN REINFORCED CONCRETE)

Kind of bars	Diameters of round bars in mm or No.. of intermittently deformed bars																									State Standard 5401-50	
	2.5	3	4	5	5.5	6	7	8	9	10	12	14	16	18	20	22	24	25	26	27	28	30	32	36	40	Symbol	Example as used in working drawings
Smooth hot-rolled; St-0 or St-3 steel				●		●		●		●	●	●	●	●	●	●	●			●		●	●	●	●	No symbol	4ø16
Hot-rolled intermittently deformed: St-5 steel										●	●	●	●	●	●	●		●			●		●	●	●	D	4ø16D
Hot-rolled intermittently deformed; grade 25Г2С steel						●	●	●	●	●	●	●	●	●	●	●		●			●		●	●	●	DA	4ø16DA
Hot-rolled intermittently deformed; grade 30ХГ2С steel										●	●	●	●	●	●	●		●			●		●			DH	4ø16DH

(Continued)

Kind of bars	Diameters of round bars in mm or No of intermittently deformed bars																									State Standard 5401-50	
	2.5	3	4	5	5.5	6	7	8	9	10	12	14	16	18	20	22	24	25	26	27	28	30	32	36	40	Symbol	Example as used in working drawings
Cold-drawn wire for prefabricated (welded) mats, State Standard 6727-58		●	●	●	●	●	●	●	●	●																C	4ø5C
Cold-drawn high strength plain wire; State Standard 7348-55	●	●	●	●		●	●	●																		CH	4ø5CH
Cold-drawn high-strength intermittently deformed wire; State Standard 8480-57	●	●	●	●		●	●	●																		CD	4ø5CD
Cold-notched bars; State Standard 6234-52						●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●			DN	4ø16DN

PROPERTIES OF PREFABRICATED (WELDED) MATS MADE OF COLD-DRAWN LOW-CARBON WIRE AND OF LOW-ALLOY INTERMITTENTLY DEFORMED WIRE (STATE STANDARD No. 8478-57) FOR REINFORCEMENT MATS

	Kind of mat	Factory index of mat	Diameter of bars, mm		Mesh size, mm		Width of mat (between centres of outer bars), mm	Weight per roll, kg
			Symbols (see Fig. 17)					
			d_1	d_2	v	u		
In rolls	Longitudinal effective bars of cold-drawn low-carbon wire	3-15/3	3	3	150	250	1,400 1,500 1,900 2,300	From 100 to 500
		4-20/3	4	3	200	250		
		4-15/3	4	3	150	250		
		5-20/4	5	4	200	250		
		5-15/4	5	4	150	250		
		5.5-15/4	5.5	4	150	250		
		5-10/4	5	4	100	250		
	Lateral effective bars of cold-drawn low-carbon wire	3/3-15	3	3	150	250	1,400 1,500 1,900 2,300	From 100 to 500
		3/4-20	4	3	200	250		
		3/4-15	4	3	150	250		
		4/5-20	5	4	200	250		
		4/5-15	5	4	150	250		
		4/5.5-15	5.5	4	150	250		
	Lateral effective bars of low-alloy intermittently deformed grade 25Г2С steel	4/5-15	6	4	150	300	2,300 and 2,650	From 200 to 500
		4/7-15	7	4	150	300		
		4/8-15	8	4	150	300		
		5/9-15	9	5	150	300		
		5/10-15	10	5	150	300		
		5/9-10	9	5	100	300		
		5/10-10	10	5	100	300		
	Effective bars in both directions, of cold-drawn low-carbon wire	4-20	4	—	200	—	1,400 and 2,300	From 200 to 500
		5-20	5	—	200	—		
		5-15	5	—	150	—	2,300 and 2,650	
		5.5-15	5.5	—	150	—		
		5-10	5	—	100	—		
		5.5-10	5.5	—	100	—		

(Continued)

Kind of mat	Factory index of mat	Diameter of bars, mm		Mesh size, mm		Width of mat (between centres of outer bars), mm	Weight per roll, kg
		Symbols (see Fig. 17)					
		d_1	d_2	v	u		
Flat mats with longitudinal effective bars of low-alloy intermittently deformed grade 25Г2C steel*	8-20/5 8-15/5 9-15/5 10-15/5.5 9-10/5.5 10-10/5.5	8 8 9 10 9 10	5 5 5 5.5 5.5 5.5	200 150 150 150 100 100	300 300 300 300 300 300	1,500 1,900 2,300	—
Flat mats, with effective bars in both directions, of low-alloy intermittently deformed grade 25Г2C steel *	8-20 8-15 9-15 10-15 9-10 10-10	8 8 9 10 9 10	— — — — — —	200 150 150 150 100 100	— — — — — —	2,300 and 2,650	—

* Length of flat mats between centres of outer lateral bars $A \leqslant 9{,}000$ mm.

DESIGN CROSS-SECTIONAL AREA OF BARS AND THEORETICAL WEIGHT PER LINEAR METRE OF PREFABRICATED MAT

Factory index of mat	Design cross-sectional area of lateral bars in cm² per linear m	Design cross-sectional area of all longitudinal bars in cm² for the following mat widths, mm					Theoretical weight per linear m of mat in kg for the following mat widths, mm				
		1,400	1,500	1,900	2,300	2,650	1,400	1,500	1,900	2,300	2,650
3-15/3	0.29	0.78	0.78	0.99	1.2	—	0.94	0.97	1.22	1.49	—
4-20/3	0.29	1.0	1.13	1.38	1.64	—	1.12	1.24	1.53	1.82	—
4-15/3	0.29	1.38	1.38	1.76	2.14	—	1.41	1.44	1.83	2.21	—
5-20/4	0.5	1.57	1.76	2.16	2.55	—	1.8	1.99	2.46	2.92	—
5-15/4	0.5	2.16	2.16	2.74	3.33	—	2.26	2.3	2.92	3.53	—
5.5-15/4	0.5	2.61	2.61	3.32	4.03	—	2.62	2.66	3.37	4.47	—
5-10/4	0.5	2.95	3.14	3.92	4.7	—	2.89	3.08	3.84	4.6	—
5.5-10/4	0.5	3.56	3.8	4.75	5.7	—	3.36	3.59	4.5	5.39	—

(Continued)

Factory index of mat	Design cross-sectional area of lateral bars in cm² per linear m	Design cross-sectional area of all longitudinal bars in cm² for the following mat widths, mm					Theoretical weight per linear m of mat in kg for the following mat widths, mm				
		1,400	1,500	1,900	2,300	2,650	1,400	1,500	1,900	2,300	2,650
3/3-15	0.47	0.5	0.57	0.64	0.79	—	0.93	1.02	1.22	1.48	—
3/4-20	0.63	0.5	0.57	0.64	0.79	—	1.11	1.21	1.47	1.78	—
3/4-15	0.84	0.5	0.57	0.64	0.79	—	1.35	1.47	1.79	2.17	—
4/5-20	0.98	0.88	1.0	1.13	1.38	—	1.81	1.97	2.39	2.88	—
4/5-15	1.3	0.88	1.0	1.13	1.38	—	2.17	2.36	2.88	3.48	—
4/5.5-15	1.58	0.88	1.0	1.13	1.38	—	2.49	2.71	3.3	4.0	—
4/6-15	1.88	—	—	—	1.26	1.39	—	—	—	4.45	5.0
4/7-15	2.57	—	—	—	1.26	1.39	—	—	—	5.73	6.53
4/8-15	3.34	—	—	—	1.26	1.39	—	—	—	7.16	8.18
5/9-15	4.24	—	—	—	1.96	2.16	—	—	—	9.47	10.7
5/10-15	5.23	—	—	—	1.96	2.16	—	—	—	11.2	12.8
5/9-10	6.35	—	—	—	1.96	2.16	—	—	—	13.3	15.2
5/10-10	7.85	—	—	—	1.96	2.16	—	—	—	16.0	18.3
4-20	0.63	1.0	—	—	1.64	1.89	1.5	—	—	2.4	2.82
5-20	0.98	1.57	—	—	2.55	2.94	2.35	—	—	3.81	4.38
5-15	1.3	—	—	—	3.33	3.73	—	—	—	5.01	5.68
5.5-15	1.58	—	—	—	4.03	4.5	—	—	—	6.07	6.87
5-10	1.96	—	—	—	4.7	5.3	—	—	—	7.3	8.32
5.5-10	2.37	—	—	—	5.7	6.4	—	—	—	8.85	10.0
8-20/5	0.65	—	4.52	5.52	6.53	—	—	4.34	5.33	6.34	—
8-15/5	0.65	—	5.52	7.03	8.55	—	—	5.13	6.52	7.92	—
9-15/5	0.65	—	7.0	8.9	10.8	—	—	6.29	7.98	9.7	—
10-15/5.5	0.79	—	8.65	11.0	13.5	—	—	7.76	9.86	12.1	—
9-10/5.5	0.79	—	10.2	12.7	15.3	—	—	8.98	11.2	13.5	—
10-10/5.5	0.79	—	12.6	15.7	18.9	—	—	10.9	13.5	16.3	—
8-20	2.51	—	—	—	6.53	7.53	—	—	—	9.75	11.3
8-15	3.35	—	—	—	8.55	9.55	—	—	—	12.9	14.6
9-15	4.24	—	—	—	10.8	12.1	—	—	—	16.3	18.4
10-15	5.23	—	—	—	13.3	15.0	—	—	—	20.1	23.7
9-10	6.35	—	—	—	15.3	17.2	—	—	—	23.7	26.9
10-10	7.85	—	—	—	18.9	21.2	—	—	—	29.3	33.2

SPECIFIED AND DESIGN LOADS AND LOAD COEFFICIENTS

Item No.	Load considered	Specified load in kg/m²	Load coefficient	Design load in kg/m²
		a	b	c
	A. Floor loads:			
1	In lofts (equipment, such as ventilation plants, water tanks, motors, etc., are not included in this category)	75	1.4	105
2	In apartments, medical institutions (with the exception of waiting rooms and auxiliary halls, where many persons may gather), kindergartens and nurseries. Weights of corresponding equipment are included	150	1.4	210
3	In hostels, offices, schoolrooms, service quarters, and factory bays, including weight of light equipment	200	1.4	280
4	In the halls of hostels, service quarters, and offices	300	1.3	390
5	In dining halls of canteens and restaurants and in auditoriums, including weight of light equipment	300	1.3	390
6	In theatres, cinemas, clubs, school auditoriums and on stages, and in waiting rooms of railway and other transport stations.	400	1.2	480
7	In industrial plants, warehouses, and shops where merchandise is sold (the load given is the minimum, but loads must be computed according to technological equipment, etc.)	400		—
8	In libraries and archives (the load given is the minimum, but loads must be computed according to actual weights)	500	At least 1.2, but the coefficient must be in accordance with either manufacturing specifications or actual data 1.2	600

(Continued)

Item No.	Load considered	Specified load in kg/m²	Load coefficient	Design load in kg/m²
		a	*b*	*c*
9	In auxiliary rooms of factories where there will be no heavy machinery or materials, and in passageways for light transport facilities (the load is given as the minimum, but loads must be computed according to actual equipment)	200	At least 1.2, but the coefficient must correspond to either manufacturing specifications or actual data	—
10	In vestibules, stairways, on balconies and verandahs:			
	a) in connection with Item Nos. 2 and 3	300	1.4	420
	b) All other rooms	400	1.4	560
	B. Miscellaneous loads:			
11	Vertical loads and horizontal thrust of cranes	According to specifications	1.3	—
12	Hydrostatic liquid pressure	Ditto	1.1	—
13	Thrust and weight of bulk material	Ditto	At least 1.2, but the coefficient must correspond to either specifications or actual data	—
14	Gas pressure	Ditto	Ditto	—
15	Dead structural weights, except those given in Item 16	Ditto	1.1	—
16	Dead weights of thermal insulation, either in the form of slabs or layers	Ditto	1.2	—

Notes. 1. Wind thrust and snow loads are dictated by geographical regions and building outlines. Wind thrust coefficient $n=1.2$. Snow load coefficient $n=1.4$. 2. Dead weights of partitions are not included in floor loads and must be computed in accordance with actual data; this must include the construction of the partition, its manner of support upon the floor, and a load coefficient of 1.1.

BENDING MOMENTS AND SHEARING FORCES IN EQUAL-SPAN CONTINUOUS BEAMS WITH BOTH UNIFORM AND CONCENTRATED LOADS

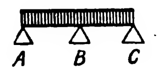
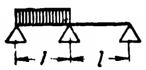
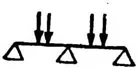
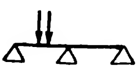
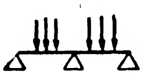
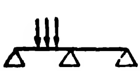
At a uniform load:

$$M = (\alpha g + \beta p)\, l^2; \quad Q = (\gamma g + \delta p)\, l.$$

At a concentrated load:

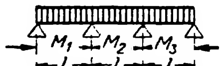
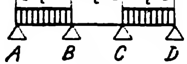
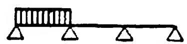
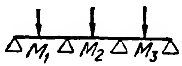
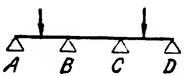
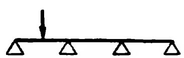
$$M = (\alpha G + \beta P)\, l; \quad Q = \gamma G + \delta P.$$

Double-Span Beams

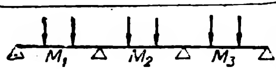
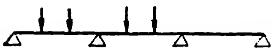
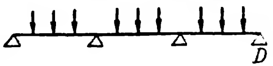
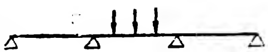
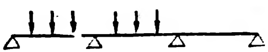
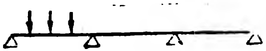
Loading diagram	Interspan moments		Moments at supports	Shearing forces			
	M_1	M_2	M_B	Q_A	Q_B^{left}	Q_B^{right}	Q_C
	0.07	0.07	−0.125	0.375	−0.625	0.625	−0.375
	0.096	−0.025	−0.063	0.437	−0.563	0.063	0.063
	0.156	0.156	−0.188	0.312	−0.688	0.688	−0.312
	0.203	−0.047	−0.094	0.406	−0.594	0.094	0.094
	0.222	0.222	−0.333	0.667	−1.334	1.334	−0.667
	0.278	−0.056	−0.167	0.833	−1.167	0.167	0.167
	0.266	0.266	−0.469	1.042	−1.958	1.958	−1.042
	0.383	−0.117	−0.234	1.266	−1.734	0.234	0.234

(Continued)

Triple-Span Beams

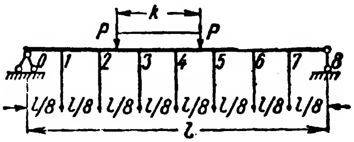
Loading diagram	Interspan moments		Moments at supports		Shearing forces					
	M_1	M_2	M_B	M_C	Q_A	Q_B^{left}	Q_B^{right}	Q_C^{left}	Q_C^{right}	Q_D
	0.08	0.025	—0.1	—0.100	0.400	—0.600	0.500	—0.500	0.600	—0.400
	0.101	—0.05	—0.050	—0.050	0.450	—0.550	0.000	0.000	0.550	—0.450
	—0.025	0.075	—0.050	—0.050	—0.050	—0.050	—0.500	—0.500	0.050	0.050
			—0.117	—0.033	0.383	—0.617	0.583	—0.417	0.033	0.033
			—0.067	0.017	0.433	—0.567	0.083	0.083	—0.017	—0.017
	0.175	0.100	—0.150	—0.150	0.350	—0.650	0.500	—0.500	—0.650	—0.350
	0.213	—0.075	—0.075	—0.075	0.425	—0.575	0.000	0.000	0.575	—0.425
	—0.038	0.175	—0.075	—0.075	—0.075	—0.075	0.500	—0.500	0.075	0.075
			—.0175	0.050	0.325	—0.675	0.625	—0.375	0.050	0.050
			—0.100	0.025	0.400	—0.600	0.125	0.125	—0.025	—0.025

Triple-Span Beams *(Continued)*

Loading diagram	Interspan moments		Moments at supports		Shearing forces					
	M_1	M_2	M_B	M_C	Q_A	Q_B^{left}	Q_B^{right}	Q_C^{left}	Q_C^{right}	Q_D
	0.244	0.067	−0.267	−0.267	0.733	−1.267	1.000	−1.000	1.267	−0.733
	0.289	−0.133	−0.133	−0.133	0.866	−1.133	0.000	0.000	1.133	−0.866
	−0.044	0.200	−0.133	−0.133	−0.133	−0.133	1.000	−1.000	0.133	0.133
			−0.311	−0.089	0.689	−1.311	1.222	−0.778	0.089	0.089
			−0.178	0.044	0.822	−1.178	0.222	0.222	−0.044	−0.044
	0.313	0.125	−0.375	−0.375	1.125	−1.875	1.500	−1.500	1.875	−1.125
	0.406	−0.188	−0.188	−0.188	1.313	−1.688	0.000	0.000	1.688	−1.313
	−0.094	0.313	−0.188	−0.188	−0.188	−0.188	1.500	−1.500	0.188	0.188
			−0.437	−0.125	1.063	−1.938	1.812	−1.188	0.125	0.125
			−0.250	−0.062	1.250	−1.750	0.312	0.312	−0.062	−0.062

DATA FOR COMPUTATION OF OVERHEAD-CRANE GIRDERS WITH ONE CRANE OPERATING

$$M=\alpha Pl$$
$$Q=\gamma P$$



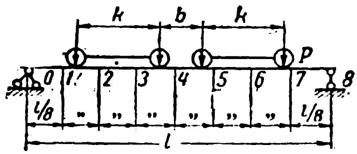
$\frac{k}{l}$	Coefficient α						
	Cross-section number						
	1	2	3	4	5	6	7
0.3	0.181	0.301	0.357	0.350	0.357	0.301	0.181
0.4	0.168	0.275	0.320	0.300	0.320	0.275	0.168
0.5	0.156	0.251	0.281	0.250	0.281	0.251	0.156
0.6	0.144	0.226	0.245	0.250	0.245	0.226	0.144
0.7	0.131	0.198	0.235	0.250	0.235	0.198	0.131
0.8	0.118	0.188	0.235	0.250	0.235	0.188	0.118
0.9	0.109	0.188	0.235	0.250	0.250	0.188	0.109

$\frac{k}{l}$	Coefficient γ								
	Cross-section number								
	0	1	2	3	4	5	6	7	8
0.3	1.700 0.000	1.450 −0.125	1.200 −0.250	0.950 −0.450	0.700 −0.700	0.450 −0.950	0.250 −1.200	0.125 −1.450	0.000 −1.700
0.4	1.600 0.000	1.350 −0.125	1.100 −0.250	0.850 −0.375	0.600 −0.600	0.375 −0.850	0.250 −1.100	0.125 −1.350	0.000 −1.600
0.5	1.500 0.000	1.250 −0.125	1.000 −0.250	0.750 −0.375	0.500 −0.500	0.375 −0.750	0.250 −1.000	0.125 −1.250	0.000 −1.500
0.6	1.400 0.000	1.150 −0.125	0.900 −0.250	0.650 −0.375	0.500 −0.500	0.375 −0.650	0.250 −0.900	0.125 −1.150	0.000 −1.400
0.7	1.300 0.000	1.050 −0.125	0.800 −0.250	0.625 −0.375	0.500 −0.500	0.375 −0.625	0.250 −0.800	0.125 −1.050	0.000 −1.300
0.8	1.200 0.000	0.950 −0.125	0.750 −0.250	0.625 −0.375	0.500 −0.500	0.375 −0.625	0.250 −0.750	0.125 −0.950	0.000 −1.200
0.9	1.100 0.000	0.875 −0.125	0.750 −0.250	0.625 −0.375	0.300 −0.500	0.375 −0.625	0.250 −0.750	0.125 −0.875	0.000 −1.100

(Continued)

DATA FOR COMPUTATION OF OVERHEAD-CRANE GIRDERS WITH TWO CRANES OPERATING

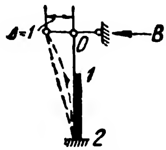
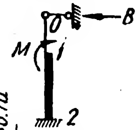
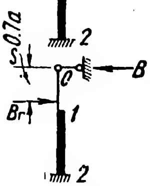
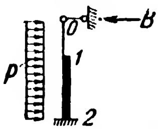
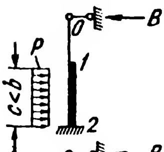
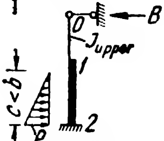
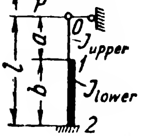
$$M = \alpha P l$$
$$Q = \gamma P$$



$\frac{b}{l}$	$\frac{k}{l}$	Coefficient α — Cross-section number 1	2	3	4	5	6	7
0.125	0.3	0.257	0.426	0.545	0.575	0.545	0.426	0.257
	0.4	0.245	0.401	0.461	0.488	0.461	0.401	0.245
	0.5	0.233	0.376	0.423	0.438	0.423	0.376	0.233
0.25	0.6	0.190	0.312	0.377	0.375	0.377	0.312	0.190
	0.7	0.187	0.312	0.377	0.375	0.377	0.312	0.187
	0.8	0.187	0.312	0.377	0.375	0.377	0.312	0.187
	0.9	0.187	0.312	0.377	0.375	0.377	0.312	0.187

$\frac{b}{l}$	$\frac{k}{l}$	Coefficient γ — Cross-section number 0	1	2	3	4	5	6	7	8
0.125	0.3	2.450	2.075	1.700	1.150	0.775	0.45	0.25	0.125	0.000
		0.000	−0.125	−0.375	−0.450	−0.775	−1.15	−1.700	−2.075	−2.450
	0.4	2.350	1.975	1.600	1.225	0.775	0.400	0.25	0.125	0.000
		0.000	−0.125	−0.25	−0.400	−0.775	−1.225	−1.600	−1.975	−2.350
	0.5	2.250	1.875	1.500	1.125	0.875	0.500	0.25	0.125	0.000
		0.000	−0.125	−0.25	−0.500	−0.875	−1.125	−1.500	−1.875	−2.250
0.25	0.6	1.900	1.525	1.250	1.000	0.750	0.500	0.250	0.125	0.000
		0.000	−0.125	−0.250	−0.500	−0.750	−1.000	−1.250	−1.525	−1.900
	0.7	1.800	1.500	1.250	1.000	0.750	0.500	0.250	0.125	0.000
		0.000	−0.125	−0.250	−0.500	−0.750	−1.000	−1.250	−1.500	−1.800
	0.8	1.750	1.500	1.250	1.000	0.750	0.500	0.250	0.125	0.000
		0.000	−0.125	−0.250	−0.500	−0.750	−1.000	−1.250	−1.500	−1.750
	0.9	1.750	1.500	1.250	1.000	0.750	0.500	0.250	0.125	0.000
		0.000	−0.125	−0.250	−0.500	−0.750	−1.000	−1.250	−1.500	−1.750

FORMULAE FOR COLUMNS WITH VARYING CROSS-SECTIONS

Loading diagrams	Reaction at supports B	Bending moments		
		M_{10}	M_{12}	M_{21}
	$\dfrac{3E_c J_{\text{lower}}}{l^3(1+k)}$	$-Ba$	$-Ba$	$-Bl$
	$\dfrac{3M(1-\alpha^2)}{2l(1+k)}$	$-Ba$	$M-Ba$	$M-Bl$
	$\dfrac{Br(1-\alpha)}{1+k}$	$0.3Bra-Ba$	$0.3Bra-Ba$	$Br(l-0.7a)-$ $-Bl$
	$\dfrac{3pl(1+\alpha k)}{8(1+k)}$	$\dfrac{pa^2}{2}-Ba$	$\dfrac{pa^2}{2}-Ba$	$\dfrac{pl^2}{2}-Bl$
	$\dfrac{pc^3(4l-c)}{8l^3(1+k)}$	$-Ba$	$-Ba$	$\dfrac{pc^2}{2}-Bl$
	$\dfrac{pc^3(5l-c)}{40l^3(1+k)}$	$-Ba$	$-Ba$	$\dfrac{pc^2}{2}-Bl$
	$\alpha=\dfrac{a}{l};\quad k=\alpha^3\left(\dfrac{J_{\text{lower}}}{J_{\text{upper}}}-1\right)$			

DATA FOR THE COMPUTATION OF VERTICAL LOADS IN MULTI-STOREY FRAMES

Table 1

Moments at the Supports in double-Span Frames (Uppermost Storey)

$$M=(\alpha g+\beta p)l^2$$

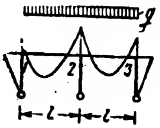
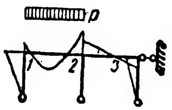
Loading and stress diagrams	$\frac{i_{lower}}{i_{girder}}$	α-and β-moment coefficients			
		M_{12}	M_{21}	M_{23}	M_{32}
	0.25 0.5 1 1.5	0.023 0.036 0.050 0.058	0.114 0.107 0.100 0.097	— — — —	— — — —
	0.25 0.5 1 1.5	0.031 0.045 0.059 0.066	0.068 0.070 0.073 0.075	0.046 0.037 0.027 0.022	0.008 0.009 0.009 0.008

Table 2

Moments at the Supports in Double-Span Frames (Intermediate Storey)

$$M = (\alpha g + \beta p)l^2$$

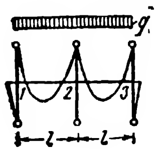
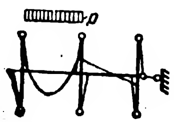
Loading and stress diagrams	$\frac{i_{lower}+i_{upper}}{i_{girder}}$	α-and β-moment coefficients			
		M_{12}	M_{21}	M_{23}	M_{32}
	0.5 1 2 3	0.036 0.050 0.063 0.068	0.107 0.100 0.094 0.092	— — — —	— — — —
	0.5 1 2 3	0.045 0.059 0.070 0.074	0.070 0.073 0.076 0.078	0.037 0.027 0.018 0.014	0.009 0.009 0.007 0.006

Table 3

Moments at the Supports in Double-Span Frames
(Ground Floor)

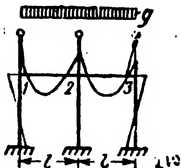
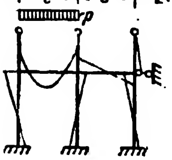
Loading and stress diagrams	$\frac{i_{lower}+1.5 i_{upper}}{i_{girder}}$	α-and β-moment coefficients			
		M_{12}	M_{21}	M_{23}	M_{32}
	0.5	0.029	0.111	—	—
	1	0.042	0.104	—	—
	2	0.056	0.098	—	—
	3	0.063	0.094	—	—
	5	0.069	0.091	—	—
	0.5	0.037	0.068	0.043	0.008
	1	0.051	0.071	0.033	0.009
	2	0.064	0.074	0.024	0.008
	3	0.070	0.076	0.018	0.007
	5	0.075	0.078	0.013	0.006

Table 4

Moments at the Supports in Triple-Span Frames
(Uppermost Storey)
$M = (\alpha g + \beta p) l^2$

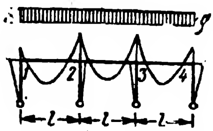
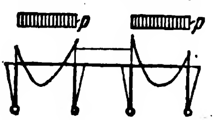
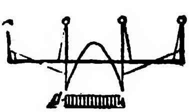
Loading and stress diagrams	$\frac{i_{lower}}{i_{girder}}$	α-and β-moment coefficients			
		M_{12}	M_{21}	M_{23}	M_{32}
	0.25	0.025	0.099	0.093	—
	0.5	0.038	0.098	0.089	—
	1	0.052	0.095	0.086	—
	1.5	0.059	0.093	0.085	—
	0.25	0.032	0.059	0.034	—
	0.5	0.047	0.064	0.026	—
	1	0.061	0.069	0.017	—
	1.5	0.067	0.072	0.013	—
	0.25	0.007	0.040	0.059	—
	0.5	0.009	0.034	0.064	—
	1	0.009	0.026	0.069	—
	1.5	0.008	0.021	0.072	—

Table 4 (Continued)

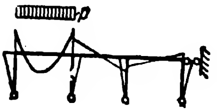
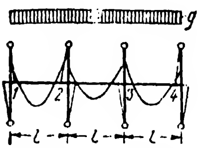
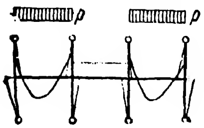
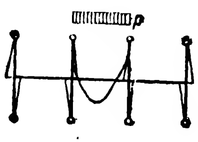
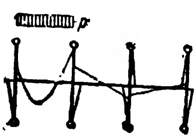
Loading and stress diagrams	$\frac{i_{lower}}{i_{girder}}$	α- and β-moment coefficients			
		M_{12}	M_{21}	M_{23}	M_{32}
	0.25	0.030	0.069	0.049	0.015
	0.5	0.045	0.071	0.039	0.013
	1	0.059	0.073	0.028	0.011
	1.5	0.066	0.075	0.022	0.009

Table 5

Moments at the Supports in Triple-Span Frames

(Intermediate Storey)

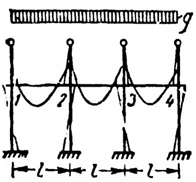
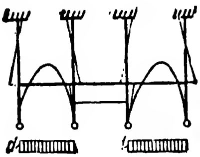
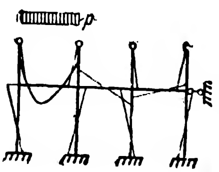
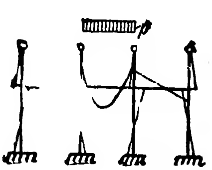
$$M = (\alpha g + \beta p)\, l^2$$

Loading and stress diagrams	$\frac{i_{lower} + i_{upper}}{i_{girder}}$	α- and β-moment coefficients			
		M_{12}	M_{21}	M_{23}	M_{32}
	0.5	0.033	0.098	0.089	—
	1	0.052	0.095	0.086	—
	2	0.063	0.092	0.085	—
	3	0.068	0.089	0.084	—
	0.5	0.047	0.064	0.026	—
	1	0.060	0.069	0.017	—
	2	0.070	0.074	0.011	—
	3	0.074	0.076	0.008	—
	0.5	0.009	0.034	0.064	—
	1	0.008	0.026	0.069	—
	2	0.007	0.018	0.074	—
	3	0.006	0.013	0.076	—
	0.5	0.045	0.070	0.039	0.013
	1	0.059	0.073	0.028	0.011
	2	0.069	0.075	0.019	0.008
	3	0.074	0.077	0.014	0.006

MOMENTS AT THE SUPPORTS IN TRIPLE-SPAN FRAMES

(Ground Floor)

$$M = (\alpha g + \beta p)\, l^2$$

Loading and stress diagrams	$\frac{i_{\text{lower}} + 1.5 i_{\text{upper}}}{i_{\text{girder}}}$	α- and β-moment coefficients			
		M_{12}	M_{21}	M_{23}	M_{32}
	0.5	0.030	0.099	0.092	—
	1	0.044	0.097	0.088	—
	2	0.057	0.094	0.086	—
	3	0.063	0.092	0.085	—
	5	0.069	0.090	0.084	—
	0.5	0.038	0.061	0.031	—
	1	0.053	0.066	0.022	—
	2	0.065	0.071	0.014	—
	3	0.070	0.074	0.011	—
	5	0.076	0.077	0.007	—
	0.5	0.008	0.038	0.061	—
	1	0.009	0.031	0.066	—
	2	0.008	0.023	0.071	—
	3	0.007	0.018	0.074	—
	5	0.007	0.013	0.077	—
	0.5	0.036	0.070	0.045	0.014
	1	0.051	0.072	0.034	0.012
	2	0.064	0.074	0.024	0.010
	3	0.070	0.076	0.018	0.007
	5	0.075	0.078	0.013	0.006